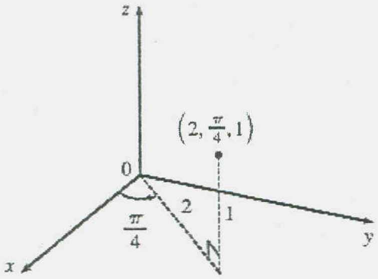
My work on later pages Not all "book" solutions

16.7 #s 1-13, 15, 18, 20, 27

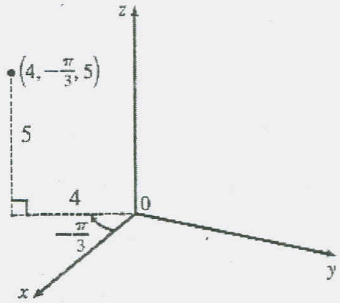
1. (a)




$x = 2\cos\frac{\pi}{4} = \sqrt{2}$, $y = 2\sin\frac{\pi}{4} = \sqrt{2}$, $z = 1$,

so the point is $(\sqrt{2}, \sqrt{2}, 1)$ in rectangular coordinates.

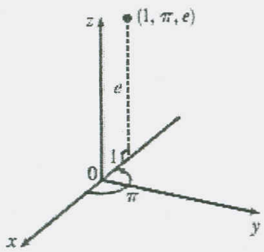
(b)




$x = 4\cos\left(-\frac{\pi}{3}\right) = 2$, $y = 4\sin\left(-\frac{\pi}{3}\right) = -2\sqrt{3}$,

and $z = 5$, so the point is $(2, -2\sqrt{3}, 5)$ in rectangular coordinates.

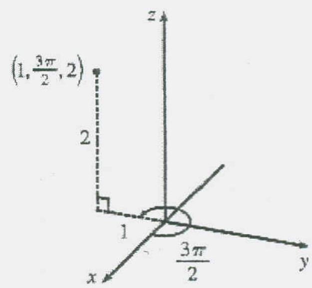
2. (a)




$x = 1\cos\pi = -1$, $y = 1\sin\pi = 0$, and $z = e$,

so the point is $(-1, 0, e)$ in rectangular coordinates.

(b)




$x = 1\cos\frac{3\pi}{2} = 0$, $y = 1\sin\frac{3\pi}{2} = -1$, $z = 2$,

so the point is $(0, -1, 2)$ in rectangular coordinates.

3. (a) $r^2 = x^2 + y^2 = 1^2 + (-1)^2 = 2$ so $r = \sqrt{2}$; $\tan\theta = \dfrac{y}{x} = \dfrac{-1}{1} = -1$ and the point $(1, -1)$ is in the fourth quadrant of the $xy$-plane, so $\theta = \frac{7\pi}{4} + 2n\pi$; $z = 4$. Thus, one set of cylindrical coordinates is $\left(\sqrt{2}, \frac{7\pi}{4}, 4\right)$.

(b) $r^2 = (-1)^2 + \left(-\sqrt{3}\right)^2 = 4$ so $r = 2$; $\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$ and the point $\left(-1, -\sqrt{3}\right)$ is in the third quadrant of the $xy$-plane, so $\theta = \frac{4\pi}{3} + 2n\pi$; $z = 2$. Thus, one set of cylindrical coordinates is $\left(2, \frac{4\pi}{3}, 2\right)$.

4. (a) $r^2 = \left(2\sqrt{3}\right)^2 + 2^2 = 16$ so $r = 4$; $\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$ and the point $\left(2\sqrt{3}, 2\right)$ is in the first quadrant of the $xy$-plane, so $\theta = \frac{\pi}{6} + 2n\pi$; $z = -1$. Thus, one set of cylindrical coordinates is $\left(4, \frac{\pi}{6}, -1\right)$.

(b) $r^2 = 4^2 + (-3)^2 = 25$ so $r = 5$; $\tan\theta = \frac{-3}{4}$ and the point $(4, -3)$ is in the fourth quadrant of the $xy$-plane, so $\theta = \tan^{-1}\left(-\frac{3}{4}\right) + 2n\pi \approx -0.64 + 2n\pi$; $z = 2$. Thus, one set of cylindrical coordinates is $\left(5, \tan^{-1}\left(-\frac{3}{4}\right) + 2\pi, 2\right) \approx (5, 5.64, 2)$.

5. Since $\theta = \frac{\pi}{4}$ but $r$ and $z$ may vary, the surface is a vertical half-plane including the $z$-axis and intersecting the $xy$-plane in the half-line $y = x$, $x \geq 0$.

6. Since $r = 5$, $x^2 + y^2 = 25$ and the surface is a circular cylinder with radius 5 and axis the $z$-axis.

203 §16.7 #s 7–13, 15, 18, 20, 27

#s 7, 8 Identify the surface

7 $z = 4 - r^2 = -x^2 - y^2 + 4$ is circular paraboloid with vertex $(0,0,4)$, axis of symmetry $z$-axis, opens down.

8 $2r^2 + z^2 = 1 \Longrightarrow 2x^2 + 2y^2 + z^2 = 1$ is an ellipsoid centered @ $(0,0,0)$.

#s 9, 10 Write eq'n in cylindrical coords.

9 (a) $z = x^2 + y^2 = r^2$ $\boxed{z = r^2}$

(b) $x^2 + y^2 = 2y$

$r^2 = 2r\sin\theta \longrightarrow \boxed{r = 2\sin\theta}$

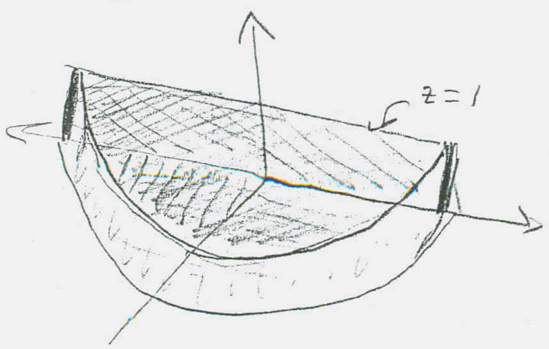
10 $3x + 2y + z = 6$

$\Longrightarrow 3r\cos\theta + 2r\sin\theta + z = 6$

$\Longrightarrow \boxed{z = 6 - r(3\cos\theta + 2\sin\theta)}$

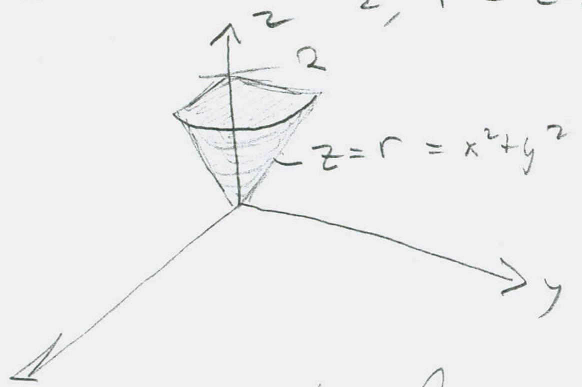
#s 11, 12 Sketch the region

11 $0 \le r \le 2,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ 0 \le z \le 1$



203 § 16.7 #s 12, 13, 15, 18, 20, 27

(12) $0 \le \theta \le \frac{\pi}{2}, \; r \le z \le 2$




part of a cone

(13) Cylindrical Shell is 20 cm long, inner radius 6 cm & outer radius 7 cm. Describe in an appropriate coordinate system

$\{(r,\theta,z) \mid 0 \le z \le 20, \; 6 \le r \le 7, \; 0 \le \theta \le 2\pi \}$

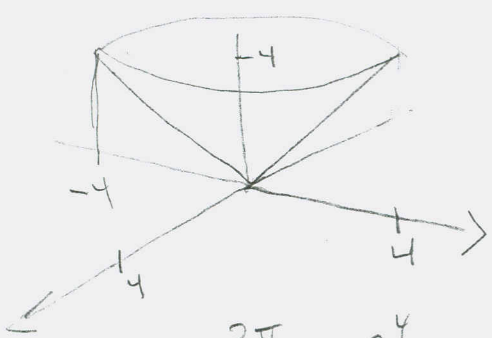
#s 15, 16 Sketch solid whose volume is given

$$\int_0^4 \int_0^{2\pi} \int_r^4 r \, dz \, d\theta \, dr$$

$r \le z \le 4$

$0 \le \theta \le 2\pi$

$0 \le r \le 4$




$$= \int_0^4 \int_0^{2\pi} \Big[ rz \Big]_{z=r}^{z=4} d\theta \, dr$$

$$= \int_0^4 \int_0^{2\pi} (4r - r^2) \, d\theta \, dr$$

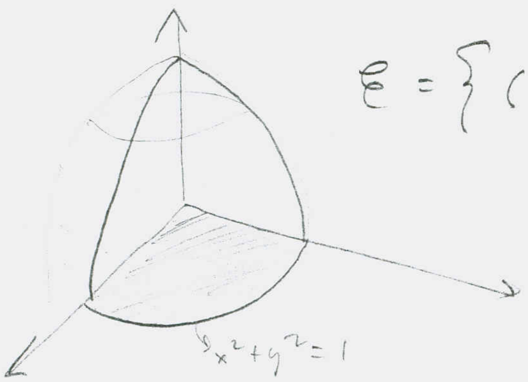
$$= \int_0^{2\pi} d\theta \int_0^4 (4r - r^2) \, dr \quad = \; \ldots \; = \boxed{\frac{64\pi}{3}}$$

203 §16.7 #s 18, 20, 27

#s 17-26 Use cylindrical coordinates

18 $\iiint_E (x^3+xy^2)\,dV$, where E

E = solid in 1st octant beneath $z = 1 - x^2 - y^2$




$E = \{(r,\theta,z) \mid 0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le 1,\ 0 \le z \le 1-r^2\}$

$$\int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=1} \int_{z=0}^{z=1-r^2} (\underbrace{r^3\cos^3\theta}_{\cos\theta(\cos^2\theta)} + r^3\cos\theta\sin^2\theta)\, r\,dz\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{1-r^2} r^4 \cos\theta \, r\,dr\,d\theta = \ldots = \boxed{\frac{2}{35}}$$

20 $\iiint_E x\,dV$ E is enclosed by the planes $z=0$, $z = x+y+5$, and the cylinders $x^2+y^2=4$ and $x^2+y^2=9$

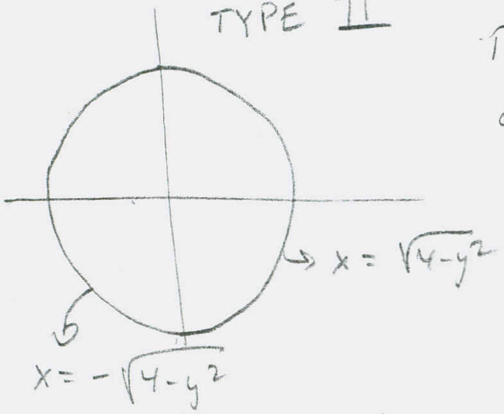
$z=0$, $z = r\cos\theta + r\sin\theta + 5$, $r=2$, $r=3$

$$\int_0^{2\pi} \int_2^3 \int_0^{r\cos\theta + r\sin\theta + 5} r\cos\theta \cdot r\,dz\,dr\,d\theta = \ldots = \boxed{\frac{65}{4}\pi}$$

203 §16.7 #27

27 Change to cylindrical coordinates and evaluate.

$$\underbrace{\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}}_{\text{TYPE II}} \int_{\underbrace{\sqrt{x^2+y^2}}}^{2} xz \, dz\, dx\, dy$$




The region above $z = \sqrt{x^2+y^2} = r$ (cone) and below $z = 2$

$$\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\int_{z=r}^{2} r\cos\theta \, z \, r \, dz\, dr\, d\theta$$

$= \ldots = 0.$