203 §17.3 #s 1, 7, 11, 12, 13, 19, 21

(1) Based on the Figure and the contour mapping for f, the fundamental theorem says $\int_C \nabla f \cdot d\bar{r}$ is $f(\bar{r}(b)) - f(\bar{r}(a)) \approx 50 - 10 = \boxed{40}$

#s 3–10 Determine if $\bar{F}$ is conservative & if it is find $f \ni \nabla f = \bar{F}$

(7) $\bar{F} = \langle ye^x + \sin y, e^x + x\cos y \rangle = \langle P, Q \rangle$

$P_y = e^x + \cos y$, $Q_x = e^x + \cos y \Rightarrow \bar{F}$ is conservative, by T6. This means $\bar{F} = \langle f_x, f_y \rangle$ for some f.

$f_x = ye^x + \sin y \Rightarrow f = \int (ye^x + \sin y)\,dx + g(y)$

$= ye^x + x\sin y + g(y) \Rightarrow$

$f_y = e^x + x\cos y + g'(y) = e^x + x\cos y$

$\Rightarrow g'(y) = 0 \Rightarrow g(y) = \text{Constant}$. Choose $C = 0$.

Then $\boxed{f(x,y) = ye^x + x\sin y}$ works as a potential function for $\bar{F}$.

(11) $\bar{F}(x,y) = \langle 2xy, x^2 \rangle = \langle P, Q \rangle$ is shown in the figure and there are 3 paths shown. Each path starts @ $(1,2)$ & ends @ $(3,2)$.

(a) Explain why $\int_C \bar{F} \cdot d\bar{r}$ has the same value for all 3 paths.

(b) What is this common value?

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(11) (a) $P_y = 2x = Q_x \Longrightarrow \bar{F}$ is conservative

$\therefore \exists f \ni f_x = P = 2xy$

$\Longrightarrow f = \int 2xy\,dx + g(y)$

$= x^2y + g(y) \Longrightarrow$

$f_y = Q = x^2 + g'(y) = Q = x^2 \Longrightarrow g'(y) = 0$

$\Longrightarrow g(y) =$ Konstant, so choose $K = 0$,

$\Longrightarrow \boxed{f(x,y) = x^2y}$ is a potential function for $\bar{F}$.

(b) The common value for all 3 paths is

$f(\bar{r}(b)) - f(\bar{r}(a)) = f(3,2) - f(1,2)$

$= (3)^2(2) - (1)^2(2) = (9-1)(2) = \boxed{16 = \int_C \bar{F}\cdot d\bar{r}}$

($\bar{r}$ = vector representation for C, where C is ANY of the paths.)

#s 12-18 (a) Find $f \ni \nabla f = \bar{F}$ & (b) Find $\int_C \bar{F}\cdot d\bar{r}$

(12) $\bar{F} = \langle x^2, y^2 \rangle$, C is arc of $y = 2x^2$ from $(-1,2) + (2,8)$.

(a) $f_x = P = x^2 \Longrightarrow$

$f = \int x^2 dx + g(y) = \frac{1}{3}x^3 + g(y) \Longrightarrow$ SET = 0

$f_y = \quad g'(y) = y^2 \Longrightarrow g(y) = \frac{1}{3}y^3 + K$

$\therefore f(x,y) = \frac{1}{3}x^3 + \frac{1}{3}y^3$ is a possibility.

ANY potential function will work, since in the $f(\bar{r}(b)) - f(\bar{r}(a))$, the K's will subtract off, but we don't want to forget the K entirely. (In the sequel (differential equations), knowing $f(x_0, y_0)$ will determine K.)

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(12) (b) $\int_C \bar{F} \cdot d\bar{r} = f(\bar{r}(b)) - f(\bar{r}(a))$

$= f(2,8) - f(-1,2) = \frac{1}{3}(2)^3 + \frac{1}{3}(8)^3 - \left(\frac{1}{3}(-1)^3 + \frac{1}{3}(2)^3\right)$

$= \frac{1}{3}\left[8^3 + 1^3\right] = \frac{1}{3}[513] = \boxed{171 = \int_C \bar{F} \cdot d\bar{r}}$ $\quad \begin{array}{r} {}^{3}64 \\ 8 \\ \hline 512 \end{array}$

(13) $\bar{F} = \langle xy^2, x^2y \rangle$, C: $\bar{r}(t) = \langle t + \sin(\frac{\pi}{2}t), t + \cos(\frac{\pi}{2}t) \rangle$
$0 \leq t \leq 1$

(a) $f_x = xy^2 \Rightarrow f = \int xy^2\,dx + g(y)$

$= \frac{1}{2}x^2y^2 + g(y) \Rightarrow$

$f_y = x^2y + g'(y) = x^2y \rightarrow g'(y) = 0$, so $g(y) = K$,

which we can throw away.

$\boxed{f(x,y) = \frac{1}{2}x^2y^2}$

(b) To find $\int_C \bar{F} \cdot d\bar{r}$, we need $\bar{r}(b)$ & $\bar{r}(a)$:

$\bar{r}(1) = \langle 1 + \sin(\frac{\pi}{2}), 1 + \cos(\frac{\pi}{2}) \rangle = \langle 2, 1 \rangle = \bar{r}(1)$

$\bar{r}(0) = \langle 0 + \sin(0), 0 + \cos(0) \rangle = \langle 0, 1 \rangle = \bar{r}(0)$

$\therefore f(\bar{r}(b)) - f(\bar{r}(a)) = \frac{1}{2}(2)^2(1)^2 - \frac{1}{2}(0)^2(1)^2 = 2$

$\Rightarrow \boxed{\int_C \bar{F} \cdot d\bar{r} = 2}$

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#s 19, 20 Show the integral is independent of path and evaluate it.

(19) $\int_C \tan y\, dx + x\sec^2 y\, dy$ C is any path from $(1,0)$ to $(2, \frac{\pi}{4})$. As long as we avoid $y = \pm(2k+1)\frac{\pi}{2}$, we're OK.

$\bar{F} = \langle P, Q \rangle$ & $P_y = \sec^2 y = Q_x$ ✓

$f_x = \tan y \Rightarrow f = x\tan y + g(y)$ $\therefore \boxed{f = x\tan y}$

$f_y = x\sec^2 y + g'(y) = x\sec^2 y \Rightarrow g'(y) = 0$

$\therefore \int_C \bar{F} \cdot d\bar{r} = f(2, \frac{\pi}{4}) - f(1,0)$

$= 2\tan\frac{\pi}{4} - 1\tan(0) = \boxed{2 = \int_C \bar{F} \cdot d\bar{r}}$

#s 21, 22 Find work done by $\bar{F}$ in moving an object from P to Q.

(21) $\bar{F}(x,y) = \langle 2y^{\frac{3}{2}}, 3x\sqrt{y} \rangle$ FROM $P(1,1)$, $Q(2,4)$

$f_x = 2y^{\frac{3}{2}} \Rightarrow f = 2xy^{\frac{3}{2}} + g(y)$ $\therefore \boxed{f = 2xy^{\frac{3}{2}}}$

$f_y = 3x\sqrt{y} + g'(y) = 3x\sqrt{y} \Rightarrow g'(y) = 0 \Rightarrow g(x,y) = K$

$\therefore \int_C \bar{F} \cdot d\bar{r} = \text{WORK} = f(2,4) - f(1,1)$

$= 2(2)(4)^{\frac{3}{2}} - 2(1)(1)^{\frac{3}{2}} = (4)(8) - 2 = 30$

$\boxed{\text{WORK} = 30}$