203 §16.2 #s 1, 4, 7, 10, 15, 19, 27, 31, 38

① Find $\int_0^5 f(x,y)\,dx$ & $\int_0^1 f(x,y)\,dy$

for $f(x,y) = 12x^2y^3$

$$\int_0^5 12x^2y^3\,dx = \frac{12}{3}\left[x^3y^3\right]_0^5 = 4\left[125y^3 - 0y^3\right]$$

$$= \boxed{500y^3}$$

$$\int_0^1 f(x,y)\,dy = \int_0^1 12x^2y^3\,dy = \frac{12}{4}x^2\left[y^4\right]_0^1 = \boxed{3x^2}$$

#s 3-14 Calculate the iterated integral.

④ $\int_0^1\int_1^2 (4x^3 - 9x^2y^2)\,dy\,dx$

$$= \int_0^1 \left[4x^3y - 3x^2y^3\right]_{y=1}^{y=2} dx$$

$$= \int_0^1 \left(8x^3 - 24x^2 - (4x^3 - 3x^2)\right) dx$$

$$= \int_0^1 (4x^3 - 21x^2)\,dx = \left[x^4 - 7x^3\right]_0^1 = 1 - 7 = \boxed{-6}$$

203 § 16.2 #s 7, 10, 15, 19, 27, 31, 38

7. $$\int_0^2 \int_0^1 (2x+y)^8 \, dx \, dy$$

$u = 2x + y$

$du = 2\,dx$ me guy

$\frac{du}{2} = dx$ you guys

$$= \int_0^2 \frac{1}{2} \int_0^1 (2x+y)^8 \cdot 2\,dx \, dy$$

$$= \frac{1}{2} \int_0^2 \frac{1}{9} \left[(2x+y)^9 \right]_0^1 dy$$

$$= \frac{1}{18} \int_0^2 \left[(2+y)^9 - y^9 \right] dy$$

$$= \frac{1}{18} \left[\frac{1}{10}(2+y)^{10} - \frac{1}{10}(y)^{10} \right]_0^2$$

$$= \frac{1}{180} \left[4^{10} - 2^{10} - (2^{10} - 0) \right]$$

$$= \frac{1}{180} \left[4^{10} - 2^{10} - 2^{10} \right] = \frac{1046528}{180} = \boxed{\frac{261,632}{45} = 5814.0\overline{4}}$$

$$= \frac{1}{180} \left[(2 \cdot 2)^{10} - 2(2^{10}) \right]$$

$$= \frac{1}{180} \left[2^{10} \cdot 2^{10} - 2 \cdot 2^{10} \right] = \frac{1}{180} \left[2^{10} (2^{10} - 2) \right]$$

203 §16.2 #s 10, 15, 19, 22, 31, 38

(10) $\int_0^1 \int_0^3 e^{x+3y}\,dx\,dy$

$= \int_0^1 \int_0^3 e^x e^{3y}\,dx\,dy$

$= \int_0^1 e^{3y}\,dy \int_0^3 e^x\,dx$

$= \left[\frac{1}{3}e^{3y}\right]_0^1 \left[e^x\right]_0^3$

$= \frac{1}{3}\left[e^3 - e^0\right]\left[e^3 - e^0\right]$

$= \boxed{\frac{1}{3}\left[e^3 - 1\right]^2} = \boxed{\frac{1}{3}\left[e^6 - 2e^3 + 1\right]}$

Either one is OK

#s 15-22 Evaluate the double integral

(15) $\iint_R (6x^2y^3 - 5y^4)\,dA$, $R = [0,3] \times [0,1]$

$= \int_0^1 \int_0^3 (6x^2y^3 - 5y^4)\,dx\,dy = \int_0^1 \left[2x^3y^3 - 5xy^4\right]_0^3 dy$

$= \int_0^1 (54y^3 - 15y^4)\,dy = \left[\frac{54}{4}y^4 - 3y^5\right]_0^1 = \frac{27}{2} - 3$

$= \boxed{\frac{21}{2}}$

203 §16.2 #s 19, 22, 31, 38

19 ——〃——

22 $\iint \frac{x}{x^2+y^2}\, dA \quad R = [1,2] \times [0,1]$

$= \int_0^1 \frac{1}{2} \int_1^2 (x^2+y^2)^{-1}\, 2x\, dx\, dy$ $\quad u = x^2+y^2$, $du = 2x\,dx$

$= \int_0^1 \frac{1}{2} \left[\ln(x^2+y^2) \right]_1^2 dy$

$= \frac{1}{2} \int_0^1 \left(\ln(y^2+4) - \ln(y^2+1) \right) dy$

$= \frac{1}{2} \int_0^1 \ln(y^2+4)\, dy \quad - \quad \frac{1}{2} \int_0^1 \ln(y^2+1)\, dy$

$u = \ln(y^2+4) \quad dv = dy$

$du = \frac{2y}{y^2+4} \quad v = y$

$$\begin{array}{r} 2 \quad r -8 \\ y^2+4 \,\big)\, \overline{2y^2+0y+0} \\ -(2y^2 \quad +8) \\ \hline -8 \end{array}$$

$uv - \int v\, du = y \ln(y^2+4) - \int \frac{2y^2}{y^2+4}\, dy$

$= y \ln(y^2+4) - \int \left(2 - \frac{8}{y^2+4} \right) dy$

$= y \ln(y^2+4) - 2y + \frac{8}{2} \arctan\left(\frac{y}{2}\right)$

$= y \ln(y^2+4) - 2y + 4 \arctan\left(\frac{y}{2}\right)$

2nd Integral By similar moves

This gives

$\iint_R f\, dA = \frac{1}{2} \left[y \ln(y^2+4) - 2y + 4 \arctan\left(\frac{y}{2}\right) \right]_0^1 - \underbrace{\frac{1}{2} \left[y \ln(y^2+1) - 2y - 2\arctan(y) \right]_0^1}_{}$

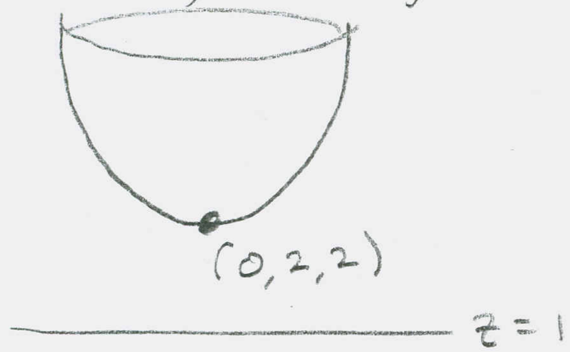
$= \frac{1}{2} \left[\ln 5 - \ln 2 + 4\arctan\left(\frac{1}{2}\right) - 2\frac{\pi}{4} \right] = \boxed{\frac{1}{2} \ln\left(\frac{5}{2}\right) + 2\arctan\left(\frac{1}{2}\right) - \frac{\pi}{2}}$

203 S" 16.2 #s 31, 38

(31) Find volume enclosed by the paraboloid

$z = 2 + x^2 + (y-2)^2$ and the planes

$z = 1$, $x = 1$, $x = -1$, $y = 0$, and $y = 4$



$x = 1$:

$z = 3 + (y-2)^2$

Trace in $x=1$ plane

$x = -1$:

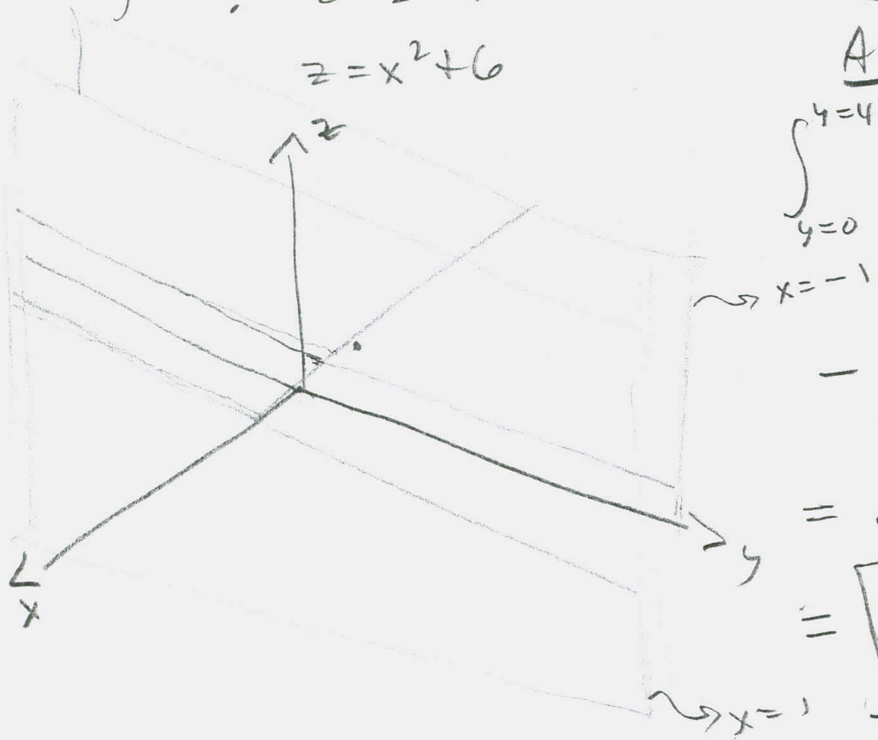
$z = 3 + (y-2)^2$ is the trace

$y = 0$: is x-z plane, ~~lying~~

Its trace is $z = x^2 + 2$

$y = 4$: $z = 2 + x^2 + 2^2$

$z = x^2 + 6$



Apparently, we're

Below the paraboloid

Above $z = 1$

$$\int_{y=0}^{y=4} \int_{x=-1}^{x=1} \left(2 + x^2 + (y-2)\right)^2 \, dx\, dy$$

$$- \int_0^4 \int_{-1}^{1} 1 \, dx\, dy$$

$= \ldots$

$= \boxed{\dfrac{64}{3}}$

203 §16.2 # 38

38 a How are Fubini's and Clairaut's Theorems similar?

$$\text{Fubini: } \iint f\,dx\,dy = \iint f\,dy\,dx$$

$$\text{Clairaut: } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

They're simply the integral & derivative versions of the same principle.

Both theorems require continuity of f. Clairaut requires smoothness, too.

b If f is cnts on $[a,b] \times [c,d]$ &

$$g(x,y) = \int_a^x \int_c^y f(s,t)\,ds\,dt \quad \text{on } (a,b) \times (c,d),$$

show that $g_{xy} = g_{yx} = f$.

$$g_x = \int_c^y f(x,t)\,ds \Longrightarrow g_{xy} = f(x,y)$$

By Fubini, we have

$$g(x,y) = \int_c^y \int_a^x f(s,t)\,ds\,dt \Longrightarrow g_y = \int_a^x f(s,y)\,ds$$

$$\Longrightarrow g_{yx} = f(x,y) = g_{xy} \; \blacksquare !$$