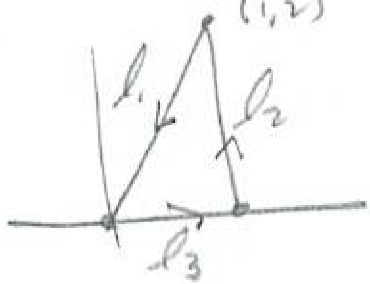
203 §16.4 #3 cont'd

(3) $\oint_C xy\,dx + x^2y^3\,dy \quad C: (0,0), (1,0), (1,2)$



$l_1: y = 2x,$

(1) $\int_1^0 x \cdot 2x\,dx + \left(\frac{1}{2}y\right)^2 y^3\,dy = \int_1^0 2x^2\,dx + \int_2^0 \frac{1}{4}y^5\,dy$

$= \frac{2}{3}x^3\Big]_1^0 + \frac{1}{24}y^6\Big]_2^0 = -\frac{2}{3} - \frac{2^6}{2^3 \cdot 3} = -\frac{2}{3} - \frac{8}{3} = -\frac{10}{3}$

(2) $\oint_C xy\,dx + x^2y^3\,dy = \int_0^2 1^2 \cdot y^3\,dy = \frac{y^4}{4}\Big]_0^2 = 4$

(3) $\oint_C xy\,dx + x^2y^3\,dy = \int_0^1 x \cdot 0\,dx + x^2 \cdot 0 \cdot 0 = 0$

$\Longrightarrow 4 - \frac{10}{3} = \frac{12-10}{3} = \frac{2}{3}$

$\int_0^1 \int_0^{2x} (2xy^3 - x)\,dA = \int_0^1 \left[\frac{2}{4}xy^4 - xy\right]_0^{2x} dx$

$= \int_0^1 \left[\frac{1}{2} \cdot 2^4 x^5 - 2x^2\right] dx = \int_0^1 (8x^5 - 2x^2)\,dx$

$= \left[\frac{8}{6}x^6 - \frac{2}{3}x^3\right]_0^1 = \frac{4}{3} - \frac{2}{3} = \boxed{\frac{2}{3}}$

203 §16.4 #s 3, 5, 7, 8, 12, 14, 27,

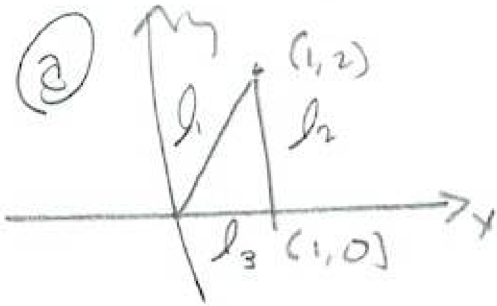
(3) Evaluate $\oint_C xy\,dx + x^2y^3\,dy$

(a) directly

(b) by Green's Thm,

where C is triangle w/ vertices $(0,0), (1,0), (1,2)$

(a)



l_1 $y = 2x$ $0 \le x \le 1$

l_2 $x = 1$ $0 \le y \le 2$

l_3 $y = 0$ $0 \le x \le 1$

(1) $\int_0^1 x \cdot 2x\,dx + \int_0^2 \left(\tfrac{1}{2}y\right)^2 (y^3)\,dy$

$= \int_0^1 2x^2\,dx + \int_0^2 \tfrac{1}{4}y^5\,dy = \tfrac{2}{3}x^3\Big]_0^1 + \tfrac{1}{24}y^6\Big]_0^2$

$= \tfrac{2}{3} + \tfrac{1}{24}\cdot 64 = \tfrac{2}{3} + \tfrac{1}{3}\cdot 8 = \tfrac{10}{3}$

(2) $\int_1^1 1y \cdot dx + \int_0^2 1^2 y^3\,dy = \frac{y^4}{4}\Big]_0^2 = \frac{16}{4} = 4$

(3) $\int_0^1 x \cdot 0\,dx + \int 0 = 0$

$\Rightarrow$ (1) + (2) + (3) $= \tfrac{10}{3} + 4\left(\tfrac{3}{3}\right) = \boxed{\tfrac{22}{3}}$

(b) $\int_0^1 \int_0^{2x} (2xy^3 - x)\,dy\,dx = \int_0^1 \left[\frac{2xy^4}{4} - xy\right]_0^{2x} dx$

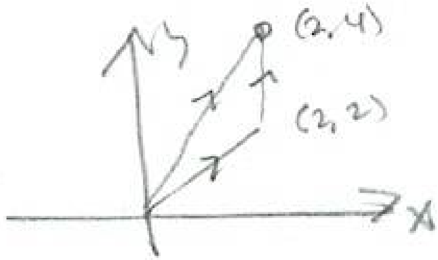
$= \int_0^1 \left[\tfrac{1}{2}x(2x)^4 - x(2x)\right] dx = \int_0^1 \left[8x^5 + 2x^2\right] dx = \left[\tfrac{8}{6}x^6 + \tfrac{2}{3}x^3\right]_0^1$

$=$

203 §16.4 #s 5, 7, 8, 12, 14, 27

⑤ #s 5-10 Use Green's to eval. the integral along the pos. oriented curve.

⑤ $\int_C xy^2\,dx + 2x^2y\,dy$ C is triangle w/ vertices (0,0), (2,2) & (2,4)



y = x from x=0 to x=2

x = 2 from y=0 to

Meh.

$$\iint_D (Q_x - P_y)\,dA = \int_0^2\int_x^{2x} (4xy - 2xy)\,dA$$

$$= \int_0^2\int_x^{2x} 2xy\,dy\,dx = \int_0^2 \Big[xy^2\Big]_x^{2x}dx = \int_0^2 \Big[x(2x)^2 - x(x)^2\Big]dx$$

$$= \int_0^2 (4x^3 - x^3)\,dx = \int_0^2 3x^3\,dx = \frac{3}{4}x^4\Big]_0^2 = \frac{3}{4}(2^4) = \boxed{12}$$

$\int_C (y + e^{\sqrt{x}})\,dx + (2x + \cos(y^2))\,dy$

C: Boundary of region enclosed by $y = x^2$ & $x = y^2$

$$\iint (Q_x - P_y)\,dA = \iint_D (2-1)\,dA$$

$$= \int_0^1\int_{x^2}^{\sqrt{x}} dy\,dx = \int_0^1 (\sqrt{x} - x^2)\,dx = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\Big]_0^1 = \boxed{\frac{1}{3}}$$

203 §16.4 #s 8, 12, 14, 27

(8)

$$\int_C xe^{-2x}\,dx + (x^4+2x^2y^2)\,dy$$

C = bdry of region between r=1 & r=2

$$\iint_D (Q_x - P_y)\,dA = \iint_D \left((4x^3+4xy^2) - 0\right)dA$$

$$= \int_1^2\int_0^{2\pi}\left(4r^3\cos^3\theta + 4r\cos\theta\, r^2\sin^2\theta\right) r\,d\theta\,dr$$

$$= \int_1^2\int_0^{2\pi}\left(4r^4(1-\sin^2\theta)\cos\theta + 4r^4\sin^2\theta\cos\theta\right)d\theta\,dr$$

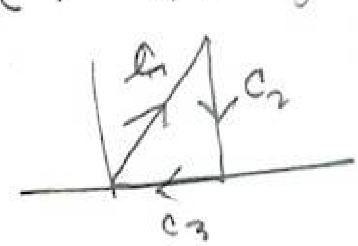
$$= 4\int_0^{2\pi}\left(\cos\theta - \sin^2\theta\cos\theta + \sin^2\theta\cos^2\theta\right)d\theta\int_1^2 r^4\,dr$$

$$= 4\sin\theta\Big]_0^{2\pi}\ \frac{r^5}{5}\Big]_1^2 = \boxed{0}$$

#s 11–14 use Green's Thm to eval $\int_C \vec{F}\cdot d\vec{r}$

(12) $\vec{F} = \langle y^2\cos x,\ x^2+2y\sin x\rangle$

C: triangle from (0,0) to (2,6) to (2,0) to (0,0)



C_1: $y=3x$ $\quad x=0$ to 2

C_2 $x=2$ $\quad y=6$ to 0

C_3 $y=0$ $\quad x=2$ to 0

203 §16.4 #5, 12, 14, 27

#12 cont'd

$\bar{F} =$

$\bar{r}_1 = \langle x, 3x \rangle$

$d\bar{r}_1 = \langle 1, 3 \rangle dx$

Nah. Just launch into Greens

$$\iint_D (Q_x - P_y)\, dA = \int_0^2 \int_0^{3x} (2x + 2y\cos x - 2y\cos x)\, dy\, dx$$

$$= \int_0^2 \int_0^{3x} 2x\, dy\, dx = \int_0^2 2xy \Big]_0^{3x} dx$$

$$= \int_0^2 2x(3x)\, dx = \int_0^2 6x^2 dx = 2x^3 \Big]_0^2 = \boxed{16}$$

(14) $\bar{F} = \langle y - \ln(x^2+y^2), 2\tan^{-1}(y/x) \rangle$

C is circle $(x-2)^2 + (y-3)^2 = 1$, oriented counter-clockwise.

$$\frac{\partial}{\partial x}\left[\arctan(yx^{-1})\right] = \left(\frac{1}{1+\left(\frac{y}{x}\right)^2}\right)\left(-\frac{y}{x^2}\right)$$

$$\iint_D (Q_x - P_y)\, dA = \iint_D -\frac{2y}{x^2} \cdot \frac{1}{1+\frac{y^2}{x^2}} - \left(1 - \frac{2y}{x^2+y^2}\right) dA \,\cancel{dy\,dx}$$

$$= \iint_D \frac{-2y}{x^2+y^2} - 1 + \frac{2y}{x^2+y^2}\, dA = \iint_D -1\, dA$$

203 §16.4 #s 14, 27

14 cnt'd

Now let's look @ C & D

$(x-2)^2+(y-3)^2=1$

$x-2=\cos\theta$ $\quad y-3=\sin\theta$

$x=2+\cos\theta$ $\quad y=\sin\theta+3$

$$\int_0^{2\pi}\int_0^1 -1\, r\,dr\,d\theta$$

$$=\int_0^{2\pi} -\frac{r^2}{2}\Big]_0^1 d\theta = -\int_0^{2\pi}\frac{1}{2}d\theta = \boxed{-\pi}$$

#13 from 8th Ed. $\quad (x-3)^2+(y+4)^2=4$ oriented clockwise.

$\langle y-\cos y,\ x\sin y\rangle$

$$\iint_D (Q_x-P_y)\,dA = \iint_D (\sin y-1-\sin y)\,dA$$

$x=2\cos\theta+3$

$y=2\sin\theta-4$

$$=\iint -1\, r\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^2 -1\, r\,dr\,d\theta = \int_0^{2\pi} -\frac{r^3}{2}\Big]_0^2 = \int_0^{2\pi} -2\,d\theta$$

$$=\boxed{-4\pi}$$

Orientation is clockwise, so our sign is off.

203 § 16.4 #27

#27 If $\bar{F}$ is the vector field of E5, show that $\int_C \bar{F} \cdot d\bar{r} = 0$ $\forall$ simple closed paths that do NOT contain the origin.

RECALL E5: $\bar{F} = \frac{1}{x^2+y^2} \langle -y, x \rangle$

$\bar{F}$ is cnt's & difbl away from the origin. $\Longrightarrow$

$\bar{F} = \langle P, Q \rangle$

$Q = x(x^2+y^2)^{-1} \Longrightarrow Q_x = (x^2+y^2)^{-1} - x(2x)(x^2+y^2)^{-2}$

$= \dfrac{-2x^2 + x^2+y^2}{(x^2+y^2)^2} = \dfrac{y^2-x^2}{(x^2+y^2)^2}$

$P = \dfrac{-y}{x^2+y^2} = -y(x^2+y^2)^{-1} \Longrightarrow$

$P_y = -1(x^2+y^2)^{-1} - y(2y)(-1)(x^2+y^2)^{-2}$

$= \dfrac{-x^2-y^2+2y^2}{(x^2+y^2)^2} = \dfrac{y^2-x^2}{(x^2+y^2)^2} = Q_x \Longrightarrow$

$Q_x - P_y = 0 \Longrightarrow \bar{F}$ is conservative away from the origin $\Longrightarrow \int_C \bar{F} \cdot d\bar{r}$ is independent of path $\forall C \Longrightarrow$

$\int_C \bar{F} \cdot d\bar{r} = 0$ $\forall$ simple closed curves C