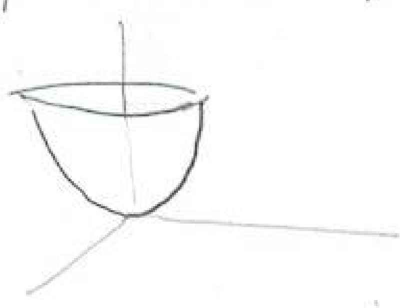
203 § 16.8 #s 1-4, 7, 13, 15-20

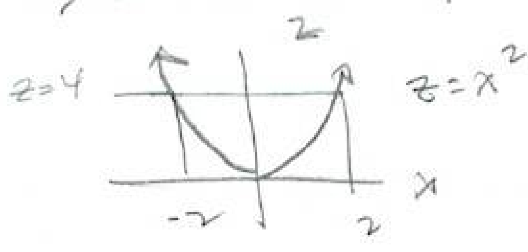
(1) $\iint_H \text{curl } \vec{F} \cdot d\vec{S} = \iint_P \text{curl } \vec{F} \cdot d\vec{S}$, because they both have same boundary, the circle C: $x^2+y^2=4$ in the xy-plane. By Stokes' Thm,

$$\iint_H \text{curl } \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r} = \iint_P \text{curl } \vec{F} \cdot d\vec{S}$$

#s 2-4 Use Stokes' to eval $\iint_S \text{curl } \vec{F} \cdot d\vec{S}$

(3) $F = \langle x^2z^2, y^2z^2, xyz \rangle$, S is the part of the paraboloid $z = x^2+y^2$ that lies inside the cylinder $x^2+y^2=4$, oriented upward.





Let $C = \{(x,y,z) \mid x^2+y^2=4, z=4\}$

Then $\iint_S \text{curl } \vec{F} \cdot d\vec{S} = \oint_C \vec{F} \cdot d\vec{r}$

$\vec{r} = \langle 2\cos\theta, 2\sin\theta, 4 \rangle$

$\|\vec{r}\,'\| = \|\langle -2\sin\theta, 2\cos\theta, 0 \rangle\| = 2$ Don't need.

$$\int_0^{2\pi} \vec{F}(\vec{r}(\theta)) \cdot \vec{r}\,'(\theta)\, d\theta$$

203 S 16.8 #s 2, 3, 4, 7, 13, 15-20

#3 cnt'd

$x^2z^2 = 4\cos^2\theta \cdot 4^2 = 32\cos^2\theta$

$y^2z^2 = 32\sin^2\theta$

$xyz = 64\sin\theta\cos\theta$

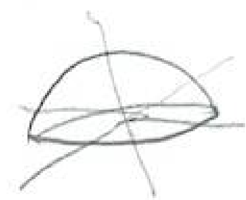
$\bar{F} \cdot \bar{r}' = \langle 32\cos^2\theta, 32\sin^2\theta, 64\sin\theta\cos\theta \rangle \cdot$
$\langle -2\sin\theta, 2\cos\theta, 0 \rangle$

$\Rightarrow = \int_0^{2\pi} (-64\cos^2\theta\sin\theta + 64\sin^2\theta\cos\theta)\, d\theta$

$= \left[\frac{64}{3}\cos^3\theta + \frac{64}{3}\sin^3\theta\right]_0^{2\pi} = \frac{64}{3} - \frac{64}{3} = \boxed{0}$

(2) $\bar{F} = \langle 2y\cos z, e^x \sin z, xe^y \rangle$

S = hemisphere $x^2+y^2+z^2 = 9$, $z \geq 0$ oriented upward.



$\iint_S \text{curl}\,\bar{F} \cdot d\bar{S} = \int_C \bar{F} \cdot d\bar{r}$, where

$\bar{r} = \langle 3\cos\theta, 3\sin\theta, 0 \rangle$

$\bar{r}' = \langle -3\sin\theta, 3\cos\theta, 0 \rangle$

$\int_C \bar{F} \cdot d\bar{r} = \int_0^{2\pi} \langle 2(3\sin\theta)\cos(0), (e^x)(0), 3\cos\theta e^{3\sin\theta} \rangle \cdot \langle -3\sin\theta, 3\cos\theta, 0 \rangle\, d\theta$

$= \int_0^{2\pi} -18\sin^2\theta\, d\theta = -\frac{18}{2}\int_0^{2\pi} (1-\cos 2\theta)\, d\theta$

203 § 16.8 #s 2, 4, 7, 13, 15-20

#2 cont'd

$$= -9\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{2\pi} = \boxed{+18\pi}$$

(4) $\bar{F} = \langle x^2y^3z, \sin(xyz), xyz\rangle$

S is part of cone $y^2 = x^2+z^2$ that lies between planes $y=0$ & $y=3$, oriented in the direction of positive y-axis

$C = \{(x,y,z) \mid x = 3\cos\theta, y=3, z=3\sin\theta\}$

$\bar{r} = \langle 3\cos\theta, 3, 3\sin\theta\rangle$

$\bar{r}' = \langle -3\sin\theta, 0, 3\cos\theta\rangle$

$\bar{F} = \langle (9\cos^2\theta)(27)(3\cos\theta), \sin((3\cos\theta)(3)(3\sin\theta)), 27\sin\theta\cos\theta\rangle$

$= 27\langle 27\cos^3\theta, \sin^2\theta\cos\theta, \sin\theta\cos\theta\rangle$

$\bar{F}\cdot\bar{r}' = 27(81\cos^4\theta + 0 + 3\sin^2\theta\cos\theta$

Scratch: $\left(\frac{\cos 2\theta + 1}{2}\right)^2 = \frac{1}{4}(\cos^2 2\theta + 2\cos 2\theta + 1)$

$= \frac{1}{8}\cos 4\theta + \frac{1}{8} + \frac{1}{2}\cos 2\theta + 1$

$= \frac{1}{8}\cos 4\theta + \frac{9}{8} + \frac{1}{2}\cos 2\theta$

203 $\quad$ S 16.8 #s 4, 7, 13, 15–20

#4 cnt'd

$$27\int_0^{2\pi}\left(81\left(\frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{9}{8}\right) + 3\sin^2\theta\cos\theta\right)d\theta$$

$$= 27\cdot 81\left[\frac{1}{8}\cdot\frac{1}{4}\sin 4\theta + \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta + \frac{9}{8}\theta + \frac{3}{3}\sin^3\theta\right]_0^{2\pi}$$

$$= \frac{27\cdot 9}{8}\cdot 2\pi = \boxed{\frac{243}{4}\pi}$$

(7) $\bar{F} = \langle y^2, x, z^2\rangle$

S = part of $z = x^2+y^2$ that's below $z=1$ oriented upward.

$x^2+y^2=1,\ z=1 \quad \bar{r} = \langle\cos\theta, \sin\theta, 1\rangle$

$$\boxed{\bar{r}' = \langle -\sin\theta, \cos\theta, 0\rangle}$$

$$\boxed{\bar{F}(\bar{r}(\theta)) = \langle \sin^2\theta, \cos\theta, 1\rangle}$$

$$\int_0^{2\pi}\bar{F}\cdot d\bar{r} = \int_0^{2\pi}\left(-\sin^3\theta + \cos^2\theta\right)d\theta$$

$$= \int_0^{2\pi}\left(-\sin\theta + \cos^2\theta\sin\theta + \frac{1}{2}(1+\cos 2\theta)\right)d\theta$$

$$= \cos\theta\Big]_0^{2\pi} - \frac{\cos^3\theta}{3}\Big]_0^{2\pi} + \frac{1}{2}\theta\Big]_0^{2\pi} + \frac{1}{4}\sin 2\theta\Big]_0^{2\pi}$$

$$= 0 - 0 + \pi + 0 = \boxed{\pi}$$

203 §16.8 #s 13, 15-20

#13 cont'd

Messy integral. Will check Maple.

$\int_C$ Heck. I didn't even do curl $\bar{F}$!

$$\begin{array}{l} \quad x \quad\; y \quad\; z \quad\;\; x \quad y \\ \bar{F} = \langle y^2, x, z^2 \rangle, y^2, x \end{array}$$

$$\operatorname{curl} \bar{F} = \langle 0-0, 0-0, 1-2y \rangle$$

$$\bullet \; \langle 2-2x, -2y, 1 \rangle$$

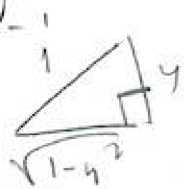
$$\operatorname{curl} \bar{F} \bullet \bar{r}' = 1-2y$$

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (1-2y)\,dx\,dy = \int_{-1}^{1} \Big[x - 2xy \Big]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}$$

$$= \int_{-1}^{1} \left(\left(\sqrt{1-y^2} - 2y\sqrt{1-y^2} \right) - \left(-\sqrt{1-y^2} - 2\left(-\sqrt{1-y^2}\right) y \right) \right) dy$$

$$= \int_{-1}^{1} \left(2\sqrt{1-y^2} + 4y(1-y^2)^{\frac{1}{2}} \right) dy = 0 + \frac{8}{3}(1-y^2)^{\frac{3}{2}} \Big]_{-1}^{1}$$

$$= 0 + 0 = \boxed{0} \,!$$



$y = \sin\theta$

$dy = \cos\theta \, d\theta$

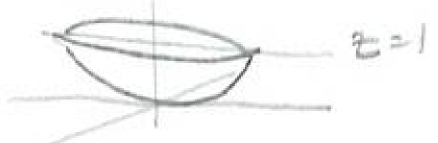
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\sin\theta\cos\theta\,d\theta$$

$$= \sin^2\theta \Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1^2 - (-1)^2 = 0$$

203 §16.8 #s 7, 13, 15-20

#s 13-15 Verify Stokes' Thm for each of the following.

13) $\bar{F} = \langle y^2, x, z^2 \rangle$, S is part of $z = x^2+y^2$ that lies below $z=1$ oriented upward.



$x^2+y^2=1$

$\bar{r} = \langle x, y, x^2+y^2 \rangle$

$\bar{r}_x = \langle 1, 0, 2x \rangle$, 1, 0

$\bar{r}_y = \langle 0, 1, 2y \rangle$, 0, 1

$\langle -2x, -2y, 1 \rangle = \bar{r}_x \times \bar{r}_y$

$\bar{F} \cdot (\bar{r}_x \times \bar{r}_y) = \langle y^2, x, (x^2+y^2)^2 \rangle \cdot \langle -2x, -2y, 1 \rangle$

$= -2xy^2 - 2xy + (x^2+y^2)^2$

$\int u\,dv = uv - \int v\,du$

$dv = dx$

$v = x$

$\int (x^2+y^2)^2 dx$ $\quad u = (x^2+y^2)^2$

$du = 2(x^2+y^2)(2x)dx$

$uv - \int v\,du = x(x^2+y^2)^2 - \int 4x^2(x^2+y^2)\,dx$

$= x(x^2+y^2)^2 - \int (4x^4 + 4x^2y^2)\,dx$

$= x(x^2+y^2)^2 - \frac{4}{5}x^5 - \frac{4}{3}x^3y^2$

$= x(x^2+y^2)^2 - \frac{4}{5}x^5 - \frac{4}{3}x^3 \Big]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}$

$\int_{-1}^{1} \Big[-x^2y^2 - x^2y + x(x^2+y^2) - \frac{4}{5}x^5 - \frac{4}{3}x^3 \Big]$

$= \int_{-1}^{1} \Big(-(1-y^2)y^2 - (1-y^2)y + \sqrt{1-y^2}\,(1-y^2+y^2) - \frac{4}{5}(1-y^2)^{\frac{5}{2}} - \frac{4}{3}(1-y^2)^{\frac{3}{2}} \Big)\,dy$

203 §16.8 #s 13, 15-20

#13 cont'd

$\bar{r} = \langle \cos\theta, \sin\theta, 1 \rangle$

$\bar{r}' = \langle -\cos\theta, \sin\theta, 0 \rangle$

$\bar{F} \cdot \bar{r}' = \langle \sin^2\theta, \cos\theta, 1 \rangle \cdot \langle -\cos\theta, \sin\theta, 0 \rangle$

$$\int_C \bar{F} \cdot d\bar{r} = \int_0^{2\pi} (-\sin^2\theta \cos\theta + \sin\theta \cos\theta)\, d\theta$$

$$= \left[-\frac{\sin^3\theta}{3} + \frac{\sin^2\theta}{2} \right]_0^{2\pi} = \boxed{0}$$

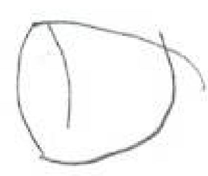
15 $\bar{F} = \langle y, z, x \rangle$ S is $x^2+y^2+z^2=1$, $y \geq 0$, oriented in direction of pos. y-axis

$\langle x, y, z \rangle$ x, y

x $\langle y$, z, x, y, z

$y = \sqrt{1-x^2-z^2}$

$\langle -1, -1, -1 \rangle = \text{curl } \bar{F}$



$\bar{r} = \langle x, \sqrt{1-x^2-z^2}, y \rangle$

Sphericals look better?

$0 \leq \phi \leq \pi$, $0 \leq \theta \leq \pi$, $\rho = 1$

$\bar{r} = \langle \sin\phi \cos\theta, \sin\phi \sin\theta, \cos\phi \rangle$ $\cos\phi\cos\theta, \cos\phi\sin\theta$

$\bar{r}_\phi = \langle \cos\phi\cos\theta, \cos\phi\sin\theta, -\sin\phi \rangle$ $-\sin\phi\sin\theta, \sin\phi\cos\theta$

$\bar{r}_\theta = \langle -\sin\phi\sin\theta, \sin\phi\cos\theta, 0 \rangle$

$\langle \sin^2\phi\cos\theta, \sin^2\phi\sin\theta, \sin\phi\cos\phi\cos^2\theta + \sin\phi\cos\phi\sin^2\theta \rangle$

$= \langle \sin^2\phi\cos\theta, \sin^2\phi\sin\theta, \sin\phi\cos\phi \rangle$

203 §16.8 #s 15-20

#15 cont'd

Meh. not sure I like sphericals, either

Let's look @ $\bar r_x \times \bar r_y$ in rectangular coords:

$$\bar r = \langle x, \sqrt{1-x^2-z^2}, z\rangle$$

$$\bar r_x = \langle 1, -x(1-x^2-z^2)^{-\frac12}, 0\rangle, \quad 1, \quad -x(1-x^2-z^2)^{-\frac12}$$

$$\times\ \bar r_z = \langle 0, -z(1-x^2-z^2)^{-\frac12}, 1\rangle, \quad 0, \quad -z(1-x^2-z^2)^{-\frac12}$$

$$\langle -x(1-x^2-z^2)^{-\frac12}, -1, -z(1-x^2-z^2)^{-\frac12}\rangle$$

$$(\operatorname{curl}\bar F)\cdot(\bar r_x \times \bar r_z) =$$

$$x(1-x^2-z^2)^{-\frac12} + 1 + z(1-x^2-z^2)^{-\frac12}$$

$$\iint_S \operatorname{curl}\bar F\cdot d\bar S = \iint_S x(1-x^2-z^2)^{-\frac12}\,dA + \iint_S x(1-x^2-z^2)^{-\frac12}\,dA$$

$$= \int_{-1}^{1}\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} -\tfrac12(1-x^2-z^2)^{-\frac12}(-2x\,dx)\,dz + \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -\tfrac12(1-x^2-z^2)^{-\frac12}(-2z)\,dz\,dx$$

$$= -\int_{-1}^{1}\Big[2(1-x^2-z^2)^{\frac12}\Big]_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}}dz = \int_{-1}^{1}\Big[2(1-1+z^2-z^2)^{\frac12} - 2(1-1+z^2-z^2)^{\frac12}\Big]dz$$

$$= \boxed{0}\,!$$

203 §16.8 #s 15-20

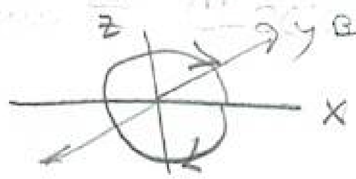
#15 cont'd We eval $\int_C \bar{F}\cdot d\bar{r}$ where

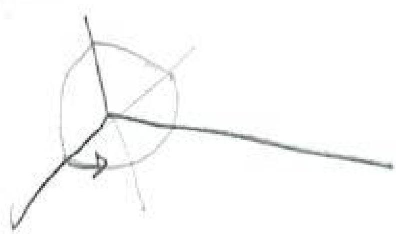
$C = x^2+z^2=1,\ y=0$

$\bar{r} = \langle \cos\theta, 0, \sin\theta \rangle$

$\bar{r}' = \langle -\sin\theta, 0, \cos\theta \rangle$

$0 \le \theta \le \pi$, where θ is measured clockwise in the xz-plane





$\bar{F} = \langle x,y,z \rangle = \langle \cos\theta, 0, \sin\theta \rangle$

$$\int_C \bar{F}\cdot d\bar{r} = \int_0^{2\pi} (-\sin\theta\cos\theta + \sin\theta\cos\theta)\, d\theta$$

$$= \boxed{0} \quad !$$

$\partial,\ \partial^2\ \exists$

simple, closed, smooth

(16) C scsc in $x+y+z=1$. Show that

$\int_C z\,dx - 2x\,dy + 3y\,dz$ depends only on area enclosed by C & not on the shape of or its location in the plane.

203 §16.8 ~~#5~~ 16 20

~~#~~16 cnt'd

$z(t)\,x'(t)\,dt - 2x(t)\,y'(t)\,dt + 3y\,z'(t)\,dt$

$= \langle z, -2x, 3y \rangle \cdot \langle x'(t), y'(t), z'(t) \rangle\, dt$

$= \bar{F} \cdot d\bar{r}$

$\bar{F} = \langle z, -2x, 3y \rangle$

curl $\bar{F}$:

$$\begin{array}{cccccc} \langle & x, & y, & z \rangle & x, & y \\ \times\ \langle & z, & -2x, & 3y \rangle & z, & -2x \\ \hline \langle & 3, & 1, & -2 \rangle & & \end{array} = \text{curl } \bar{F}$$

so $\iint_{S'} \text{curl } \bar{F} \cdot d\bar{S}' =$

$= \iint_{S'} \langle 3, 1, -2 \rangle \cdot \langle 1, 1, 1 \rangle\, dA$

$= \iint_{S'} (3 + 1 - 2)\, dA$

$= 2 \iint_{S'} dA = 2$ Area of S'

(depends only on the area of S')

$x + y + z = 1$

$z = 1 - x - y$

$\bar{r} = \langle x, y, 1 - x - y \rangle$

$$\begin{array}{l} \bar{r}_x = \langle 1, 0, -1 \rangle, 1, 0 \\ \times\ \bar{r}_y = \langle 0, 1, -1 \rangle, 0, 1 \\ \hline \langle 1, 1, 1 \rangle \end{array}$$

Right here, you know it comes down to the area, since $\bar{F}$ & $\bar{r}_x \times \bar{r}_y$ are constant and can be factored out of the double integral.

203 §16.8 #s17-20

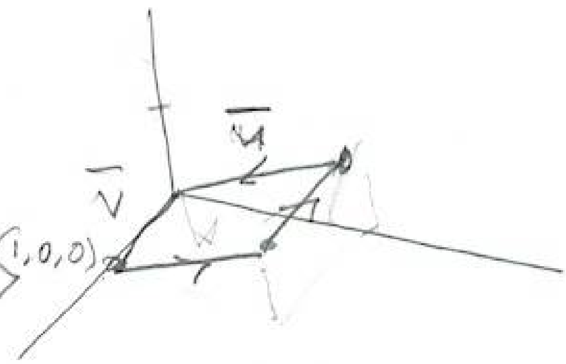
(17) A particle moves from (the origin (0,0,0) to) (1,0,0) to (1,2,1) to (0,2,1) back to (0,0,0).

under a force field $\bar{F} = \langle z^2, 2xy, 4y^2 \rangle$

Find the work done

$\bar{v} = \langle 1, 0, 0 \rangle$

$\bar{u} = \langle 0, 2, 1 \rangle$



$\bar{r} = s\bar{u} + t\bar{v}$

$= \langle 0, 2s, s \rangle + \langle t, 0, 0 \rangle$

$= \langle t, 2s, s \rangle$

$$\begin{array}{rl} \bar{r}_s = \langle 0, 2, 1 \rangle & 0, 2 \\ \times \quad \bar{r}_t = \langle 1, 0, 0 \rangle & 1, 0 \\ \hline \langle 0, 1, -2 \rangle = \bar{n}_1 & \end{array}$$

is Downward normal. We want upward normal, in keeping w/ counter-clockwise traverse of the closed curve C.

$= C_1 \cup C_2 \cup C_3 \cup C_4$, where C_i is i^{th} line segment.

203 §16.8 #s 17-20

#17 cnt'd

---

we want $\bar{n} = -\bar{n}_1 = \langle 0,-1,2 \rangle$

Now, if $(x,y,z) \in$ Plane P, then

$\bar{x} = \langle x-0, y-0, z-0 \rangle = \langle x,y,z \rangle$ is $\in P$.

& $\bar{n} \cdot \bar{x} = -y + 2z = 0 \rightarrow y = 2z$

or $z = \frac{1}{2}y$ is more natural.

$\langle x, y, \frac{1}{2}y \rangle$ Going in circles

Get back to $\bar{F} = \langle z^2, 2xy, 4y^2 \rangle$

By Stokes, we want

$$\iint_S \text{curl } \bar{F} \cdot d\bar{S}$$

$$\begin{array}{rl} & \langle z^2, 2xy, 4y^2 \rangle, z^2, 2xy \\ \hline \nabla \times \bar{F} & \langle 8y, 2z, 2y \rangle, \cancel{8y, 2z} \\ \bar{r}_s \times \bar{r}_t & \langle 0, -1, 2 \rangle \cancel{, 0, -1} \\ \hline \end{array}$$

$-2z + 4y = -2(s) + 4(2s)$

$= -2s + 8s = 6s$

$$\int_0^1\int_0^1 6s \, ds \, dt = \int_0^1 [3s^2]_0^1 dt$$

$= [s^3]_0^1 = \boxed{1}$

$x = 0..1$

$t = 0..1$

$y = 0..2$

$2s = 0..2$

$s = 0..1$

203 § 16.8 #s 18-20

(18) Eval $\int_C (y+\sin x)dx + (z^2+\cos y)dy + x^3 dz$

when C is given by $\bar{r} = \langle \sin t, \cos t, \sin 2t \rangle$

$0 \le t \le 2\pi$ (observe C lies on $z = 2xy$.)

Since this is Stokes time, look @

curl!

$\bar{F} = \langle y + \sin x, z^2 + \cos y, x^3 \rangle$

$\langle y+\sin x, z^2+\cos y, x^3 \rangle$ $y+\sin x$, $z^2+\cos y$

$\langle -2z, -3x^2, -1 \rangle$ is nice & simple!

Now. $\bar{r} = \langle \sin t, \cos t, \sin 2t \rangle$

$= \langle \sin t, \cos t, 2\sin t \cos t \rangle$

curl $\bar{F} \cdot d\bar{S}$

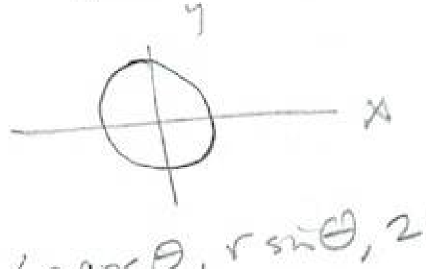
S is $\bar{r} = \langle x, y, 2xy \rangle$

$x = r\cos\theta$, $y = r\sin\theta$

$z = 2r^2 \sin\theta\cos\theta$

$r^2 \sin 2\theta$

$2r^2 \cos 2\theta$



$\bar{r} = \langle r\cos\theta, r\sin\theta, 2r^2\sin\theta\cos\theta \rangle$ $\cos\theta, \sin\theta$

$\bar{r}_r = \langle \cos\theta, \sin\theta, 4r\sin\theta\cos\theta \rangle$, $-r\sin\theta, r\cos\theta$

$\bar{r}_\theta = \langle -r\sin\theta, r\cos\theta, 2r^2\cos(2\theta) \rangle$

$2r^2\cos^2\theta\sin\theta$,

$2r^2\sin\theta\cos 2\theta - 4r^2\cos^2\theta\sin\theta$

$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$

$\bar{r}_r \times \bar{r}_\theta$:

$\langle 2r^2\sin\theta\cos 2\theta - 4r^2\cos^2\theta\sin\theta, -4r^2\sin^2\theta\cos\theta - 2r^2\cos 2\theta\cos\theta,$

$r\cos^2\theta + r\sin^2\theta \rangle$

$= \langle 2r^2\sin\theta(\cos^2\theta - \sin^2\theta) - 4r^2\cos^2\theta\sin\theta,$

$-4r^2\sin^2\theta\cos\theta - 2r^2(\cos^2\theta - \sin^2\theta)\cos\theta, \ r \rangle$

$= \langle -2r^2\cos^2\theta\sin\theta - 2r^2\sin^3\theta, -2r^2\sin^2\theta + 2r^2\cos^3\theta, r \rangle$

~~Now,~~

$$\int_0^{2\pi}\int_0^1 \operatorname{curl}\bar{F} \cdot (\bar{r}_r \times \bar{r}_\theta)\, dr\, d\theta$$

$= \langle -2r^2\sin\theta(\cos^2\theta + \sin^2\theta), -2r^2$

$-2r^2\sin^2\theta\cos\theta - 2r^2\cos^3\theta, r \rangle = \bar{r}_r \times \bar{r}_\theta$

$= \langle -2r^2\sin\theta, -2r^2\sin^2\theta\cos\theta - 2r^2\cos^3\theta$

?!

$$= \int_0^{2\pi}\int_0^1 \langle -2z, -3x^2, 1 \rangle \cdot$$

203 §16.8 #s 18-20

<u>#18 cont'd</u> That led to a real mess!

$\text{curl}\,\bar{F} = \langle -2z, -3x^2, 1\rangle$

$\bar{r} = \langle x, y, 2xy\rangle$

$-1 \le x \le 1,\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \langle -2z, -3x^2, 1\rangle \cdot (\bar{r}_x \times \bar{r}_y)\, dy\, dx$$

$\bar{r}_x = \langle 1, 0, 2y\rangle,\ 1, 0$

$\times\ \bar{r}_y = \langle 0, 1, 2x\rangle,\ 0, 1$

$\langle -2y, -2x, 1\rangle$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-2y + 2y)\, dy\, dx = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 0\, dy\, dx \boxed{=0}!$$

(19) If S is a sphere & $\bar{F}$ satisfies the hypotheses of Stokes' Thm, show that $\iint_S \text{curl}\,\bar{F} \cdot d\bar{S} = 0$.

Let S_1 = Top hemisphere, C_1 its boundary.

" S_2 = Bottom " , C_2 " "

Then $\iint_S \text{cu} = \iint_{S_1} \text{cu} + \iint_{S_2} \text{cu} = \oint_{C_1} \text{cu} + \int_{C_2} \text{cu} = \int_{C_1} - \int_{C_1}$

$= 0$, b/c

~~C_1 & C_2 have opp~~

203 §16.8 #s 19, 20

#19 cont'd.

C_1 & C_{12} will have opposite directions

but they're the same closed curve.

$\therefore \oint_{C_1} \bar{F} \cdot d\bar{r} = -\int_{C_{12}} \bar{F} \cdot d\bar{r}$ so

$$\iint_{S'} \text{curl}\,\bar{F} \cdot d\bar{S} = \iint_{S_1'} \text{curl}\,\bar{F} \cdot d\bar{S} + \iint_{S_2'} \text{curl}\,\bar{F} \cdot d\bar{S}$$

$$= \oint_{C_1'} \bar{F} \cdot d\bar{r} + \oint_{C_{12}'} \bar{F} \cdot d\bar{r} = \oint_{C_1'} \bar{F} \cdot d\bar{r} - \oint_{C_1} \bar{F} \cdot d\bar{r}$$

$$= 0 \quad \blacksquare$$

20 $\mathcal{S}$ S & C satisfy Stokes' hypo,

and f, g have cnts 2nd-order partials.

use 16.5 #s 24, 26 to show

(a) $\int_C (f \nabla g) \cdot d\bar{r} = \iint_{S} (\nabla f \times \nabla g) \cdot d\bar{S}$

(b) $\int_C (f \nabla f) \cdot d\bar{r} = 0$

(c) $\int_{C} (f \nabla g + g \nabla f) \cdot d\bar{r} = 0$

203 §16.8 #20

#20 cont'd

(a) $\int_{C'} (f \nabla g) \cdot d\bar{r} = \iint_{S'} (\nabla f \times \nabla g) \cdot d\bar{S}$

$\int_{C'} (f \nabla g) \cdot d\bar{r}$

$= \iint_{S} \text{curl}(f \nabla g) \cdot d\bar{S}$

by #26:

$\text{curl}(f \nabla g) = f \,\text{curl} \nabla g + \nabla f \times \nabla g$

$= 0 + (\nabla f) \times (\nabla g)$ ■

(b) $\int_{C'} (f \nabla f) \cdot d\bar{r} = \iint_{S} \text{curl}(f \nabla f) \cdot d\bar{S}$

$= f \,\text{curl} \nabla f + \nabla f \times \nabla f$

$= \bar{0} + \bar{0} = \bar{0}$ by below:

$$\begin{array}{r} \langle f_x, f_y, f_z \rangle \; f_x, f_y \\ \times \; \langle f_x, f_y, f_z \rangle \; f_x, f_y \\ \hline \langle f_y f_z - f_y f_z, f_x f_z - f_x f_z, f_x f_y - f_x f_y \rangle = \bar{0} \end{array}$$

■

203 §16.8 #20

#20 cont'd

Just check curl $(\nabla f) = \bar{0}$

$$\begin{array}{cccccc} & x & y & z & x & y \\ x & \langle f_x, & f_y, & f_z \rangle & f_x, & f_y \end{array}$$

$\langle f_{zy} - f_{yz}, f_{xz} - f_{zx}, f_{yx} - f_{xy} \rangle = \bar{0}$ by

Clairaut.

(c) $\int_{C} (f\nabla g + g\nabla f) \cdot d\bar{r} = 0$

curl $(f\nabla g + g\nabla f) =$

curl $(f\nabla g)$ + curl $(g\nabla f)$

$= f$ curl $(\nabla g) + \nabla f \times \nabla g$

$+ g$ curl $(\nabla f) + \nabla g \times \nabla f$

$= \quad 0 + \nabla f \times \nabla g + 0 - \nabla f \times \nabla g = \bar{0}$

$(A \times B = -B \times A)$

So $\iint_S$ curl $(f\nabla g + g\nabla f) \cdot d\bar{S} = 0$

Could've just used part (b) of #24 §16.5.