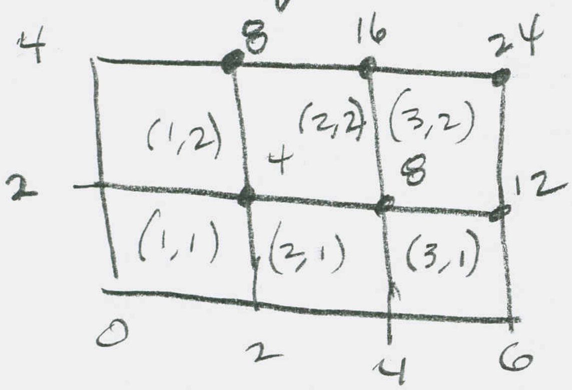
203 §16.1 #5 1, 5, 7, 8, 11, 14

① (a) Estimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle $R = \{(x,y) \mid 0 \le x \le 6, 0 \le y \le 4\}$. Use a Riemann sum, with $m=3$, $n=2$ & take the sample point to be the upper right corner of each square



$\Delta A = \Delta x \Delta y = (2)(2) = 4$

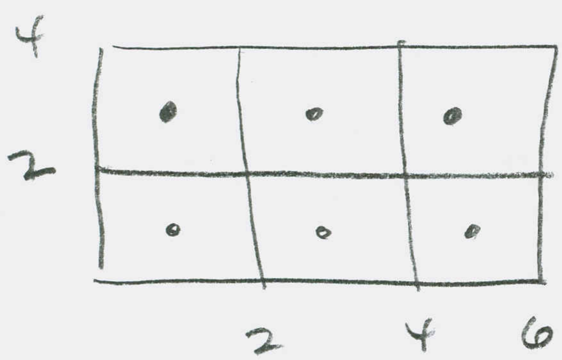
$$V \approx \sum_{i=1}^{3} \sum_{j=1}^{2} f(x_i^*, y_j^*) \Delta A$$

$$= \left(f(x_1,y_1) + f(x_1,y_2) + f(x_2,y_1) + \cdots + f(x_3,y_2) \right) \Delta A$$

$$V \approx (4+8+8+16+12+24)(4)$$

$$= (72)(4) = \boxed{288 \approx V}$$

(b) Use Midpoint Rule:



$f(1,1) = 1$

$f(1,3) = 3$

$f(3,1) = 3$

$f(3,3) = 9$

$f(5,1) = 5$

$f(5,3) = 15$

Pretty big discrepancy!

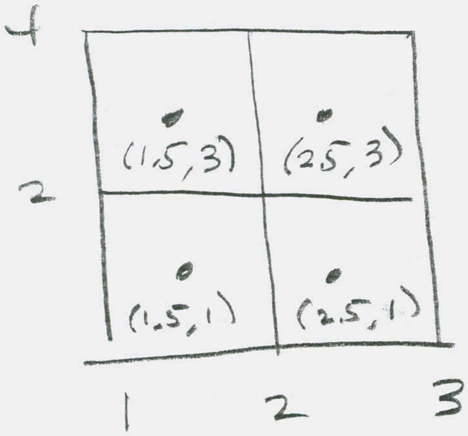
$$\sum_{i=1}^{3} \sum_{j=1}^{2} f(\bar{x}_i, \bar{y}_i) \Delta A = (36)(4) = \boxed{144 \approx V}$$

203 §16.1 #s 5, 7, 8, 11, 14

(5) A table is given for $f(x,y)$ on $[1,3]\times[0,4] = R$

(a) Estimate $\iint_R f(x,y)\,dA$ with $m=n=2$

x \ y	0	1	2	3	4
1.0	2	0	-3	-6	-5
1.5	3	1	-4	-8	-6
2.0	4	3	0	-5	-8
2.5	5	5	3	-1	-4
3.0	7	8	6	3	0



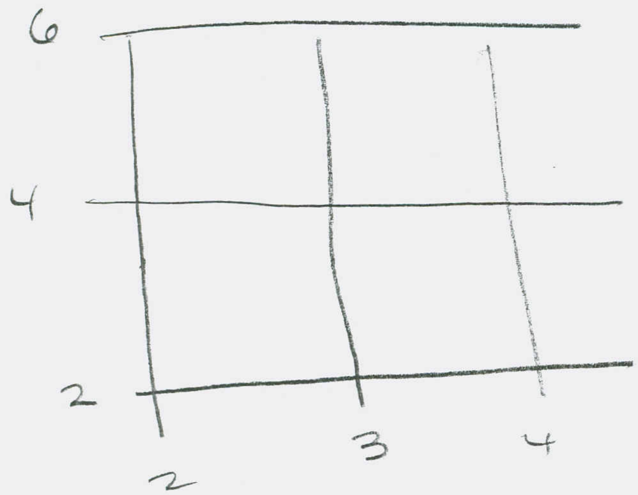
$$\sum_{i=1}^{2}\sum_{j=1}^{2} f(x_i, y_j)\,\Delta A$$

$$= \big(f(1.5,1) + f(1.5,3) + f(2.5,1) + f(2.5,3)\big)(2)$$

$$= \big(1 + (-8) + 5 + (-1)\big)(2) = (-3)(2) = \boxed{-6 \approx \iint_R f(x,y)\,dA}$$

203 § 16.1 #s 7, 8, 11, 14

(7) Let V = Volume beneath $f(x,y)=\sqrt{52-x^2-y^2}$ and above the rectangle $[2,4]\times[2,6]$

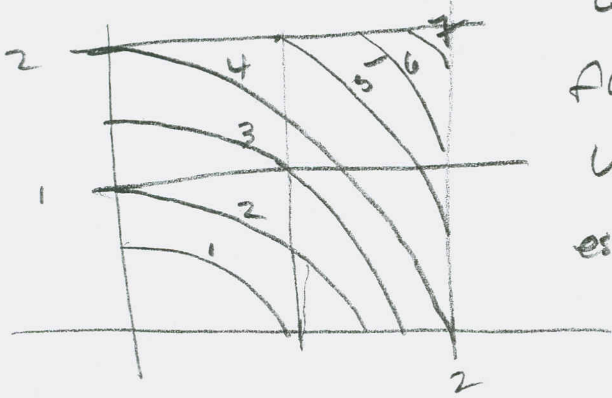


L = Riemann sum for lower left corners.

U = Riemann sum for upper right corners

$U < V < L$, because $f(x,y)$ decreases as x, y increase.

(8) Level Curves for $f(x,y)$ on $[1,2]\times[1,2]$ Use midpoint rule to estimate $\iint_R f(x,y)\,dA$, with $m=n=2$

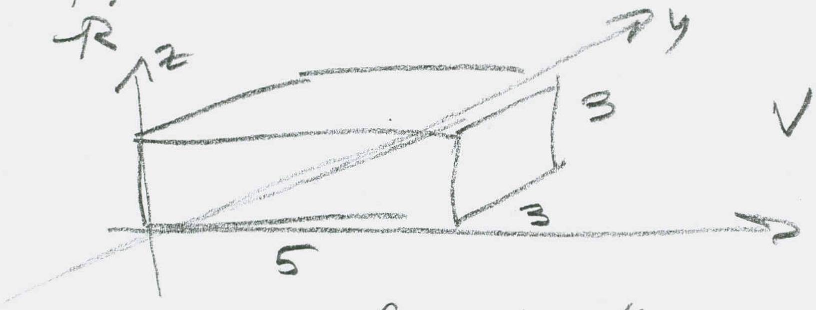


$$\sum_{i=1}^{2}\sum_{j=1}^{2} f(\bar{x}_i,\bar{y}_j)\,\Delta A \approx (1.3+3.3+3.2+5.2)(1)$$
$$=(13.0)(1)=\boxed{13 \approx \iint_R f(x,y)\,dA}$$

203 §16.1 #s 11, 14

#s 11-13 Evaluate the integral by identifying it as the volume of a solid.

(11) $\iint_R 3\,dA$, where $R = \{(x,y) \mid 0 \le x \le 5, 0 \le y \le 3\}$



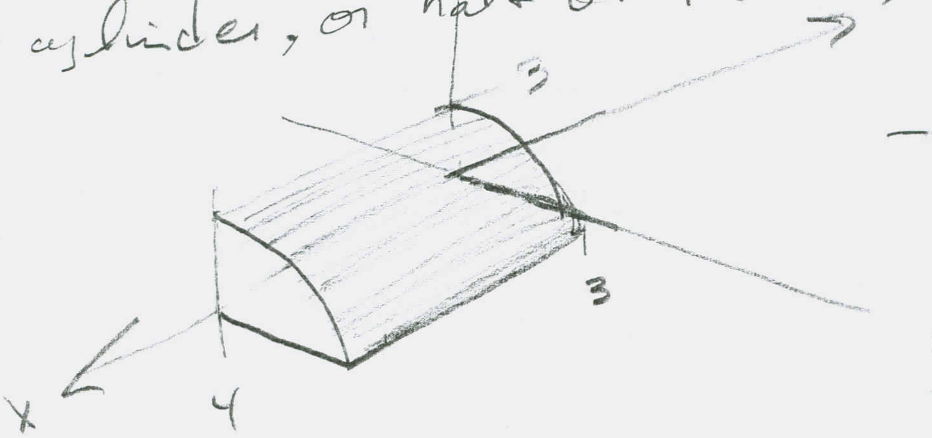
$V = (3)(3)(5)$
$= \boxed{30 = V}$

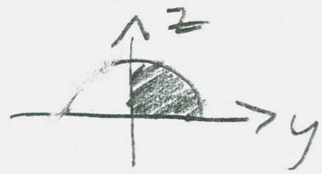
Rectangular "box"

(14) $\iint_R \sqrt{9-y^2}\,dA$, $R = [0,4] \times [0,2]$

is volume of a solid. What's the solid? Sketch it.

$f(x,y) = \sqrt{9-y^2}$ is top of a circular cylinder, or half of the top, any way.





y-z cross-section

$z = \sqrt{9-y^2}$