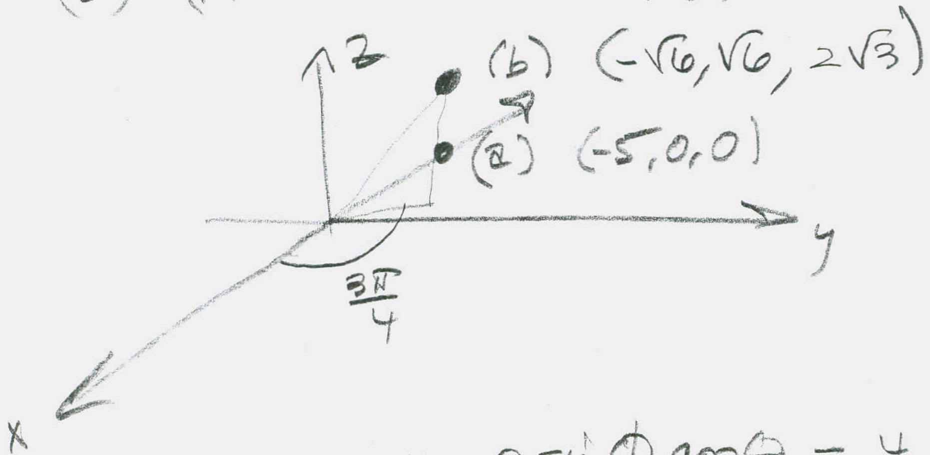
203 § 16.8 #s 2, 4, 6, 8, 12, 14, 15, 18, 26, 36

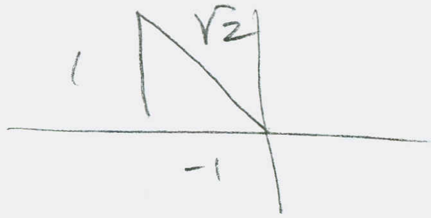
#36 is an orange slice.

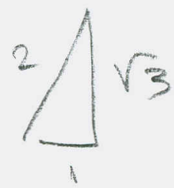
We plot. Then find rectangular coords

(2) (a) $(\rho, \theta, \phi) = (5, \pi, \frac{\pi}{2})$

(b) $(\rho, \theta, \phi) = (4, \frac{3\pi}{4}, \frac{\pi}{3})$







$$x = \rho \sin\phi \cos\theta = 4 \sin\frac{\pi}{3} \cos\frac{3\pi}{4}$$
$$= 4 \cdot \frac{\sqrt{3}}{2} \cdot \left(-\frac{1}{\sqrt{2}}\right)$$
$$= -\frac{4\sqrt{3}}{2\sqrt{2}} \text{ OR } -\frac{4\sqrt{6}}{4} = -\sqrt{6} = x$$

$$y = \rho \sin\phi \sin\theta = 4 \cdot \sin\frac{\pi}{3} \sin\frac{3\pi}{4}$$
$$= 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$
$$= \frac{4\sqrt{3}\sqrt{2}}{4} = \sqrt{6} = y$$

$$z = \rho \sin\phi = \frac{4\sqrt{3}}{2} = 2\sqrt{3} = z$$

(4) Change from rectangular to spherical

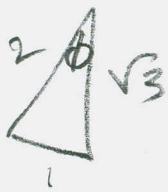
(a) $(x, y, z) = (0, \sqrt{3}, 1) = (2, \frac{\pi}{2}, \frac{\pi}{6})$

$\rho = \sqrt{3+1} = 2 = \rho$

203 §16.8 #s 4, 6, 8, 12, 14, 15, 18, 26, 36

(4b) $(x,y,z) = (-1, 1, \sqrt{6})$

$\rho = \sqrt{1+1+6} = \sqrt{8} = \boxed{2\sqrt{2} = \rho}$



$\cos\phi = \frac{z}{\rho} = \frac{\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{12}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} =$

$\cos\phi \Longrightarrow \boxed{\phi = \frac{\pi}{6}}$

$x = \rho \sin\phi \cos\theta$

$x = 2\sqrt{2} \cdot \frac{1}{2} \cos\theta$

$-1 = \sqrt{2}\cos\theta \Longrightarrow$

$-\frac{1}{\sqrt{2}} = \cos\theta \Longrightarrow \boxed{\theta = \frac{3\pi}{4}}$

$\Longrightarrow \boxed{(\rho, \theta, \phi) = \left(2\sqrt{2}, \frac{3\pi}{4}, \frac{\pi}{6}\right)}$

(6) $\rho = 3$ is a sphere of radius 3, centered at the origin.

(8) Identify $\rho^2(\sin^2\phi \sin^2\theta + \cos^2\phi) = 9$

$\longrightarrow \rho^2 \sin^2\phi \sin^2\theta + \rho^2\cos^2\phi = 9$

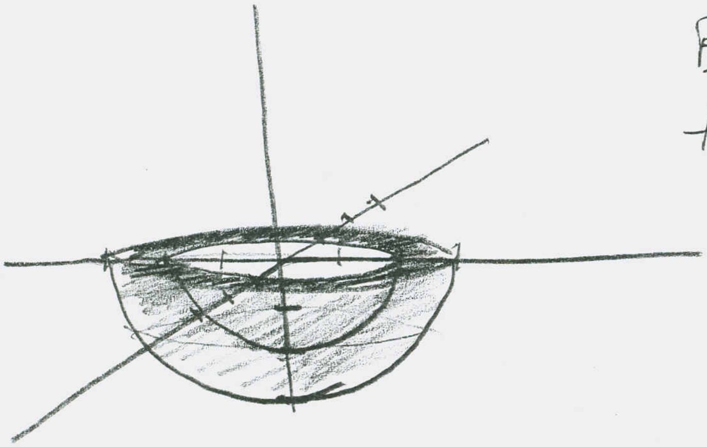
$\Longrightarrow y^2 + z^2 = 9$

Is a right circular cylinder of radius 3, whose long axis is the x-axis.

203 §16.8 #s 12, 14, 15, 18, 26, 36

(12) Sketch the solid #s 11-14

$2 \le \rho \le 3, \quad \frac{\pi}{2} \le \phi \le \pi$



Bottom half of the the region between the two spheres $\rho = 2$ & $\rho = 3$

(14) $\rho \le 2, \; \rho \le \csc\phi$

$\rho \le \csc\phi \Rightarrow \rho \sin\phi \le 1$, i.e.,

$\sin\phi \le \frac{1}{2}$ when $\rho = 2$

No help.

We know we're inside the sphere

$\rho = 2$. Now let's play with

$\rho \le \csc\phi \Rightarrow$

$\rho \sin\phi \le 1 \Rightarrow$

$\rho^2 \sin^2\phi \le 1$ Aha!

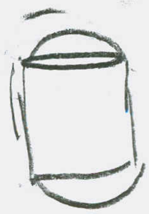
$\rho^2 \sin^2\phi \, (1) \le 1$

$\rho^2 \sin^2\phi \, (\sin^2\theta + \cos^2\theta) \le 1 \Rightarrow$

$x^2 + y^2 \le 1$

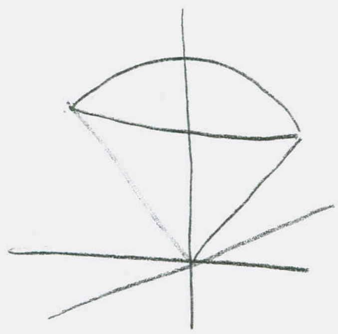
Inside sphere and inside the cylinder

$x^2 + y^2 = 1$



203 § 16.8 #s 15, 18, 26, 36

(15) A solid lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$.
Write a description in spherical coords.



$$x^2+y^2+z^2-z+\frac{1}{4}=\frac{1}{4}$$
$$x^2+y^2+(z-\tfrac{1}{2})^2=\frac{1}{4}$$

Need outer ϕ

$$x^2+y^2+z^2=\rho^2=z=\rho\sin\phi$$

Above the cone

$$z\geq\sqrt{x^2+y^2} \implies z^2\geq x^2+y^2$$
$$\implies z^2+z^2\geq x^2+y^2+z^2$$
$$\implies 2z^2\geq\rho^2$$
$$\implies 2\rho^2\cos^2\phi\geq\rho^2$$
$$\implies \cos^2\phi\geq\frac{1}{2}$$
$$\implies \cos\phi\geq\frac{1}{2}$$
$$\implies \boxed{0\leq\phi\leq\frac{\pi}{4}}$$

In sphericals, the sphere

$z=x^2+y^2+z^2$, is

$\rho\cos\phi=\rho^2$

$\implies \rho=\cos\phi$

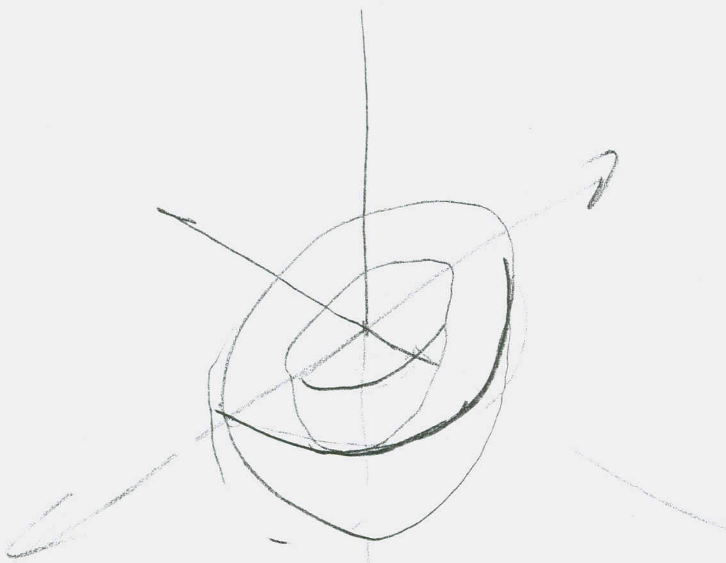
Solid ~~below~~ the sphere: $\rho\leq\cos\phi$

and so $\boxed{0\leq\rho\leq\cos\phi}$

The two key descriptors

203 §16.8

18 $\int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi}\int_1^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

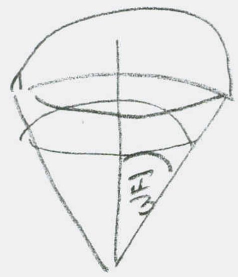


$\int_0^{2\pi}$ all way around wrt θ

$\int_{\frac{\pi}{2}}^{\pi}$ Below xy-plane wrt ϕ

$\int_1^2$ Between spheres $\rho = 1$ & $\rho = 2$

36 Evaluate $\iiint_E xyz \, dV$, where E lies between $\rho = 2$, $\rho = 4$ & above the cone $\phi = \frac{\pi}{3}$



$\int$

$\int_0^{\frac{\pi}{3}}\int_0^{2\pi}\int_2^4 (\rho \sin\phi \cos\theta)(\rho \sin\phi \sin\theta) \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$\cdot \, \rho \cos\phi$