203 §15.9 #15

We ignore the suggested substitution and make our own, based on wanting a vertical edge & a horizontal edge.

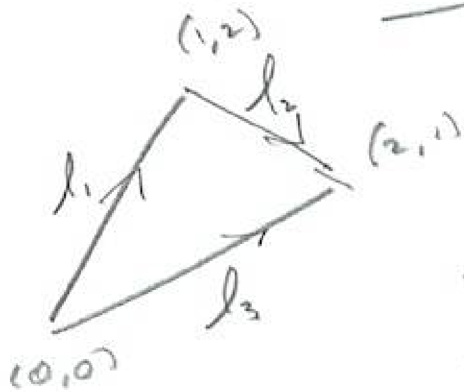
(0,0), (1,2), (2,1) are vertices of a triangular region R over which we integrate the function $f(x,y) = x-3y$

$$\iint_R (x-3y)\,dA$$

$\frac{2-1}{1-2} = -1$

$-1(x-1)+2$

$= -x+3$



$l_1 : y = 2x$

$l_2 : -x+3 = y$

$l_3 : \frac{1}{2}x = y$

I want l_1 to map to v-axis

" l_3 " " u-axis

$l_1 : 2x-y=0$

$l_2 : x+y=3$

$l_3 : x-2y=0$

Let $u = 2x-y$

$v = x-2y$

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The line segment from (0,0) to (1,2)

$u(0,0) = 0, \; u(1,2) = 2(1) - 2 = 0$ ✓

$v(0,0) = 0, \; v(1,2) = 1 - 2(2) = -3$

l_2 from (1,2) to (2,1)

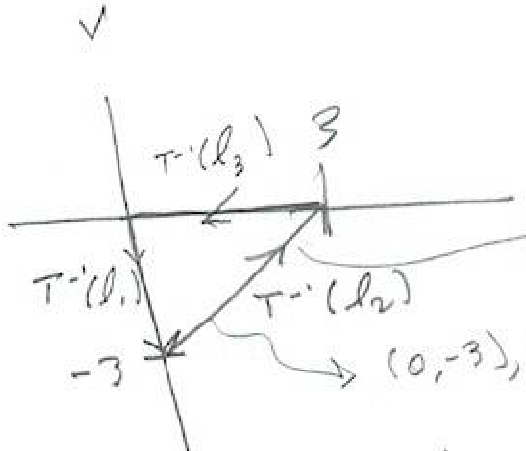
$u(1,2) = 0, \; u(2,1) = 3$

$v(1,2) = -3, \; v(2,1) = 0$

l_3 from (2,1) to (0,0)

$u(2,1) = 3, \; u(0,0) = 0$

$v(2,1) = 0, \; v(0,0) = 0$



$(0,-3), (3,0) \Rightarrow \boxed{v = u - 3}$

Now, find $T(u,v)$ from $T^{-1}(u,v)$

$x - 2y = v$

$2x - y = u$

$\left[\begin{array}{cc|c} 1 & -2 & v \\ 2 & -1 & u \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & v \\ 0 & 3 & u - 2v \end{array}\right]$

$y = \frac{1}{3}(u - 2v)$

$x - 2\left(\frac{1}{3}(u - 2v)\right) = v$

$x - \frac{2}{3}u + \frac{4}{3}v = v$

$x = \frac{2}{3}u - \frac{1}{3}v = \frac{1}{3}(2u - v)$

$\left|\begin{bmatrix} \vec{r}_u \\ \vec{r}_v \end{bmatrix}\right| = \left|\begin{matrix} \langle \frac{2}{3}, \frac{1}{3} \rangle \\ \langle -\frac{1}{3}, -\frac{2}{3} \rangle \end{matrix}\right| = -\frac{4}{9} + \frac{1}{9} = -\frac{3}{9} = -\frac{1}{3}$

$= -\frac{1}{3} \Rightarrow -\frac{1}{3} = \left|\frac{\partial(x,y)}{\partial(u,v)}\right|$

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So now all we need is $f(x(u,v), y(u,v))$

$= x(u,v) - 3y(u,v) = \frac{2u-v}{3} - 3\left(\frac{u-2v}{3}\right)$

$= \frac{2u - v - 3u + 6v}{3} = \frac{-u+5v}{3} \rightarrow$

$$\frac{1}{3}\int_0^3 \int_{u-3}^{0} (-u+5v)\,dv\,du$$

$$= \frac{1}{3}\int_0^3 \left[-uv + \frac{5}{2}v^2\right]_{u-3}^{0} du$$

$$= \frac{1}{3}\int_0^3 -\left[\frac{5}{2}(u^2 - 6u + 9) - u(u-3)\right) du$$

$$= \frac{1}{3}\int_0^3 \left(-\frac{5}{2}u^2 + 15u - \frac{45}{2} + u^2 - 3u\right) du$$

$$= \frac{1}{3}\int_0^3 \left(-\frac{3}{2}u^2 + 12u - \frac{45}{2}\right) du$$

$$= \frac{1}{3}\left[-\frac{1}{2}u^3 + 6u^2 - \frac{45}{2}u\right]_0^3$$

$$= \frac{1}{3}\left[-\frac{27}{2} + 54 - \frac{45}{2}(3)\right]$$

$$= -\frac{9}{2} + 18 + \frac{45}{2} = \frac{36+36}{2} = \boxed{36}$$

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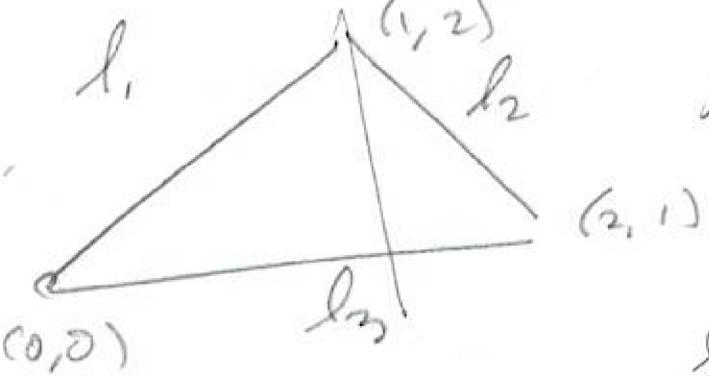
Just work it as given.



$l_1 = 2x = y$

$l_2 = -x + 3 = y$

$l_3 = y = \frac{1}{2}x$

$$\int_0^1 \int_{\frac{1}{2}x}^{2x} (x - 3y)\,dy\,dx + \int_1^2 \int_{\frac{1}{2}x}^{3-x} (x - 3y)\,dy\,dx$$

$$= \int_0^1 \left[xy - \tfrac{3}{2}y^2 \right]_{\frac{1}{2}x}^{2x} dx + \int_1^2 \left[xy - \tfrac{3}{2}y^2 \right]_{\frac{1}{2}x}^{3-x} dx$$

$$= \int_0^1 \left[2x^2 - \tfrac{3}{2}(4x^2) - \left(\tfrac{1}{2}x^2 - \tfrac{3}{2}\left(\tfrac{1}{4}x^2\right) \right) \right] dx$$

$\frac{1}{2}x^2 - \frac{3}{8}x^2$

$$+ \int_1^2 \left[x(3-x) - \tfrac{3}{2}(x^2 - 6x + 9) - \left(\tfrac{1}{2}x^2 - \tfrac{3}{2}\left(\tfrac{1}{4}x^2\right) \right) \right] dx$$

$$= \int_0^1 \left[2x^2 - 6x^2 - \tfrac{1}{8}x^2 \right] dx + \int_1^2 \left[3x - x^2 - \tfrac{3}{2}x^2 + 9x - \tfrac{27}{2} - \tfrac{1}{8}x^2 \right] dx$$

$$= \int_0^1 \frac{16 - 48 - 1}{8} x^2\,dx + \int_1^2 \frac{-x^2 - 12x^2 + 72x^2 - 108x^2 - x^2}{8} dx$$

$$= -\frac{33}{8}\int_0^1 x^2\,dx + \int_1^2 \frac{-50x^2}{8}\,dx = -\frac{33}{24}x^3 \Big]_0^1 + \frac{-50}{24}x^3 \Big]_1^2$$

$$= -\frac{33}{24} - \frac{50}{24}[7] = \frac{33 + 350}{24}$$