203 TEST 2 FALL '20

(1) $f(x,y)=\sqrt{x+1}+\sqrt{25-x^2}-\sqrt{y^2-1}$

$\mathcal{D}(f)$: $x+1\geq 0$, $x\geq -1$

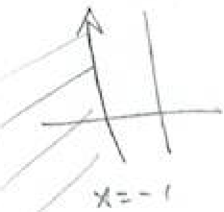
$25-x^2\geq 0$

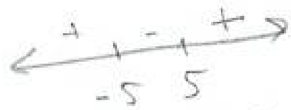
$x^2-25\leq 0$

$(x-5)(x+5)\leq 0$

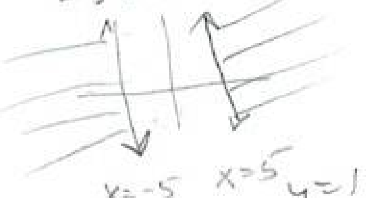
$y^2-1\geq 0$

$(y-1)(y+1)\geq 0$

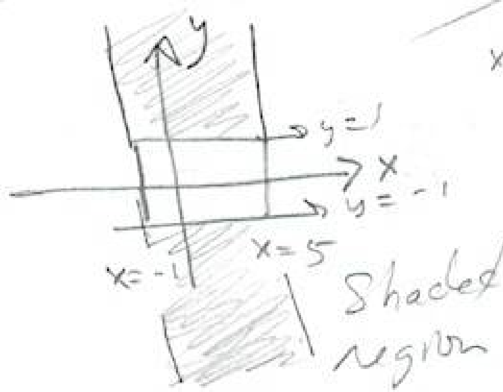


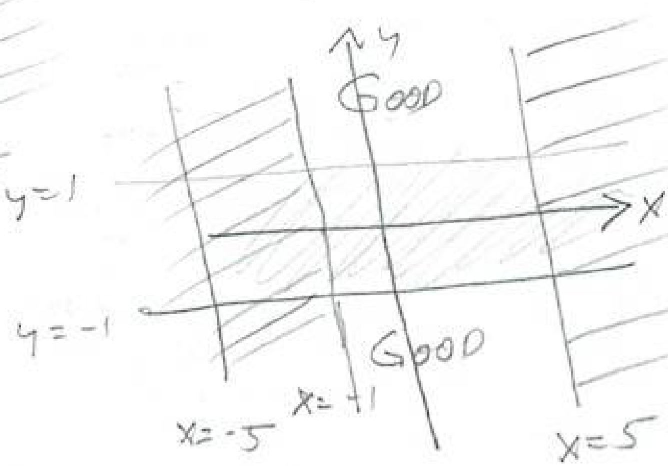


$-5\leq x\leq 5$



STUDENT STYLE





(2) (a) $f(x,y)=(2x^2-3x^2y+5y^4)^3$

$$\boxed{\begin{aligned} f_x &= 3(2x^2-3x^2y+5y^4)^2(4x-6xy)\\ f_y &= 3(2x^2-3x^2y+5y^4)^2(-3x^2+20y^3)\end{aligned}}$$

10 pts

(b) $f(x,y)=\displaystyle\int_0^{\sin(x)-5x}\frac{y^2\sinh(\tau)\cos(\tau)}{\tau^2+\pi}\,d\tau$

$$\boxed{\begin{aligned} f_x &= \frac{y^2\sinh(\sin(x)-5x)\cos(\sin(x)-5x)}{(\sin(x)-5x)^2+\pi}\\ f_y &= 2y\,f(x,y)\end{aligned}}$$

Not on TEST 2 & Given in class.

203 TEST 2

3 $\cancel{y\tan(x^2y)+}\,2x^2yz^2 = 2x^2y^3z$

(a) 5pts $\frac{\partial z}{\partial x}$:

$$4xyz^2 + 2x^2y(2zz_x) = 4xy^3z + 2x^2y^3z_x$$

$$\Rightarrow (4x^2yz - 2x^2y^3)z_x = 4xy^3z - 4xyz^2$$

$$\Rightarrow z_x = \frac{\partial z}{\partial x} = \frac{4xy^3z - 4xyz^2}{4x^2yz - 2x^2y^3} = \boxed{\frac{2y^2z - 2z^2}{2xz - xy^2} = z_x}$$

(b) 5pts $F(x,y,z) = 2x^2yz^2 - 2x^2y^3z$

$$F_x = 4xyz^2 - 4xy^3z \Rightarrow$$

$$F_z = 4x^2yz - 2x^2y^3$$

$$z_x = -\frac{F_x}{F_z} = -\frac{4xyz^2 - 4xy^3z}{4x^2yz - 2x^2y^3} = \boxed{\frac{2y^2z - 2z^2}{2xz - xy^2} = z_x}$$

4 $f(x,y) = \frac{2}{\pi}\cos\left(\frac{\pi}{6}x\right) + \frac{1}{\pi}\sin\left(\frac{\pi}{3}xy\right) - \frac{3\sqrt{3}}{2\pi} + \frac{7}{6}$

$$f_x = \frac{\pi}{6}\cdot\frac{2}{\pi}\left(-\sin\left(\frac{\pi}{6}x\right)\right) + \frac{\pi}{3}\cdot\frac{1}{\pi}y\cos\left(\frac{\pi}{3}xy\right)$$

$$\Rightarrow f_x(1,2) = -\frac{2}{6}\sin\left(\frac{\pi}{6}\right) + \frac{1}{3}(2)\cos\left(\frac{\pi}{3}(1)(2)\right)$$

$$= -\frac{1}{3}\left(\frac{1}{2}\right) + \frac{2}{3}\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{6} + \frac{2}{3}\left(-\frac{1}{2}\right)$$

$$= \frac{-1-2}{6} = -\frac{1}{2} = \boxed{-\frac{1}{2} = f_x(1,2)}$$

203 TEST 2 FALL '20 KEY

4 $f(x,y) = 2x^2y^3 - \frac{4}{\pi}\sin(\pi xy)$ a $(1,2,16)$

2 $f_x = 4xy^3 - \frac{4}{\pi}\cdot\pi y\cos(\pi xy) \longrightarrow$

5pts $= 4xy^3 - 4y\cos(\pi xy)$

$f_x(1,2) = 32 - 4(2)\cos(\pi(1)(2)) =$

$= 32 - 8(1) = \boxed{24 = f_x(1,2)}$

$f_y = 6x^2y^2 - \frac{4}{\pi}\cdot\pi x\cos(\pi xy) \longrightarrow$

$f_y(1,2) = 6(1)(2)^2 - 4(1)(1) = 24 - 4 = \boxed{20 = f_y(1,2)}$

$f(1,2) = 2(1)(2)^3 - \frac{4}{\pi}\sin(\pi(1)(2)) = \boxed{16 = f(1,2)}$

$\boxed{L(x,y) = f_x(1,2)(x-1) + f_y(1,2)(y-2) + f(1,2)}$

$= \boxed{24(x-1) + 20(y-2)} + \boxed{16}$

$= 24x - 24 + 20y - 40 + 16$

$= 24x + 20y - 48$

b 5pts $L(1.2, 1.9) = 24(.2) + 20(-.1) + 16$

$= 4.8 - 2 + 16 = 2.8 + 16 = \boxed{18.8 = L(1.2, 1.9)}$

c 5pts $f(1.2, 1.9) = 2(1.2)^2(1.9)^3 - \frac{4}{\pi}\sin(\pi(1.2)(1.9))$

$\approx 18.77287207 \approx f(1.2, 1.9)$

d $\Delta z = f(1.2, 1.9) - f(1,2) \approx \boxed{2.77287207 \approx \Delta z}$

203 T2

~~#~~4 cont'd

e dz = L(1.2, 1.9) − f(1,2)

5 pts = 18.8 − 16 = $\boxed{2.8 = dz}$ ≈ Δz

dz = $f_x(1,2)\Delta x + f_y \Delta y = f_x\,dx + f_y\,dy$

$= 24(.2) + 20(-.1) = 4.8 - 2 = 2.8$ ✓

f 5 pts $\nabla f(1,2,16) = \langle f_x(1,2), f_y(1,2)\rangle$

$= \boxed{\langle 24,20\rangle = \nabla f(1,2)}$

g 5 pts $\bar{u} = \langle 3,-4\rangle \longrightarrow \|\bar{u}\| = \sqrt{9+16} = 5$

$\longrightarrow D_{\bar{u}} f(1,2) = (\nabla f)\cdot \dfrac{\bar{u}}{\|\bar{u}\|} = \dfrac{1}{\|\bar{u}\|}(\nabla f)\cdot \bar{u}$

$= \dfrac{1}{5}\langle 24,20\rangle \cdot \langle 3,-4\rangle = \dfrac{4}{5}\langle 6,5\rangle \cdot \langle 3,-4\rangle$

$= \dfrac{4}{5}(18-20) = \dfrac{4}{5}(-2) = \boxed{-\dfrac{8}{5} = D_u f(1,2)}$

:)

203 T 2

5 Distance from plane P : 3x-4y+12z=24

to p(13,-6,39)

a 10 pts $d = \sqrt{(x-13)^2+(y+6)^2+(z-39)^2}$

Define $f(x,y,z) = (x-13)^2+(y+6)^2+(z-39)^2$

Using P to solve for z, we have

$12z = 24-3x+4y \longrightarrow$

$z = 2-\frac{1}{4}x+\frac{1}{3}y \longrightarrow z-39 = -\frac{1}{4}x+\frac{1}{3}y-37$

f(x,y,z) is actually

$f(x,y) = (x-13)^2+(y+6)^2+(-\frac{1}{4}x+\frac{1}{3}y-37)^2$

$\longrightarrow f_x = 2(x-13)+2(-\frac{1}{4}x+\frac{1}{3}y-37)(-\frac{1}{4})$

$=2x-26-\frac{1}{2}(-\frac{1}{4}x+\frac{1}{3}y-37)$

$=2x-26+\frac{1}{8}x-\frac{1}{6}y+\frac{37}{2} \overset{SET}{=} 0 \longrightarrow$

$\frac{17}{8}x-\frac{1}{6}y = \frac{-37+52}{2} = \frac{15}{2}$ LCD=24

$\boxed{51x-4y=180}$

203 T2

~~#~~5a cnt'd

Likewise for f_y!

$$f_y = 2(y+6) + 2\left(-\tfrac{1}{4}x + \tfrac{1}{3}y - 37\right)\left(\tfrac{1}{3}\right)$$

$$= 2y + 12 + \tfrac{2}{3}\left(-\tfrac{1}{4}x + \tfrac{1}{3}y - 37\right)$$

$$= 2y + 12 - \tfrac{1}{6}x + \tfrac{2}{9}y - \tfrac{74}{3} \overset{SET}{=} 0 \Longrightarrow$$

$$= -\tfrac{1}{6}x + \tfrac{20}{9}y = \tfrac{74}{3} - \tfrac{36}{3} = \tfrac{38}{3} \qquad LCD = 18 \Longrightarrow$$

$$\boxed{-3x + 40y = 228} \quad \text{So}$$

$$51x - 4y = 180$$

$$(-3x + 40y = 228)(17)$$

$$-51x + 680y = 3876$$

$$51x - 4y = 180$$

$$676y = 4056$$

$$\boxed{y = 6}$$

Now $51x - 4y = 51x - 24 = 180 \Longrightarrow$

$$51x = 204 \Longrightarrow \boxed{x = 4}$$

$$z = 2 - \tfrac{1}{4}(4) + \tfrac{1}{3}(6) = 2 - 1 + 2 = \boxed{3 = z}$$

203 T2

#5a cnt'd

$(x,y,z) = (4,6,3) \Longrightarrow$

$d = \sqrt{(4-13)^2 + (6+6)^2 + (3-39)^2}$

$= \sqrt{9^2 + 12^2 + 36^2} = \sqrt{(3^2)^2 + 3^2 \cdot 4^2 + 3^2 \cdot 12^2}$

$= 3\sqrt{3^2+4^2+12^2} = 3\sqrt{169} = 3(13) = \boxed{39 = d}$

Dylan used

203 T2

#5b 10 pts $\mathcal{P}: 3x-4y+12z=24$

$P(13,-6,39)$

Let $Q(0,0,2) \in \mathcal{P}$. Then

$\overrightarrow{QP} = \langle 13,-6,37\rangle = \bar{u}$.

$$\text{comp}_{\bar{n}}\,\bar{u} = \frac{\bar{n}\cdot\bar{u}}{\|\bar{n}\|}$$

$$= \frac{\langle 3,-4,12\rangle\cdot\langle 13,-6,37\rangle}{\sqrt{3^2+4^2+12^2}}$$

$$= \frac{|(3)(13)-4(-6)+12(37)|}{\sqrt{169}} = \frac{507}{13} = 39$$

$$\boxed{d = 39}$$

BOOK FORMULA:

$3x-4y+12z-24=0$

$ax+by+cz+d=0$

$$\frac{|3(13)-4(-6)+12(39)-24|}{\|\bar{n}\|^2}$$

$$= \frac{|\bar{n}\cdot\bar{x}+d|}{\|\bar{n}\|}, \text{ where } \bar{x} = \langle 13,-6,39\rangle$$

$$= 39$$

203 T2

5c 5pB $f(x,y,z) =$

$d = \sqrt{(x-13)^2 + (y+6)^2 + (z-39)^2}$

Let $f(x,y) = (x-4)^2 + (y+6)^2 + (z-39)^2$

to be minimized, s.t.

$g(x,y,z) = 3x - 4y + 12z = 24$

$f_x = 2(x-13) = 2x - 26 = 3\lambda$

$f_y = 2(y+6) = 2y + 12 = -4\lambda$

$f_z = 2(z-39) = 2z - 78 = 12\lambda$

So $\lambda = \frac{2x-26}{3} = \cancel{\frac{2y+12}{-4}} = \frac{-y-6}{2} = \frac{z-39}{6}$

gives symmetric eq'ns for a line.

Find that line's intersection with P.

P. Parametric Eq'ns:

$x = \frac{3t+26}{2}$, $y = \frac{2t+6}{-1} = -2t-6$,

$z = 6t+39$ Sub:

P: $3\left(\frac{3t+26}{2}\right) - 4\left(\frac{2t+6}{-1}\right) + 12(6t+39) = 24$

$\frac{9}{2}t + 39 + 8t + 24 + 72t + 468 = 24$

$84.5t = -507 \Rightarrow \boxed{t = -6}$

203 T2

5c cont'd

$t = -6 \longrightarrow$

$$x = \frac{3(-6)+26}{2} = \frac{-18+26}{2} = \frac{8}{2} = 4$$

$$y = \frac{2(-6)+6}{-1} = \frac{-6}{-1} = 6$$

$$z = 6(-6) + 39 = -36 + 39 = 3$$

$$\boxed{(x, y, z) = (4, 6, 3)}$$

$$d(4,6,3) = \sqrt{(4-13)^2 + (6+6)^2 + (3-39)^2}$$

$$= \sqrt{9^2 + 12^2 + 36^2}$$

$$= \sqrt{(3^2)^2 + 3^2 \cdot 4^2 + 3^2 \cdot 12^2}$$

$$= \sqrt{3^2 \cdot 3^2 + 3^2 \cdot 4^2 + 3^2 \cdot 12^2}$$

$$= 3\sqrt{3^2 + 4^2 + 12^2} = 3\sqrt{169} = 3 \cdot 13 = 39$$

$\boxed{39 \text{ is min distance}}$

203 T2

B1 $f(x,y) = \int_0^{\sin(x)-5x} \frac{y^2 \sinh(\tau)\cos(\tau)}{\tau^2+11}\, d\tau$

$$\boxed{f_x = \left(\frac{y^2 \sinh(\sin(x)-5x)\cos(\sin(x)-5x)}{(\sin(x)-5x)^2+11}\right)(\cos(x)-5)}$$

$$\boxed{f_y = 2y\int_0^{\sin(x)-5x} \frac{\sinh(\tau)\cos(\tau)}{\tau^2+11}\, d\tau}$$

y^2 may pass in & out of $\int_0^{\sin(x)} y^2 f(\tau)\, d\tau$

$y^2 f(\tau)\, d\tau$

B2

a

$x+2y-4z=7$

$2x+3y+2z=11$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 7 \\ 2 & 3 & 2 & 11 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & 7 \\ 0 & -1 & 10 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 16 & 1 \\ 0 & 1 & -10 & +3 \end{array}\right]$$

$x = -16z+1$

$y = 10z+3$

$z =$ any

Check: $z=1$, $x=-15$, $y=13$

$-15+2(13)-4$

$=-15+26-4=7$ ✓

$2(-15)+3(13)+2=$

$=-30+39+2=11$ ✓

$$\boxed{x=-16t+1,\ y=10t+3,\ z=t}$$

vector: $\boxed{\bar{r} = \langle 1, 3, 0\rangle + t\langle -16, 10, 1\rangle}$

203 T2

P32 (b) Dylan's way?

$\bar{n}_1 \times \bar{n}_2 = \langle 1, 2, -4 \rangle \times \langle 2, 3, 2 \rangle$

$$\begin{array}{r} \langle 1, 2, -4 \rangle, 1, 2 \\ \times \ \langle 2, 3, 2 \rangle, 2, 3 \\ \hline \langle 16, -10, -1 \rangle \end{array}$$

Now all we need is a point on the line. Since None of the planes are parallel to xy-plane, let $z = 0$.

// to xy-plane $\Rightarrow$ $z = k$ is its equation! That's $x + 2y = 7$ not

Then $x + 2y = 7$
$2x + 3y = 11$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 2 & 3 & 11 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & -1 & -3 \end{array}\right] \Rightarrow y = 3$$

$x + 2(3) = x + 6 = 7$

$\Rightarrow x = 1$

So $\bar{r}_0 = \langle 1, 3, 0 \rangle$

& $\bar{u} = \langle 16, -10, -1 \rangle$ gives

$\bar{r} = \bar{r}_0 + t\bar{u} = \langle 1, 3, 0 \rangle + \langle 16, -10, -1 \rangle t$

$= \langle 16t + 1, -10t + 3, -t \rangle$ ~~OR~~

& $\boxed{x = 16t + 1, \ y = -10t + 3, \ z = -t}$

203 J2

B3 $A(2,-3,7)$, $B(-3,2,1)$

Line segment from A to B is given by

$t\langle -3,2,1\rangle + (1-t)\langle 2,-3,7\rangle$, $0 \le t \le 1$

w/o the restriction on t, you get the entire line!

$\langle -3t+2-2t, 2t-3+3t, t+7-7t\rangle = \langle -5t+2, 5t-3, -6t+7\rangle$

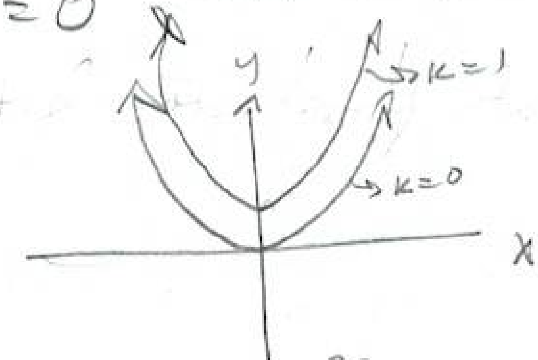
B4 $9x^2+4z^2-25y=0$

$z=k$:

$9x^2-25y=-4k^2$

$-25y=-9x^2-4k^2$

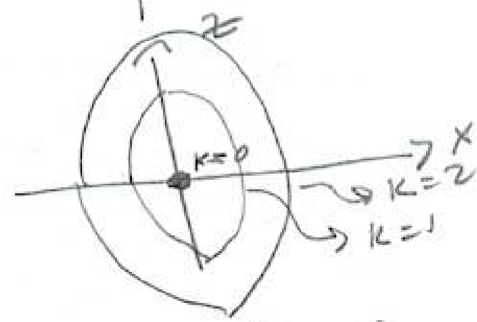
$y=\frac{9}{25}x^2+4k^2$



$y=k$ $\quad 9x^2+4z^2=25k$

$\frac{9}{25k}x^2+\frac{4}{25k}z^2=1$

$k=1 \quad \frac{x^2}{\frac{25}{9}}+\frac{z^2}{\frac{25}{4}}=1$



$x=k$: $-25y=-9x^2-4z^2$

$y=\frac{4}{25}z^2+\frac{9}{25}k^2$