203 § 16.2 #s 5, 11, 15, 23, 27, 32, 37, 43

8 #s 1-16 Eval the line integral.

5 $\int_C (x^2 y^3 \sqrt{x})\,dy$ C: $\sqrt{x} = y$ from (1, 1) to (4, 2)

$y = \sqrt{x} \rightarrow y^2 = x$

$dy = \frac{1}{2\sqrt{x}}$

$\int_C (x^2y^3 - \sqrt{x})\,dy = \int_1^2 (y^4 y^3 - y)\,dy$

$= \frac{1}{7}y^7 - \frac{1}{2}y^2\Big]_1^2 = \frac{1}{7}\cdot 128 - \frac{1}{2}(4) - \left(\frac{1}{7} - \frac{1}{2}\right)$

$= \frac{128}{7} - \frac{1}{7} - \frac{4}{2} + \frac{1}{2} = \frac{127}{7} - \frac{3}{2} = \frac{254-21}{14}$

$= \boxed{\frac{233}{14}}$

$x = t$ $\quad y = \sqrt{x} = \sqrt{t}$

$dx = dt$ $\quad dy = \frac{1}{2\sqrt{t}}\,dt$

$1 \le t \le 4$

$\int_C (x^2y^3 - \sqrt{x})\,dy$

$= \int_1^4 \left(t^2 (\sqrt{t})^3 - \sqrt{t}\right)\left(\frac{1}{2\sqrt{t}}\,dt\right)$

$= \int_1^4 \left(t^2 \cdot t^{\frac{3}{2}} \cdot \frac{1}{2t^{1/2}} - \frac{t^{\frac{1}{2}}}{2t^{1/2}}\right)dt$

$= \int_1^4 \left(\frac{t^3}{2} - \frac{1}{2}\right)dt = \left[\frac{t^4}{8} - \frac{t}{2}\right] = \frac{4^4}{8} - \frac{4}{2} - \frac{1}{8} + \frac{1}{2}$

203 §16.2 #s 5, 11, 15, 23, 27, 32, 37, 43

\# 5 cont'd

$y = \sqrt{x}$

$dy = \frac{1}{2}x^{-\frac{1}{2}}dx$

$\frac{64}{256}$

$$\int_C (x^2y^3 - \sqrt{x})\,dy = \int_1^4 (x^2x^{\frac{3}{2}} - x^{\frac{1}{2}})(\frac{1}{2}x^{-\frac{1}{2}}dx)$$

$$= \int_1^4 (\frac{1}{2}x^3 - \frac{1}{2})dx = \left[\frac{1}{8}x^4 - \frac{1}{2}x\right]_1^4 = \frac{1}{8}(4^4) - \frac{1}{2}(4)$$

$$- \frac{1}{8}(1) + \frac{1}{2}(1) = \frac{256}{8} - 2 - \frac{1}{8} + \frac{1}{2} = \frac{255}{8} - \frac{16}{8} + \frac{4}{8}$$

$$= \boxed{\frac{243}{8}}$$

$x = y^2 \quad 0 \le y \le 2$

$$\int_C (x^2y^3 - x^{\frac{1}{2}})\,dy = \int_1^2 (y^4y^3 - y)dy$$

$$= \int_1^2 (y^7 - y)\,dy = \left[\frac{y^8}{8} - \frac{y^2}{2}\right]_1^2 = \frac{256}{8} - \frac{4}{2}$$

$$- \frac{1}{8} + \frac{1}{2} = \frac{256 - 1 - 3(4)}{8} = \frac{255-12}{8} = \boxed{\frac{243}{8}}$$

203 § 16.2 ~~#s~~ 5, 11, 15, 23, 27, 32, 37, 43

(11) $\int_c xe^{yz}\, ds$ C: line segment from $(0,0,0)$ to $(1,2,3)$

$\bar{r} = (1-t)\langle 0,0,0\rangle + t\langle 1,2,3\rangle = \langle t, 2t, 3t\rangle$

$||\bar{r}'|| = ||\langle 1,2,3\rangle|| = \sqrt{1^2+2^2+3^2}$

$= \sqrt{1+4+9} = \sqrt{14}$

$\int_c xe^{yz}\, ds = \int_0^1 te^{(2t)(3t)}\sqrt{14}\, dt = \sqrt{14}\int_0^1 te^{6t^2}\, dt$

$= \frac{\sqrt{14}}{12}\int_0^1 e^{6t^2}(12t\, dt) = \frac{\sqrt{14}}{12}\left[e^{6t^2}\right]_0^1 =$

$\frac{\sqrt{14}}{12}\left[e^6 - e^0\right] = \boxed{\frac{\sqrt{14}}{12}\left[e^6 - 1\right]}$

(15) $\int_C (x+yz)\,dx + 2x\,dy + xyz\,dz$

from $(1,0,1)$ to $(2,3,1)$ to $(2,5,2)$

$\bar{r}_1 = (1-t)\langle 1,0,1\rangle + t\langle 2,3,1\rangle,\ 0 \le t \le 1$

$= \langle 1-t+2t,\ 0+3t,\ 1-t+t\rangle = \langle 1+t, 3t, 1\rangle$

$x = 1+t \Rightarrow dx = dt$

$y = 3t \Rightarrow dy = 3\,dt$

$z = 1 \Rightarrow dz = 0$

$\int_{C_1}$ ~~~

203 §16.2 #s 15, 23, 27, 32, 37, 43

#15 cont'd

$\int_{C_1} (x+yz)\,dx + 2x\,dy + xyz\,dz$

$= \int_0^1 ((t+1) + (3t)(1))\,dt + \int_0^1 2(t+1)\cdot 3\,dt + \int_0^1 0$

$= \int_0^1 (4t+1)\,dt + \int_0^1 (6t+6)\,dt = \int_0^1 (10t+7)\,dt$

$= \left[5t^2 + 7t\right]_0^1 = 5+7 = \boxed{12} = \int_{C_1}$

$\int_{C_2}$: $(1-t)\langle 2,3,1\rangle + t\langle 2,5,2\rangle$

$= \langle 2-2t+2t,\ 3-3t+5t,\ 1-t+2t\rangle$

$= \langle 2,\ 3+2t,\ 1+t\rangle$

$x = 2 \Rightarrow dx = 0$

$y = 3+2t \Rightarrow dy = 2\,dt$

$z = t+1 \Rightarrow dz = dt$

$\int_{C_2} = \int_{C_2} (x+yz)\,dx + 2x\,dy + xyz\,dz$

$= \int_0^1 2+(2t+3)(t+1)\cdot 0 + \int_0^1 2(2)\cdot 2\,dt + \int_0^1 2(2t+3)(t+1)\,dt$

$= \int_0^1 8\,dt + \int_0^1 2(2t^2+5t+3)\,dt = 8 + 2\left[\frac{2}{3}t^3 + \frac{5}{2}t^2 + 3t\right]_0^1$

$= 8 + 2\left[\frac{2}{3} + \frac{5}{2} + 3\right] = 8 + 2\left[\frac{4+15+18}{6}\right]$

$= 8 + \frac{27}{3} = 8 + 9 = \boxed{17}$

203 §16.2 #s 23, 27, 32, 37, 43

#s 23-26 Use calculator or CAS to eval to 4 places

23) $\int_C \bar{F} \cdot d\bar{r}$, where $\bar{F} = \langle xy, \sin y\rangle$ and

$\bar{r} = \langle e^t, e^{-t^2}\rangle, \quad 1 \le t \le 2$

$\bar{r}' = \langle e^t, -2te^{-t^2}\rangle$

$\int_1^2 \bar{F} \cdot \bar{r}'\,dt = \int_1^2$

$\bar{F}(\bar{r}(t)) = \langle e^t(e^{-t^2}), \sin(e^{-t^2})\rangle$

$\Rightarrow \int_1^2 \bar{F} \cdot \bar{r}'\,dt = \int_1^2 \langle e^{t-t^2}, \sin(e^{-t^2})\rangle \cdot \langle e^t, -2te^{-t^2}\rangle\,dt$

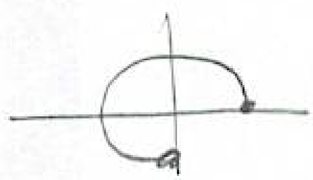
$= \int_1^2 \left(e^{2t-t^2} - 2te^{-t^2}\sin(e^{-t^2})\right)dt$

203 §16.2 #s 27, 32, 37, 43

~~27~~ #s 27-28 use graph of $\bar{F}$ & curve to guess if $\int_C \bar{F}\cdot d\bar{r}$ is $+, -, 0$.

27) $\bar{F} = \langle (x-y), xy \rangle$

C: arc of $x^2+y^2=4$ traversed counter-clockwise from $(2,0)$ to $(0,-2)$



$\bar{r} = \langle 2\cos\theta, 2\sin\theta \rangle$

$\bar{F} = \langle 2(\cos\theta - \sin\theta), 4\cos\theta\sin\theta \rangle$

$0 \le \theta \le \frac{3\pi}{2}$ $\quad \bar{r}' = \langle -2\sin\theta, 2\cos\theta \rangle$

$$\int_C \bar{F}\cdot d\bar{r} = \int_0^{\frac{3\pi}{2}} \langle 2\cos\theta - 2\sin\theta, 4\cos\theta\sin\theta \rangle \cdot \langle -2\sin\theta, 2\cos\theta \rangle$$

$$= \int_0^{\frac{3\pi}{2}} \left(-4\sin\theta\cos\theta + 4\sin^2\theta + 8\cos^2\theta\sin\theta\right) d\theta$$

$$= \left[\frac{4\cos^2\theta}{2}\right]_0^{\frac{3\pi}{2}} + \int_0^{\frac{3\pi}{2}} \frac{4}{2} - \frac{4}{2}\cos(2\theta) - \left[\frac{8\cos^3\theta}{3}\right]_0^{\frac{3\pi}{2}}$$

$$= 2(0) - 2 + \left[2\theta - \sin(2\theta)\right]_0^{\frac{3\pi}{2}} - \frac{8}{3}(0) + \frac{8}{3}$$

$$= -2 + 3\pi - 0 + 0 - 0 + \frac{8}{3} = \frac{8-6}{3} + \frac{9\pi}{3} = \boxed{\frac{9\pi + 2}{3}}$$

203 §16.2 #s 32, 37, 43

(32) Find the work done by $\bar{F} = \langle x^2, xy \rangle$ on a particle that moves once around the circle $x^2+y^2=4$ oriented clockwise

$$\bar{r} = \langle 2\cos\theta, 2\sin\theta \rangle$$

$$\bar{F}(\bar{r}(\theta)) = \langle 4\cos^2\theta, 4\sin\theta\cos\theta \rangle$$

$$\bar{F}\cdot d\bar{r} = \langle 4\cos^2\theta, 4\sin\theta\cos\theta \rangle \cdot \langle -2\sin\theta, 2\sin\theta \rangle$$

$$= (4\cos^2\theta)(-2\sin\theta) + 8\sin^2\theta\cos\theta)$$

$$\int_C \bar{F}\cdot d\bar{r} = \int_0^{2\pi} (-8\cos^2\theta\sin\theta + 8\sin^2\theta\cos\theta)\,d\theta$$

$$= \left[\frac{8}{3}\cos^3\theta + \frac{8}{3}\sin^3\theta\right]_0^{2\pi} = \frac{8}{3} + 0 - \left(\frac{8}{3} + 0\right) = 0 \,!$$

(37) Moments of Inertia $I_x = \int_C y^2 \rho(x,y)\,ds$

$I_y = \int_C x^2 \rho(x,y)\,ds$, where ρ is linear density of wire

E3: $x^2+y^2=1$: C, $y \ge 0$,

linear density is $\rho(x,y) = k(1-y)$

$$\bar{r} = \langle \cos\theta, \sin\theta \rangle$$

$$\bar{r}' = \langle -\sin\theta, \cos\theta \rangle$$

203 § 16.2 #s 37, 43

37 cont'd

$$I_x = \int_C y^2 (k(1-y))\, ds \quad ds = \|\bar{r}'\|\, d\theta$$

$$= \int_0^{\pi} k \sin^2\theta \,(1-\sin\theta)\, |\sin^2\theta + \cos^2\theta|\, d\theta$$

$$= k\int_0^{\pi} \sin^2\theta - (1-\cos^2\theta)\sin\theta \, d\theta$$

$$= k\int_0^{\pi} \left(\tfrac{1}{2}(1-\cos(2\theta)) - \sin\theta + \cos^2\theta \sin\theta\right) d\theta$$

$$= k\left[\int_0^{\pi} \tfrac{1}{2}\, d\theta - \tfrac{1}{2}\int_0^{\pi} \cos(2\theta)\, d\theta - \int_0^{\pi} \sin\theta\, d\theta + \int_0^{\pi} \cos^2\theta \sin\theta\, d\theta\right]$$

$$= k\left[\tfrac{1}{2}\pi - \tfrac{1}{4}\sin 2\theta\Big]_0^{\pi} + \cos\theta\Big]_0^{\pi} - \frac{\cos^3\theta}{3}\Big]_0^{\pi}\right]$$

$$= k\left(\left[\tfrac{1}{2}\pi - 0 + -1 - 1 - \left[\left(-\tfrac{1}{3} - \tfrac{1}{3}\right)\right]\right]\right)$$

$$= k\left[\frac{\pi}{2} - 2 + \frac{2}{3}\right] = \frac{3\pi - 12 + 4}{6} = \boxed{\frac{3\pi - 8}{6}}$$

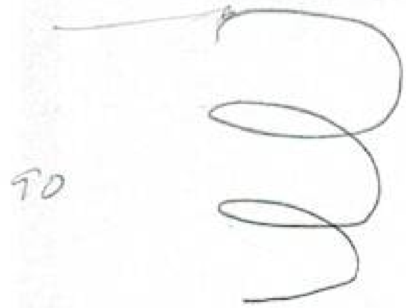
Not interested in repeat

for I_y.

203 § 16.2 # 43

(43) Simple answer: $(185)(90)$

$= 16,650$ ft-lbs.



$$x^2 + y^2 = 20^2$$

$$x = \cancel{40}\cos\theta,\ y = 20\sin\theta$$

$$0 \le \theta \le 6\pi$$

$$z(0) = 0,\ z(6\pi) = 90$$

$$(0,0),\ (6\pi, 90)$$

$$\frac{90}{6\pi} = \frac{30}{2\pi} = \frac{15}{\pi}$$

$$z = \frac{15}{\pi}\theta$$

$$\bar{r} = \langle 20\cos\theta, 20\sin\theta, \tfrac{15}{\pi}\theta \rangle$$

$$\bar{r}' = \langle -20\sin\theta, 20\cos\theta, \tfrac{15}{\pi} \rangle$$

$$\bar{F} = \langle 0, 0, -185 \rangle$$

$$\int_C \bar{F} \cdot d\bar{r} = \int_0^{6\pi} -185 \cdot \frac{15}{\pi}\, d\theta$$

$$= -\frac{185 \cdot 15}{\pi}\theta \Big]_0^{6\pi} = -\frac{185 \cdot 15 \cdot 6\pi}{\pi}$$

$$= -185 \cdot 90 = \boxed{16,650 \text{ ft-lbs}}$$