203 § 16.6 #s 1, 3, 5, 13, 14 − 18, 23, 29, 33, 39, 47,

① Determine if P & Q lie on

$\bar{r} = \langle 2u+3v,\ 1+5u-v,\ 2+u+v \rangle$

$P(7, 10, 4)$, $Q(5, 22, 5)$

$2u+3v = 7$

$5u - v = 10 - 1 = 9$

$u + v = 4 - 2 = 2$

$u = 2 - v$

$8(2-v) - v = 9$

$16 - 8v - v = 9$

$-9v = -7$

$v = \frac{7}{9} \Rightarrow$

$u = 2 - \frac{7}{9} = \frac{11}{9} = u$

$2u+3v = 2\left(\frac{11}{9}\right) + 3\left(\frac{7}{9}\right)$

$= \frac{22+21}{9} = \frac{43}{9} \neq 7$

$\boxed{\text{No. P not on } \bar{r}}$

$2u+3v = 5$

$1 + 5u - v = 22$

$2 + u + v = 5$

$2u+3v = 5$

$5u - v = 21$

$u + v = 3$

$u = 3 - v$

$5(3-v) - v = 21$

$15 - 5v - v = 21$

$-6v = 6$

$\boxed{v = -1}$ $\quad u = 3 - v = 3 + 1 = \boxed{4 = u}$

Check

$2(4) + 3(-1) = 8 - 3 = 5$ ✓

Checks

$\boxed{\text{Yes Q's on } \bar{r}}$

203 S' 16.6 #s 3, 5, 13-18, 23, 29, 33

#s 3-6 Identify the surface

(3) $\bar{r}(u,v) = \langle u+v, 3-v, 1+4u+5v \rangle$

$= \langle 0,3,1\rangle + \langle 1,0,4\rangle u + \langle 1,-1,5\rangle v$

$$\begin{array}{r} \langle 1,0,4\rangle, 1, 0 \\ \times \ \langle 1,-1,5\rangle, 1, -1 \\ \hline \end{array}$$

$\bar{n} = \langle 4,-1,-1 \rangle$

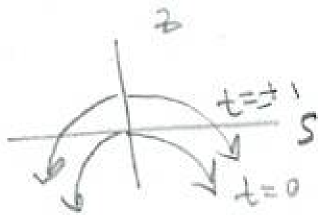
Let $\bar{x} = \langle x, y-3, z-1 \rangle \Rightarrow$

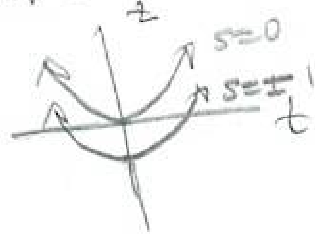
$\bar{n} \cdot \bar{x} = 0 \Rightarrow$

$4x - (y-3) - (z-1) = 0 \Rightarrow$

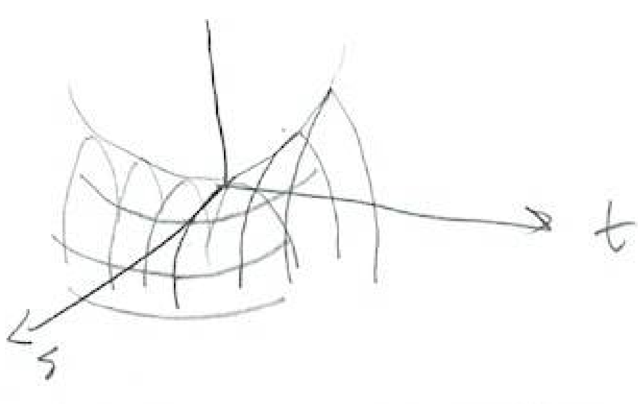
$4x - y - z = -3 - 1 = -4$

(5) $\bar{r}(s,t) = \langle s, t, t^2 - s^2 \rangle$




Hyperbolic paraboloid?



203 §16.6 #s 13-18, 23, 29, 33

(13) $\bar{r} = \langle u\cos v,\ u\sin v,\ v\rangle$ $\boxed{IV}$

(14) $\bar{r} = \langle u\cos v,\ u\sin v,\ \sin u\rangle$

(15) $\bar{r} = \langle \sin v,\ \cos u \sin(2v),\ \sin u \sin(2v)\rangle$ $\boxed{I}$

(16)
$x = (1-u)(3+\cos v)\cos 4\pi u$

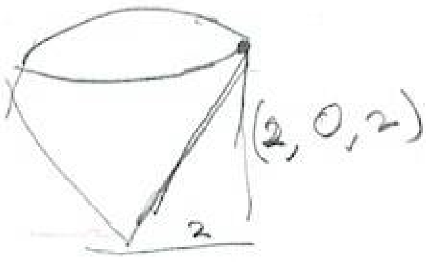
$y = (1-u)(3+\cos v)\sin 4\pi u$

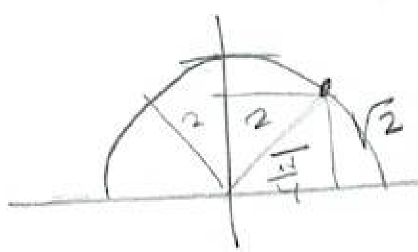
$z = 3u + (1-u)\sin v$

(17) $x = \cos^3 u \cos^3 v,\ y = \sin^3 u \cos^3 v,\ z = \sin^3 v$ $\boxed{III}$

(18) $x = (1-|u|)\cos v,\ y = (1-|u|)\sin v,\ z = u.$

(23) The part of the sphere $x^2+y^2+z^2=4$ that lies above $z=\sqrt{x^2+y^2}$

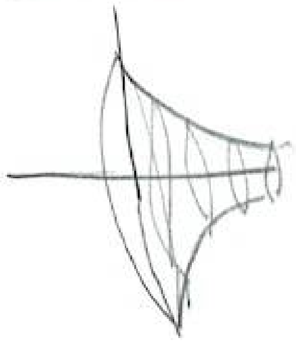

$(\sqrt{2}, 0, \sqrt{2})$

$x = 2\sin\phi\cos\theta,\ y = 2\sin\phi\sin\theta,$

$0 \le \phi \le \frac{\pi}{4},\ 0 \le \theta \le 2\pi$

203 §16.6 #s 29, 33

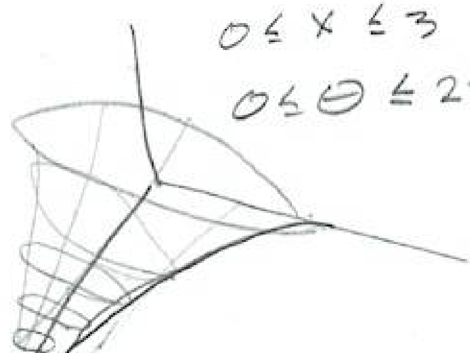
(29) Find parametric eq'ns for the surface obtained by rotating the curve $y = e^{-x}$, $0 \le x \le 3$ about the x-axis & use them to graph the surface.?



$\bar{r} = \langle x, \cos\theta\, e^{-x}, \sin\theta\, e^{-x} \rangle$

$0 \le x \le 3$

$0 \le \theta \le 2\pi$



#s 33-36

~~33~~ Find eq'n of tan plane to given surface @ given pt. If you have CAS, use CAS to graph surface & tan plane.

(33) $x = u+v$, $y = 3u^2$, $z = u-v$, $(2,3,0)$

$\bar{r}_u = \langle 1, 6u, u \rangle$, 1, 6u

$\times\ \bar{r}_v = \langle 1, 0, -1 \rangle$, 1, 0

$\langle -6u, u+1, -6u \rangle$

$u - v = 0$

$u = v$

$u + v = 2u = 2 \Rightarrow u = 1$

$\Rightarrow v = 1 \Rightarrow$

$u = v = 1 \Rightarrow$

$(\bar{r}_u \times \bar{r}_v)(1,1) = \langle -6, 2, -6 \rangle = \bar{n}$

$-6(x-2) + 2(y-3) - 6(z-0) = 0$

$-6x + 12 + 2y - 6 - 6z = 0 \Rightarrow -6x + 2y - 6z = -6$

$\boxed{3x - y + 3z = 3}$

203 §16.6 #s 39, 47

#s 39-47 Find surface area.

39 The surface $z = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}})$, $0 \le x \le 1$, $0 \le y \le 1$

$\bar{r} = \langle x, y, \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}}) \rangle$

$$A(S) = \iint_S \|\bar{r}_u \times \bar{r}_v\| \, dA$$

$$\Rightarrow \quad \bar{r}_x = \langle 1, 0, x^{\frac{1}{2}} \rangle \quad 1, 0$$

$$\times \quad \bar{r}_y = \langle 0, 1, y^{\frac{1}{2}} \rangle \quad 0, 1$$

$$\langle -\sqrt{x}, -\sqrt{y}, 1 \rangle$$

$$\Rightarrow \|\bar{r}_x \times \bar{r}_y\| = \sqrt{x+y}$$

$$\Rightarrow A(S) = \int_0^1 \int_0^1 (x+y)^{\frac{1}{2}} \, dy \, dx$$

$$= \int_0^1 \frac{2}{3}(x+y)^{\frac{3}{2}} \Big]_0^1 dx = \int_0^1 \left[\left(\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}\right)\right] dx$$

$$= \frac{2}{3} \cdot \frac{2}{5}(x+1)^{\frac{5}{2}} \Big]_0^1 - \frac{2}{3} \cdot \frac{2}{5}(x)^{\frac{5}{2}} \Big]_0^1$$

$$= \frac{4}{15}(2^{\frac{5}{2}}) - \frac{4}{15} = \frac{4}{15}(4)(\sqrt{2}) - \frac{4}{15}$$

$$= \boxed{\frac{16\sqrt{2} - 4}{15}}$$

203 §16.6 # 47

47

$x = u^2, y = uv, z = \frac{1}{2}v^2, 0 \le u \le 1, 0 \le v \le 2$

$\vec{r}_u = \langle 2u, v, 0 \rangle, 2u, -v$

$\times\ \vec{r}_v = \langle 0, u, v \rangle, 0, u$

$\langle v^2, -2uv, 2u^2 \rangle$

$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{v^4 + 4u^2v^2 + 4u^2v^2}$

$= \sqrt{v^4 + 8u^2v^2}$

$= v\sqrt{v^2 + 8u^2} \longrightarrow$

$A(S) = \int_0^1 \int_0^2 v\sqrt{v^2 + 8u^2}\, dv\, du$

$= \frac{1}{2}\int_0^1 \int_0^2 (v^2 + 8u^2)^{\frac{1}{2}} (2v\, dv)\, du$

$= \frac{1}{2}\int_0^1 \frac{2}{3}(v^2 + 8u^2)^{\frac{3}{2}} \Big]_0^2 du =$

$= \frac{1}{2}\int_0^1 \left(\frac{2}{3}(8u^2 + 4)^{\frac{3}{2}} - (8u^2)^{\frac{3}{2}}\right) du$

$= \frac{1}{3}\int_0^1 \left((8u^2+4)^{\frac{1}{2}} - (8u^2)^{\frac{1}{2}}\right)\left((8u^2+4) + (8u^2)^{\frac{1}{2}}(8u^2+4)^{\frac{1}{2}} + (8u^2)^3\right)$

203 §16.6 #47

$x = u^2,\ y = uv,\ z = \frac{1}{2}v^2,\ 0 \le u \le 1,\ 0 \le v \le 2$

$\bar{r}_u = \langle 2u, v, 0 \rangle, 2u, v$

$\times\ \bar{r}_v = \langle 0, u, v \rangle, 0, u$

$\langle v^2, -2uv, 2u^2$

$\| \bar{r}_u \times \bar{r}_v \| = \sqrt{v^4 + 4u^2v^2 + 4u^4}$

$x^2 + 4xy + 4y^2 = (x + \cancel{2}y)^2 =$

$x^2 + 4xy + 4y^2$

$= x^2 + 4y\ x + 4y^2$

$= x^2 +$ $\qquad \| \bar{r}_u \times \bar{r}_v \| = \sqrt{(v^2 + 2u^2)^2}$

$= v^2 + 2u^2$

$\int_0^1 \int_0^2 [v^2 + 2u^2]\, dv\, du = \int_0^1 \left[\frac{1}{3}v^3 + 2vu^2\right]_0^2 du$

$= \int_0^1 (\frac{1}{3}2^3 + 2(2)u^2 - 0)\, du = \frac{8}{3}u + \frac{4}{3}u^3 \Big]_0^1$

$= \frac{8}{3} + \frac{4}{3} = \frac{12}{3} = \boxed{4}$