203 §16.3 #s 1, 8, 15, 19, 20, 31, 48, 53

① Evaluate $\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx \, dy$

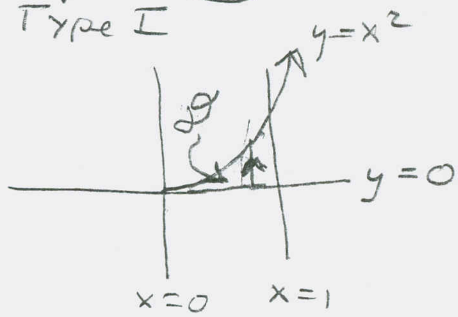
$$= \int_0^4 \left[\tfrac{1}{2} x^2 y^2 \right]_{x=0}^{x=\sqrt{y}} dy = \int_0^4 \tfrac{1}{2} \left[(\sqrt{y})^2 y^2 - 0 \right] dy$$

$$= \tfrac{1}{2} \int_0^4 y \cdot y^2 \, dy = \tfrac{1}{2} \int_0^4 y^3 \, dy = \tfrac{1}{8} \left[y^4 \right]_0^4 = \tfrac{1}{8} \left[256 - 0 \right] = \boxed{32}$$

#s 7–18 Evaluate the double integral

⑧ $\iint_{\mathcal{D}} \frac{y}{x^5+1} \, dA$, where $\mathcal{D} = \{ (x,y) \mid \underbrace{0 \le x \le 1,\ 0 \le y \le x^2}_{\text{Type I}} \}$

$$\int_{x=0}^{x=1} \int_{y=0}^{y=x^2} \frac{y}{x^5+1} \, dy \, dx = \int_{x=0}^{x=1} \left[\tfrac{1}{2} y^2 \cdot \frac{1}{x^5+1} \right]_{y=0}^{y=x^2} dx$$



$$= \tfrac{1}{2} \int_0^1 \frac{1}{x^5+1} \left((x^2)^2 - (0^2)^2 \right) dx$$

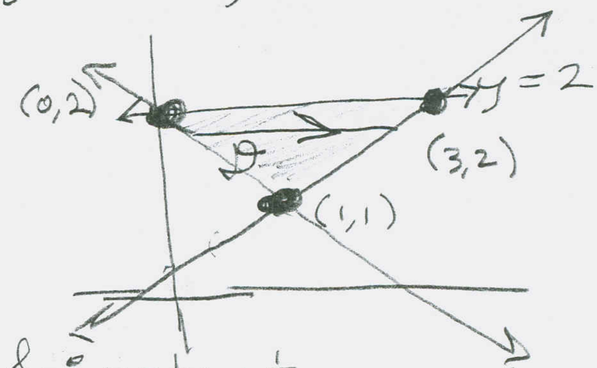
$$= \tfrac{1}{2} \cdot \tfrac{1}{5} \int_0^1 \frac{5x^4 \, dx}{x^5+1} = \tfrac{1}{10} \left[\ln(x^5+1) \right]_{x=0}^{x=1}$$

$$= \tfrac{1}{10} \left[\ln 2 - \ln 1 \right] = \boxed{\tfrac{1}{10} \ln 2}$$

We got lucky with the y^2 ending up as an x^4 to be the $du = 5x^4 \, dx$ to go with the x^5+1 in the denominator. Otherwise, we might try re-writing the whole thing as Type II.

203 § 16.3 #s 15, 19, 20, 31, 48, 53

15 $\iint_D y^3\,dA$, where D is right triangle with vertices $(0,2)$, $(1,1)$, $(3,2)$.



Equation for l_1:

$(1,1)$, $(3,2)$

$m = \frac{2-1}{3-1} = \frac{1}{2}$

$y = \frac{1}{2}(x-1)+1$

$y = \frac{1}{2}x + \frac{1}{2}$

l_2:

$(0,2)$, $(1,1)$

$m = \frac{1-2}{1-0} = -1$

$y = -x+2$

l_1: $y = \frac{1}{2}x + \frac{1}{2}$

l_2: $y = -x+2$

Should've used the picture to be thinking Type II sooner, Steve!

Type II lets us use one double integral

l_1: $y = \frac{1}{2}x+\frac{1}{2} \Rightarrow 2y = x+1 \Rightarrow \underline{x = 2y-1}$ right end

l_2: $y = -x+2 \Rightarrow \underline{x = -y+2}$ left end

$$\int_{y=1}^{y=2}\int_{x=-y+2}^{x=2y-1} y^3\,dx\,dy = \int_{y=1}^{y=2} y^3\Big[x\Big]_{x=-y+2}^{x=2y-1}\,dy$$

$$= \int_{y=1}^{y=2} y^3\Big[2y-1-(-y+2)\Big]\,dy = \int_{y=1}^{y=2} y^3(3y-3)\,dy$$

$$= \int_{y=1}^{y=2}(3y^4-3y^3)\,dy = 3\left[\frac{1}{5}y^5 - \frac{1}{4}y^4\right]_1^2$$

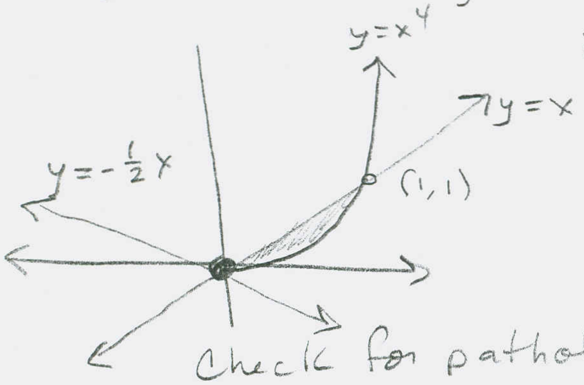
$$= \frac{3}{5}(2^5) - \frac{3}{4}(2^4) - \left(\frac{3}{5}-\frac{3}{4}\right) = \frac{96}{5} - \frac{48}{4} - \frac{3}{5} + \frac{3}{4}$$

$$= \frac{93}{5} - \frac{45}{4} = \boxed{\frac{147}{20}}$$

203 § 16.3 #s 19, 20, 31, 48, 53

#s 19–28 Find the volume of the given solid.

(19) under $x+2y-z=0$ and above $y=x$ & $y=x^4$



$z = x+2y = f(x,y)$

TYPE I

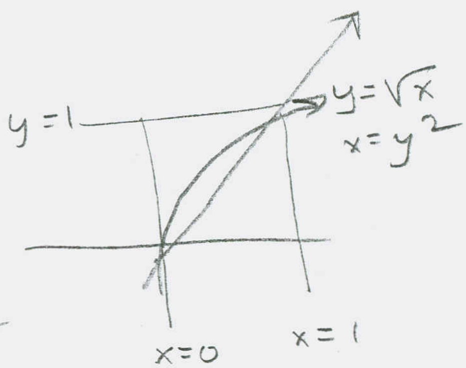
$$\int_{x=0}^{x=1}\int_{y=x^4}^{y=x} (x+2y)\,dy\,dx$$

Check for pathology: Is $z=x+2y$ actually above the xy-plane? Solve $z=0$, find this line of intersection in the xy-plane: $x+2y=0 \Longrightarrow y=-\frac{1}{2}x$. It doesn't impinge on $\mathcal{D}$. <u>GOOD</u>.

$$\iint = \int_{x=0}^{x=1}\left[xy+y^2\right]_{y=x^4}^{y=x} dx = \int_{x=0}^{x=1}\left[(x^2+x^2)-(x^5+x^8)\right]dx$$

$$= \left[\frac{2}{3}x^3-\frac{1}{6}x^6-\frac{1}{9}x^9\right]_0^1 = \frac{2}{3}-\frac{1}{6}-\frac{1}{9} = \frac{12-3-2}{18} = \boxed{\frac{7}{18}}$$

(20) We started this in class.



Type I:

$$\int_{x=0}^{x=1}\int_{y=x}^{y=\sqrt{x}} (2x+y^2)\,dy\,dx = \ldots \boxed{\frac{11}{60}}$$

Type II

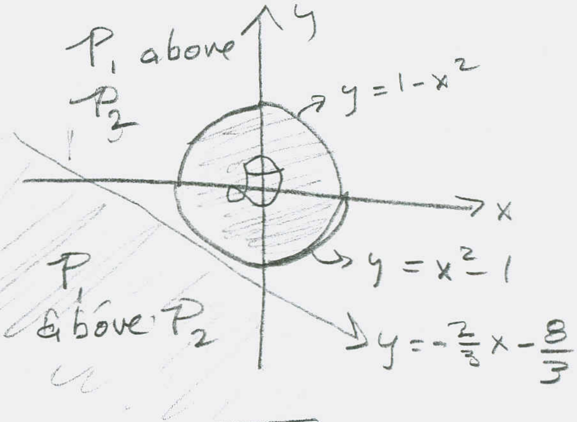
$$\int_{y=0}^{y=1}\int_{x=y^2}^{x=y} (2x+y^2)\,dx\,dy = \ldots = \boxed{\frac{11}{60}}$$

203 § 16.3 #s 31, 48, 53

(31) Find the volume by subtracting the volumes, where the solid is the one enclosed by the parabolic cylinders $y = 1 - x^2$, $y = x^2 - 1$ and the planes $x + y + z = 2$, $2x + 2y - z + 10 = 0$

P_1: $f(x,y) = z = -x - y + 2$

P_2: $f(x,y) = z = 2x + 2y + 10$



Check for pathology by finding where P_1 & P_2 meet and comparing this to what's happening in the xy-plane:

$-x - y + 2 = x + 2y + 10 \Longrightarrow \ldots$

$\Longrightarrow y = -\frac{2}{3}x - \frac{8}{3}$

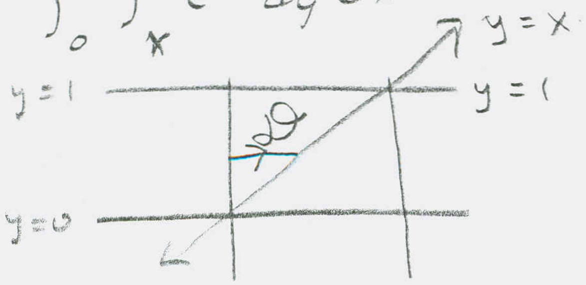
$\boxed{\text{TYPE I}}$

$$\text{Volume} = \int_{x=-1}^{x=1} \int_{y=x^2-1}^{y=1-x^2} \left(2x + 2y + 10 - (-x - y + 2)\right) dy\, dx$$

$$= \int_{x=-1}^{x=1} \int_{y=x^2-1}^{y=1-x^2} (3x + 3y + 8)\, dy\, dx = \ldots = \boxed{\frac{64}{3}}$$

(48) Switch order (makes integral more "doable") and evaluate

$\int_0^1 \int_x^1 e^{\frac{x}{y}}\, dy\, dx$ is evidently Type I. Re-do as II:



$$\int_{y=0}^{y=1} \int_{x=0}^{x=y} e^{\frac{x}{y}}\, dx\, dy$$

203 §16.3 #s 48, 53

(48) Cnt'd

$$= \int_{y=0}^{y=1} \int_{u=0}^{u=1} y e^{u} \, du \, dy = \int_{y=0}^{y=1} \left[y e^{u} \right]_{u=0}^{u=1} dy = \ldots = \boxed{\frac{e-1}{2}}$$

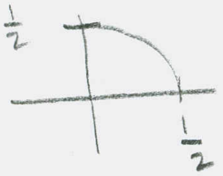
$u = \frac{x}{y}$, $du = \frac{1}{y} dx$, $y \, du = dx$

$x = 0 \Rightarrow u = 0$

$x = y \Rightarrow u = \frac{y}{1} = 1$

(53) Use properties of (double) integrals to estimate $\iint_Q e^{-(x^2+y^2)^2} dA$, where $Q = \{(x,y) \mid x^2+y^2 \le \frac{1}{4}, \ x, y \ge 0\}$

Q is "the quarter circle with center (0,0) and radius $\frac{1}{2}$ in the first quadrant.



$$\int_{x=0}^{x=\frac{1}{2}} \int_{y=0}^{y=\sqrt{\frac{1}{4}-x^2}} e^{-(x^2+y^2)^2} \, dy \, dx$$

e^{-u} is decreasing $\forall \, u \in \mathbb{R}$. We see that $u = x^2+y^2$ ranges from $u = 0$ to $u = \frac{1}{2}$ on the quarter-disk of $x^2+y^2 \le \frac{1}{4}$

The area of $\mathcal{D}$ is $\frac{1}{4}(\pi r^2) = \frac{1}{4}\pi\left(\frac{1}{4}\right) = \frac{\pi}{16}$

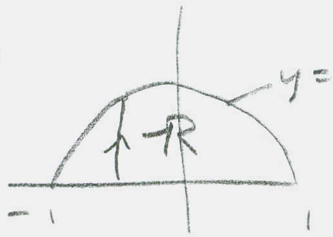
$$\therefore \boxed{e^{-\frac{1}{16}} \cdot \frac{\pi}{16} \le \iint_{\mathcal{D}} e^{-(x^2+y^2)^2} dA \le e^{-0} \frac{\pi}{16} = \frac{\pi}{16}}$$

$e^{-\left(\frac{1}{4}\right)^2} = e^{-\frac{1}{16}}$

MAT 203 §16.4 #s 2, 4, 5, 8, 11, 15, 18, 32, 36a

#s 1-4 A region is given. Decide whether rect. or polar coordinates is better for $\iint_R f(x,y)\,dA$.

(2)

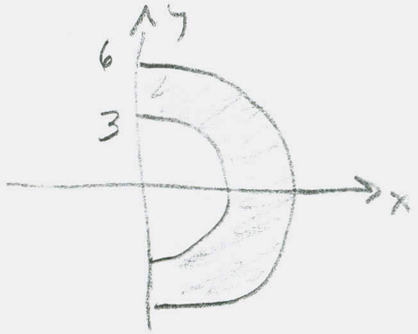


Rectangular

$R = \{(x,y) \mid -1 \le x \le 1,\ 0 \le y \le 1-x^2\}$

$$\iint_R f(x,y)\,dA = \int_{-1}^{1}\int_0^{1-x^2} f(x,y)\,dy\,dx$$

(4)



Polar.

$R = \{(r,\theta) \mid 3 \le r \le 6,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\}$

$$\iint_R f(x,y)\,dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_3^6 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

(5) Sketch region whose area is given by the integral & eval the int.

$$\int_{\pi}^{2\pi}\int_4^7 r\,dr\,d\theta$$