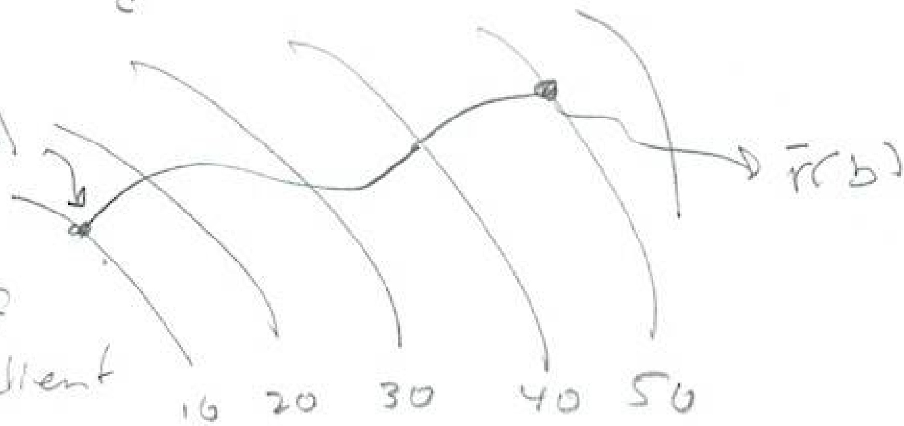
203 S 16.3 #'s 1, 7, 11, 12, 13, 15, 19, 21

(1) Find $\int_C \nabla f \cdot d\bar{r}$



Contour map of cnt^s gradient

$$\int_C \nabla f \cdot d\bar{r} = f(\bar{r}(b)) - f(\bar{r}(a)) = 50 - 10 = 40,$$

by FTLI.

#'s 3-10 Determine if $\bar{F}$ is cnsrvtv. If it is, find $f \ni \nabla f = \bar{F}$

(7) $\bar{F} = \langle ye^x + \sin y, e^x + x\cos y \rangle$

$\bar{F}$ is cnt^{s}ly dif'b^{le} on $\mathbb{R}^2$. $\bar{F} = \langle P, Q \rangle$

$Q_x - P_y = e^x + \cos y - (e^x + \cos y) = 0 \Longrightarrow$

$\bar{F}$ is conservative.

$f_x = ye^x + \sin y \Longrightarrow f = ye^x + x\sin y + g(y)$

$f_y = e^x + x\cos y = e^x + x\cos y + g_y(y) \Longrightarrow$

$g_y = 0 \Longrightarrow$

$g = K \equiv 0$

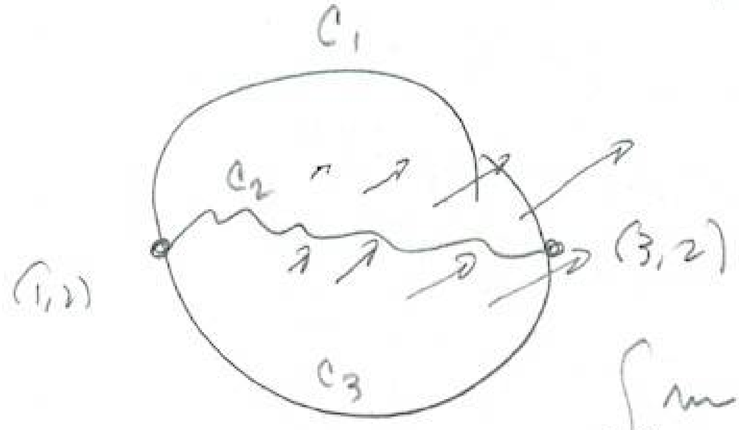
$\boxed{f = e^x + x\cos y}$

203 §16.9 #s 11, 13, 15, 19, 21

(11) The figure shows $\bar{F} = \langle 2xy, x^2 \rangle$ &

3 curves that start @ $(1,2)$ & end @ $(3,2)$

(a) Explain why $\int_C \bar{F} \cdot d\bar{r}$ has same value $\forall$ 3 curves



$\bar{F}$ cnt's'ly difb'l on $\mathbb{R}^2$

$Q_x - P_y = 2x - 2x = 0$

$\Longrightarrow$ independent of path $\Longrightarrow$

$\int_{C_1} \sim = \int_{C_2} \sim = \int_{C_3} \sim$.

(b) What is the common value?

$\bar{F}$ conservative $\Longrightarrow \bar{F} = \nabla f$ for some f:

$f_x = 2xy \Longrightarrow f = x^2 y + g(y) \Longrightarrow$

$f_y = x^2 = x^2 + g_y(y) \Longrightarrow g(y) = K \equiv 0 \Longrightarrow$

$f = x^2 y \Longrightarrow f(3,2) - f(2,2)$

$= 3^2 \cdot 2 - 2^2 \cdot 2 = 2(9-4) = \boxed{10}$

203 §16.3 #s 12, 13, 15, 19, 21

12 #s 12, 13 (a) Find $f \ni \bar{F} = \nabla f$

(b) use (a) to eval $\int_C \bar{F} \cdot d\bar{r}$ along C.

~~12~~ 13 $\bar{F} = \langle x^2, y^2 \rangle$, C = arc of parabola

$y = 2x^2$ from $(-1, 2)$ to $(2, 8)$

(a) $f_x = x^2 \Rightarrow f = \frac{1}{3}x^3 + g(y) \Rightarrow$

$f_y = y^2 = g_y(y) \rightarrow g(y) = \frac{1}{3}y^3 + K$

Let $K \equiv 0 \Rightarrow \boxed{f = \frac{1}{3}(x^3 + y^3)}$

$$\begin{array}{r} 64 \\ 8 \\ \hline 512 \end{array}$$

(b) $\rightarrow \int_C \bar{F} \cdot d\bar{r} = \frac{1}{3}(x^3 + y^3)\Big]_{(-1,2)}^{(2,8)}$

$= \frac{1}{3}\left[(2^3 + 8^3) - ((-1)^3 + 2^3)\right] = \frac{1}{3}\left[8 + 512 - (-1 + 8)\right]$

$= \frac{1}{3}[520 - 7] = \frac{1}{3}[513] = \boxed{171}$

13 $\bar{F} = \langle xy^2, x^2y \rangle$

$\bar{r}(t) = \langle t + \sin(\frac{\pi}{2}t), t + \cos(\frac{\pi}{2}t) \rangle \quad 0 \le t \le 1$

(a) $f_x = xy^2 \rightarrow f = \frac{1}{2}x^2y^2 + g(y)$

$f_y = x^2y = x^2y + g_y(y) \rightarrow g_y(y) = 0$

$\Rightarrow \boxed{f = \frac{1}{2}x^2y^2}$

(b) $f(\bar{r}(1)) - f(\bar{r}(0)) = \frac{1}{2}(t + \sin(\frac{\pi}{2}t))^2(t + \cos(\frac{\pi}{2}t))^2\Big]_0^1$

$= \frac{1}{2}(1+1)^2(1+0)^2 - \frac{1}{2}(0)^2(1)^2 = \boxed{2}$

203 §16.3 #s 9, 15, 19, 21

#s' 15, 16 (a) Find $f \ni \nabla f = \bar{F}$

(b) Use (a) to eval $\int_C \bar{F} \cdot d\bar{r}$ along C.

15 16 $\bar{F} = \langle yz, xz, xy + 2z \rangle$

C is ~~line~~ segment from $(1, 0, -2)$ to $(4, 6, 3)$

(a) $f_x = yz \Rightarrow f = xyz + g(y, z) \Rightarrow$

$f_y = xz = xz + g_y(y, z) \Rightarrow g_y(y, z) = 0 \Rightarrow$

$g(y, z) = g(z) \Rightarrow$

$f_z = xy + 2z = xy + g'(z) \Rightarrow$

$g'(z) = 2z \Rightarrow g(z) = z^2 + K$. Let $K \equiv 0$.

Then $\boxed{f = xyz + z^2}$

(b) $f(4, 6, 3) - f(1, 0, -2)$

$= (4)(6)(3) + 3^2 - ((1)(0)(-2) + (-2)^2)$

$= 72 + 9 - 4 = \boxed{77}$

203 §16.3 #s 19, 21

#s 19–20 Show the line integral is independent of path & eval the integral.

(19) $\int_C \tan y\,dx + x\sec^2 y\,dy$, C = any path from (1,0) to $(2, \pi/4)$

$= \int_C \langle \tan y, x\sec^2 y\rangle \cdot \langle x', y'\rangle\,dt$

$\bar{F} = \langle \tan y, x\sec^2 y\rangle$

$f_x = \tan y \Rightarrow f = x\tan y + g(y) \rightarrow$

$f_y = x\sec^2 y \doteq x\sec^2 y + g'(y) \rightarrow g'(y) = 0$

$\rightarrow g(y) = K \equiv 0 \rightarrow f = x\sec^2 y \rightarrow$

$\int_C F\cdot d\bar{r} = f(2, \frac{\pi}{4}) - f(1,0)$

$= 2\sec^2(\frac{\pi}{4}) - 1\sec^2(0) = 2(\sqrt{2}) - 1$

$= \boxed{2\sqrt{2} - 1}$

(21) Find work done by Force Field $\bar{F}$ in moving an object from P(1,1) to Q(2,4), if $\bar{F} = \langle 2y^{\frac{3}{2}}, 3x\sqrt{y}\rangle$

$Q_x - P_y = 3\sqrt{y} - 2(\frac{3}{2})(\sqrt{y}) = 0 \Rightarrow$ conservative $\Rightarrow f_x = 2y^{\frac{3}{2}} \Rightarrow f = 2xy^{\frac{3}{2}} + g(y)$

$\Rightarrow f_y = 3xy^{\frac{1}{2}} = 3xy^{\frac{1}{2}} + g'(y) \Rightarrow f = 3xy^{\frac{1}{2}}$ works $(K \equiv 0)$

203 § 16.3 #21

#21 cont'd

$$\int_C \vec{F} \cdot d\vec{r} = f(2,4) - f(1,1)$$

$$= 3(2)(4)^{\frac{1}{2}} - 3(1)(1)^{\frac{1}{2}} = 6 \cdot 2 - 3 = \boxed{9}$$