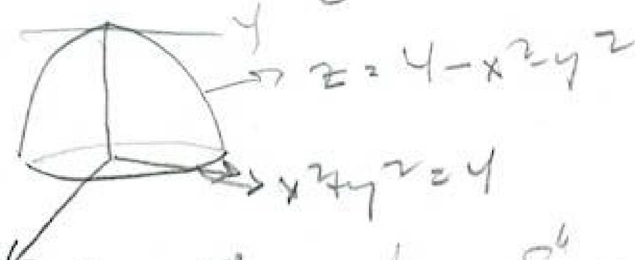
203 §16.9 #s 2, 5, 8, 9, 10

(2) Verify that the Divergence Theorem is true for the vector field $\bar{F}$ & solid E.

$\ni$ $\bar{F} = \langle x^2, xy, z \rangle$, E = solid bdd by $z = 4 - x^2 - y^2$ & the xy-plane.

$$\iint_S \bar{F} \cdot d\bar{S} = \iiint_E \operatorname{div} \bar{F} \, dV$$



$S = S_1 \cup S_2$, where S_1 = Paraboloid = $\{(x,y,z) \mid z = 4 - x^2 - y^2,\ x^2 + y^2 \le 4\}$,

S_2 = disk of radius $r = 4$. = $\{(x,y,z) \mid x^2 + y^2 \le 4,\ z = 0\}$

$\iint_{S_1} \bar{F} \cdot d\bar{S}$: $z = 4 - x^2 - y^2$

$\bar{r} = \langle x, y, 4 - x^2 - y^2 \rangle$

$\bar{r}_x = \langle 1, 0, -2x \rangle, 1, 0$

$\bar{r}_y = \langle 0, 1, -2y \rangle, 0, 1$

$\langle 2x, 2y, 1 \rangle = \bar{r}_x \times \bar{r}_y$

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#2 cont'd

$$\iint_{S_1} \bar{F} \cdot d\bar{S} = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \langle x^2, xy, z\rangle \cdot \langle 2x, 2y, 1\rangle \, dy\, dx$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x^3 + 2x^2y + z)\, dy\, dx$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x^3 + 2x^2y + 4 - x^2 - y^2)\, dy\, dx$$

$= \int_{-2}^{2}$ ugh!

$x = 2\cos\theta$
$y = 2\sin\theta$
$z = 4 - (4\cos^2\theta + 4\sin^2\theta)$
$= 0$?! No.

$$= \int_{-2}^{2} \left[2x^3y + x^2y^2 + 4y - x^2y - \tfrac{1}{3}y^3\right]_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$$

$$= \int_{-2}^{2} \left[4x^3\sqrt{4-x^2} + 8\sqrt{4-x^2} - 2x\sqrt{4-x^2}\right] dx$$

$I_1 + I_2 + I_3$

$=$

I_1 : $dv = -2x(4-x^2)^{\frac{1}{2}}$ $\quad u = -2x^2$
$v = \frac{2}{3}(4-x^2)^{\frac{3}{2}}$ $\quad du = -4x^2\, dx$

wait. I_1 & I_3 have odd funcs as integrands

So it reduces to $\int_{-2}^{2} 8\sqrt{4-x^2}\, dx$

$2\sin\theta = 2 \quad \sin\theta = 1 \quad \theta = \frac{\pi}{2}$
$2\sin\theta = -2, \quad \sin\theta = -1 \quad \theta = -\frac{\pi}{2}$

$=$ $\quad x = 2\sin\theta \Rightarrow$
$dx = 2\cos\theta\, d\theta$

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#2 cont'd

$$8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos\theta \cdot 2\cos\theta \, d\theta$$

$$= 8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\cos^2\theta \, d\theta = 16\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+\cos 2\theta)\, d\theta$$

$$= 16\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 16\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \boxed{16\pi}$$

#s 5–15 Use Divergence Thm to calculate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$, i.e., find the flux of $\vec{F}$ across $\vec{S}$

(5) $\vec{F} = \langle e^x \sin y, e^x \cos y, yz^2 \rangle$. S is the surface of the box bdd by the planes $x=0, x=1, y=0, y=1, z=0, z=2$.

$$\operatorname{div}\vec{F} = e^x \sin y - e^x \sin y + 2yz$$

$$\int_0^1\int_0^1\int_0^2 2yz \, dz\, dy\, dx = \int_0^1\int_0^1 yz^2\Big]_0^2 dy\, dx$$

$$= \int_0^1\int_0^1 4y\, dy\, dx = \int_0^1 2y^2\Big]_0^1 dx = \int_0^1 2\, dx$$

$$= 2x\Big]_0^1 = \boxed{2}$$

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#5 cont'd

Try $\iint_{S} \vec{F} \cdot d\vec{S}$

$\sum_{k=1}^{6} \iint_{S_k} \vec{F} \cdot d\vec{S}$

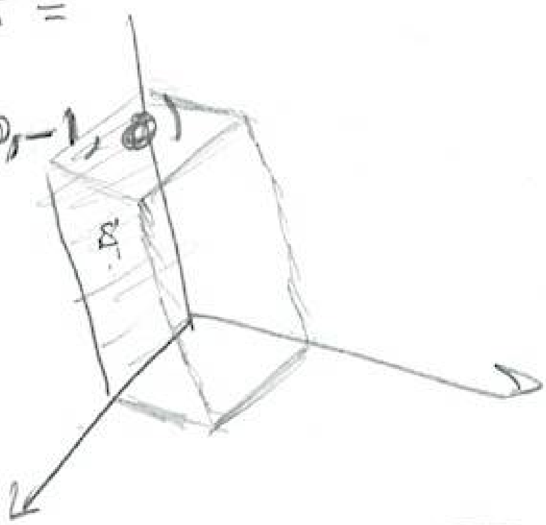
$x=0$: $y = 0..1$, $z = 0..2$

$\vec{r}_1 = \langle 0, y, z \rangle$

$\vec{r}_{1y} = \langle 0, 1, 0 \rangle$

$\vec{r}_{1z} = \langle 1$

$\int$ $y=0$: nah. $\vec{n} =$



$\iint \langle e^x \sin y, e^x \cos y, yz^2 \rangle \cdot \langle 0, -1, 0 \rangle$

$= \int_0^1 \int_0^2 -e^x \cos y \, dz\,dx$

$= \int_0^1 \int_0^2 -e^x \cos(0) \, dz\,dx$

$= -\int_0^1 e^x \Big]_0^2 dx = -\int_0^1 (e^2 - 1)\,dx = \boxed{e^2 - 1}$ I_1

$y = 1$:

I_2: $\int_0^1 \int_0^2 e^x \cos y \, dz\,dx = \int_0^1 \int_0^2 e^x \cos(1) \, dz\,dx$

No way I'm doin' 6 of these out of curiosity!

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(8) $\bar{F} = \langle x^3y, -x^2y^2, -x^2yz \rangle$

S is surface bdd by hyperboloid

$x^2+y^2-z^2=1$ & the planes $z=\pm 2$.

~~$z^2 = 1-x^2-y^2$~~ $\quad z^2 = x^2+y^2-1$

~~$z = \pm\sqrt{1-x^2-y^2}$~~ $\quad z = \pm\sqrt{x^2+y^2-1}$

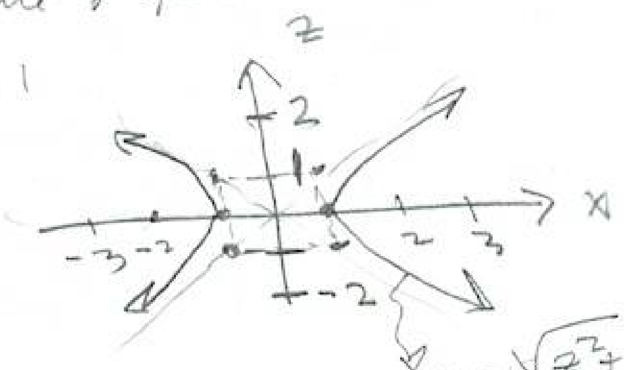
z = constant :
circle of radius $\sqrt{z^2+1}$

$r(z) =$
It's a solid of revolution.

xz-plane : $y=0$

$x^2-z^2=1$

$z=b=1$



$\operatorname{div}\bar{F} = 3x^2y - 2x^2y - x^2y = 0$!

LOL!

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(9) $\bar{F} = \langle xy\sin z, \cos(xz), y\cos z \rangle$

Using divergence Thm, start by finding div $\bar{F}$!

div $\bar{F} = y\sin z - y\sin z = 0$! LOL!

(10) $\bar{F} = \langle x^2 y, xy^2, 2xyz \rangle$

div $\bar{F} = 2xy + 2xy + 2xy = 6xy \neq 0$ *sigh*

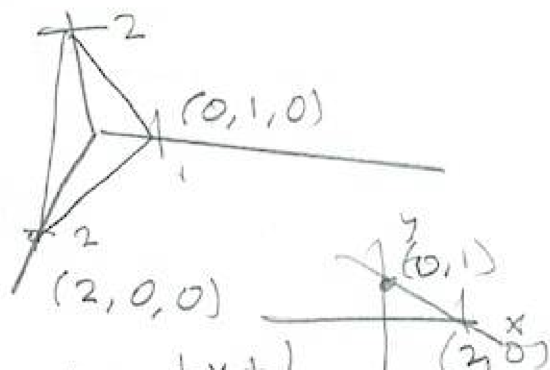
S is surface of tetrahedron bdd by

$x=0, y=0, z=0,$ & $x+2y+z=2$

$(2,0,0), (0,1,0), (0,0,2)$

$z = 2-x-2y$

$$\int_0^2 \int_0^{-\frac{1}{2}x+1} \int_0^{2-x-2y} 6xy \, dz\, dy\, dx$$



$$= \int_0^2 \int_0^{-\frac{1}{2}x+1} 6xyz \Big]_0^{2-x-2y} dy\, dx$$

$$= \int_0^2 \int_0^{-\frac{1}{2}x+1} \left[6xy(2-x-2y) \right. dy\, dx$$

$$= \int_0^2 \int_0^{-\frac{1}{2}x+1} \left[12xy - 6x^2y - 12xy^2 \right] dy\, dx$$

203 §16.9 #10

#10 cnt'd

$$= \int_0^2 \left[6xy^2 - 3x^2y^2 - 4xy^3 \right]_0^{-\frac{1}{2}x+1} dx$$

$$= \int_0^2 \left[6x\left(-\frac{x}{2}+1\right)^2 - 3x^2\left(-\frac{x}{2}+1\right)^2 - 4x\left(-\frac{1}{2}x+1\right)^3 \right] dx$$

$$= \int_0^2 \Big[6x\left(\frac{x^2}{4} - x + 1\right) - 3x^2\left(\frac{x^2}{4} - x + 1\right)$$

$$+ 4x\left(\left(\frac{x}{2}\right)^3 - 3\left(\frac{x}{2}\right)^2(1) + 3\left(\frac{x}{2}\right)(1)^2 - 1\right) \Big] dx$$

$$= \int_0^2 \Big[\frac{3}{2}x^3 - 6x^2 + 6x - \frac{3}{4}x^4 + 3x^3 - 3x^2$$

$\left(-\frac{1}{2}x+1\right)^3$ See next page

→ No. It's OK

$$+ \frac{x^4}{2} - 3x^3 + \frac{3x}{2} + 6x^2 - 4x \Big] dx$$

$$= \int_0^2 \left[\frac{3}{2}x^3 - 3x^2 + \frac{7}{2}x \right] dx = \left[\frac{3}{8}x^4 - x^3 + \frac{7}{4}x^2 \right]_0^2$$

$$= \frac{3}{8}[2]^4 - (2)^3 + \frac{7}{4}(2)^2 = \frac{3}{8}(16) - 8 + 7$$

$$= 6 - 8 + 7 = \boxed{5}$$

203 §16.7 #10 errata

$(-\frac{1}{2}x+1)^2 = (\frac{1}{2}x-1)^2$ is OK, but I also did $(-\frac{1}{2}x+1)^3 = (\frac{1}{2}x-1)^3$, so I'm off by a sign (at least) in the expansion of the cubic in the integrand

$$= \int_0^2 \Big[\frac{3}{2}x^3 - 6x^2 + 6x - \frac{3}{4}x^4 + 3x^3 - 3x^2$$

$$- x$$

Wait. No. It looks like I handled the signs $-4x(-\frac{1}{2}x+1)^3 = 4x(-1)^3(-\frac{1}{2}x+1)^3$

$$= 4x(-1(-\frac{1}{2}x+1))^3 = 4x(\frac{1}{2}x-1)^3$$

= etc., so I think it's as OK as the work leading up to it is OK.