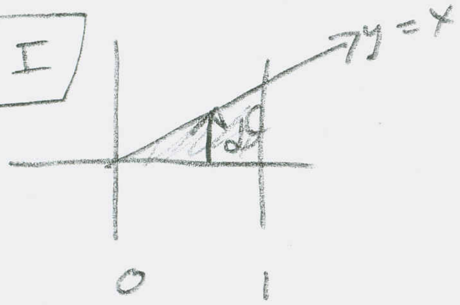
203 §16.6 #5 10, 12, 13, 15, 19, $\boxed{26B}$

#s 9–18 Evaluate the triple integral

(10) $\iiint_{E} yz\cos(x^5)\,dV$, where

$E = \{(x,y,z) \mid 0 \le x \le 1,\ 0 \le y \le x,\ x \le z \le 2x\}$

$\boxed{\text{TI}}$



$$\int_{x=0}^{x=1}\int_{y=0}^{y=x}\int_{z=x}^{z=2x} yz\cos(x^5)\,dz\,dy\,dx$$

$$= \int_{x=0}^{x=1}\int_{y=0}^{y=1}\left[\tfrac{1}{2}yz^2\cos(x^5)\right]_{z=x}^{z=2x} dy\,dx$$

$$= \tfrac{1}{2}\int_{x=0}^{x=1}\int_{y=0}^{y=x}\left(y(2x)^2\cos(x^5) - yx^2\cos(x^5)\right)dy\,dx$$

$$= \tfrac{1}{2}\int_0^1\int_0^x x^2 y\cos(x^5)\,dy\,dx = \tfrac{1}{4}\int_0^1\left[x^2y^2\cos(x^5)\right]_{y=0}^{y=x} dx$$

$$= \tfrac{1}{4}\int_0^1 x^4\cos(x^5)\,dx = \tfrac{1}{4}\cdot\tfrac{1}{5}\int_0^1 \cos(x^5)(5x^4dx)$$

$$= \tfrac{1}{20}\left[\sin(x^5)\right]_0^1 = \boxed{\tfrac{1}{20}\sin(1)}$$

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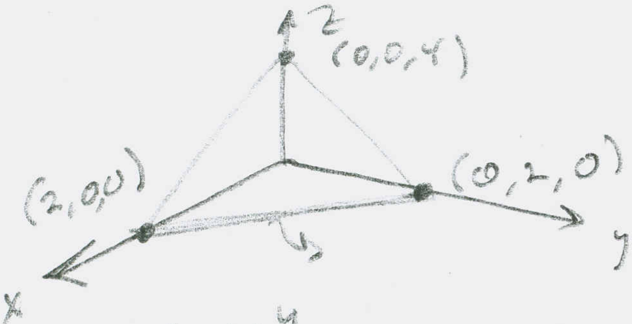
(12) $\iiint_E y \, dV$, E is bdd by the planes

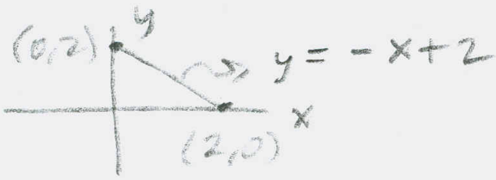
$x=0,\ y=0,\ z=0$ & $2x+2y+z=4$

$x=0: \ z=-2y+4$

$y=0: \ z=-2x+4$

$z=0: \ y=-x+2$





$$\therefore \int_0^2 \int_0^{-x+2} \int_0^{-2x-2y+4} y \, dz \, dy \, dx$$

$$= \int_0^2 \int_0^{-x+2} \Big[yz \Big]_{z=0}^{-2x-2y+4} dy \, dx = \int_0^2 \int_0^{-x+2} y(-2x-2y+4) \, dy \, dx$$

$$= \int_0^2 \int_0^{-x+2} (-2xy-2y^2+4y) \, dy \, dx = \int_0^2 \left(-xy^2 - \frac{2}{3}y^3 + 2y^2\right)\Big|_0^{-x+2} dx$$

$$= \int_0^2 \left(-x(-x+2)^2 - \frac{2}{3}(-x+2)^3 + 2(-x+2)^2\right) dx$$

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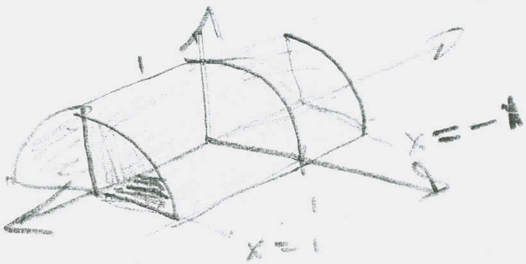
$$= \int_0^2 \left((-x+2)^2(-x+2) - \frac{2}{3}(-x+2)^3 \right) dx$$

$$= \int_0^2 (-x+2)^3 \left(1 - \frac{2}{3}\right) dx = \frac{1}{3}\int_0^2 (2-x)^3 dx$$

$$= \left(\frac{1}{3}\right)\left(-\frac{1}{4}\right)\left[(2-x)^4\right]_0^2 = -\frac{1}{12}(0-16) = \boxed{\frac{4}{3}}$$

(13) $\iiint_E x^2 e^y \, dV$ E is bdd by $z = 1-y^2$, $z=0$, $x=1$, $x=-1$



$$\int_{-1}^1 \int_0^1 \int_0^{1-y^2} x^2 e^y \, dz \, dy \, dx$$

$$= \int_{-1}^1 x^2 \int_{-1}^1 e^y \int_0^{1-y^2} dz \, dy \, dx = \int_{-1}^1 x^2 \int_{-1}^1 e^y(1-y^2) \, dy \, dx$$

$$= \int_{-1}^1 x^2 \, dx \int_{-1}^1 (e^y - y^2 e^y) \, dy = \dots$$

$$= 2\int_0^1 x^2 \, dx \left[e^y - (y^2 - 2y + 2)e^y\right]_{-1}^1 = \dots = \boxed{\frac{8}{3e}}$$

(15) Very similar to #10 only You have to build eqn of plane thru 3 pts.

ANS: $\boxed{\frac{1}{60}}$

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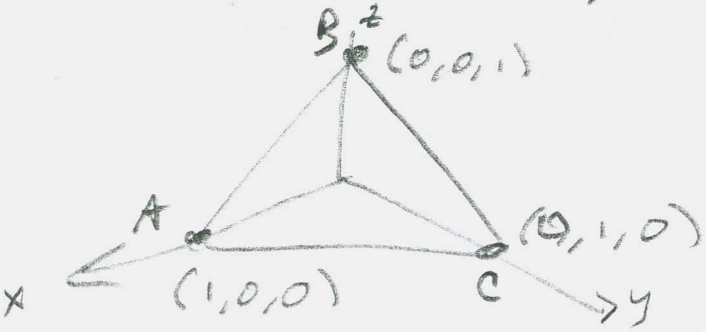
$\boxed{\text{15 cnt'd}}$

(0,0,1



$\overrightarrow{AB} = \langle -1, 0, 1 \rangle = \bar{u}$

$\overrightarrow{AC} = \langle -1, 1, 0 \rangle = \bar{v}$

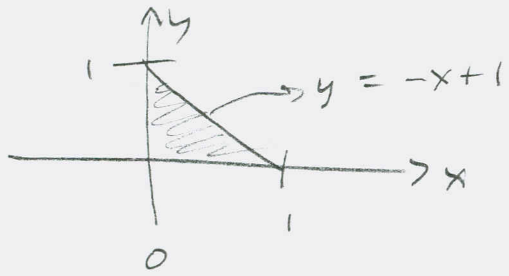
$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix} = -1i - 1j - 1k = \langle -1, -1, -1 \rangle$$

$= \bar{n} =$ normal vec.

$-1(x-1) - 1y - 1z = 0 \longrightarrow z = -x - y + 1$

is upper func.

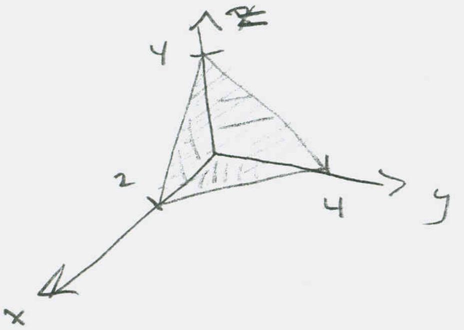
In xy-plane:



$$V = \int_{x=0}^{x=1} \int_{y=0}^{y=1-x} \int_{z=0}^{z=1-x-y} x^2 \, dz \, dy \, dx$$

$$= \ldots = \boxed{\frac{1}{60}}$$

$\boxed{\#19}$ Similar to what went before.



$$V = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz \, dy \, dx$$

$$= \boxed{\frac{16}{3}}$$