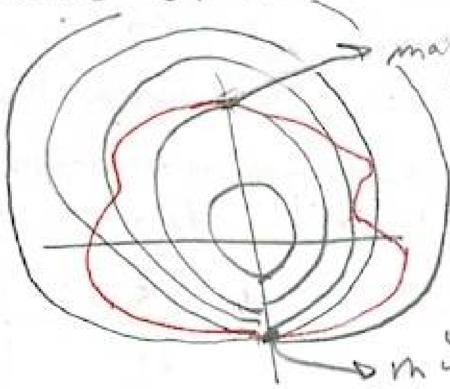
203 §14.8 #s 1, 3, 6, 10, 13, 14, 27, 28, 41, 45, 46

1, 3, 6, 10, 25, 27, 45 looks better.

Worked w/ graphing calculator.

(1) $g(x,y) = 8$ & contour map of $f(x,y)$ are shown. Estimate max & min.



max of 60

Looks like a min of 30 and max of 60

Constraint in RED.

$g(x,y)=8$

min of 30

(2) Graph $x^2+y^2=1$ & $x^2+y=c$ (i.e. $y=c-x^2$)

Find 2 curves that just touch the circle

Clearly $y=1-x^2$ is one.

$y=\sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}}$

$y_1' = (1-x^2)^{-\frac{1}{2}}(\frac{1}{2})(-2x) = \frac{-x}{\sqrt{1-x^2}}$

$y_2' = -2x$

$-2x = \frac{-x}{\sqrt{1-x^2}} \Rightarrow \frac{x-2x\sqrt{1-x^2}}{\sqrt{1-x^2}} = 0$

.69021243

My idea was to find $x \ni \sqrt{1-x^2}$ & $c-x^2$ had same slope. Then find $c \ni$ they touched there.

Not sure why it's not working.

$x(1-2\sqrt{1-x^2}) = 0$ $\quad x=0$ OR

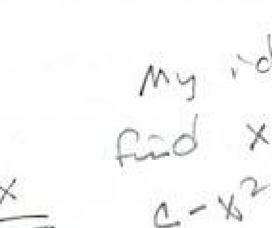
$2\sqrt{1-x^2} - 1 = 0$

$4(1-x^2) = 1$

$4-4x^2-1=0$

$-4x^2+3=0$

$x = \pm\frac{\sqrt{3}}{2}$



$y=c-x^2$

$y(\frac{\sqrt{3}}{2}) = c - \frac{3}{4} = \frac{1}{2} \Rightarrow$

$c = \frac{5}{4}$

203 §4.14.0 #s 2, 3, 6, 10, 13, 14, 27, 29, 41, 45, 46

#2 cont'd

$y_1 = \sqrt{1-x^2}$ = top ½ of circle $x^2+y^2=1$

$y_1 = (1-x^2)^{\frac{1}{2}} \Rightarrow y_1' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}}$

$y_2 = c - x^2 \Rightarrow y_2' = -2x$

Set $-2x = \frac{-x}{\sqrt{1-x^2}} \Rightarrow$

$\frac{x}{\sqrt{1-x^2}} - \frac{2x}{1} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} = \frac{x(1-2\sqrt{1-x^2})}{LCD} = 0 \Rightarrow$

$2\sqrt{1-x^2} - 1 = 0 \Rightarrow$

$2\sqrt{1-x^2} = 1 \Rightarrow 4 - 4x^2 = 1 \Rightarrow -4x^2 = -3$

$\Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \pm\frac{\sqrt{3}}{2}$ SAME.

Want $c - x^2 = \sqrt{1-x^2}$ @ $x = \pm\frac{\sqrt{3}}{2} \Rightarrow$

$c - \frac{3}{4} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow$

$c = \frac{3}{4} + \frac{1}{2} = \frac{3+2}{4} = \frac{5}{4}$. Same as before.

So $y_2 = \frac{5}{4} - x^2$

$y_2' = -2x$

$y_2'\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$y_1'\left(\frac{\sqrt{3}}{2}\right) = \frac{-\frac{\sqrt{3}}{2}}{\sqrt{1-\frac{3}{4}}} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$ ✓

OK. ⊥ checks.

$y_1\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2}$

$y_2\left(\frac{\sqrt{3}}{2}\right) = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$ ✓

203 §14.8 #s 2, 3, 6, 10, 13, 14, 27, 29, 41, 45, 46

#2 cntd

(b) maximize/minimize $f(x,y) = x^2+y$, s.t.

$x^2+y^2=1 \quad g(x,y) = x^2+y^2$

$f_x = 2x = \lambda 2x \Rightarrow x(2-2\lambda) = 0$

$f_y = 1 = 2\lambda y \longrightarrow y = \frac{1}{2\lambda}$

$\Rightarrow x = 0$ OR $\lambda = 1$

$\lambda = 1 \Rightarrow y = \frac{1}{2}$. $y = \frac{1}{2} \Rightarrow x^2 + \frac{1}{4} = 1 \Rightarrow$

$\Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \pm\frac{\sqrt{3}}{2}$

Max of $\frac{5}{4}$ @ $(\pm\frac{\sqrt{3}}{2}, \frac{1}{2})$

Min of 1 @ $(0,1)$.

Answers to a & b are the same, basically.

#s 3-17 Use Lagrange multipliers to find max & min values

(3) $f(x,y) = x^2+y^2$; $xy = 1$

$f_x = 2x \overset{SET}{=} \lambda y \Rightarrow x = \frac{\lambda y}{2}$ OR $\lambda = \frac{2x}{y}$

$f_y = 2y = \lambda x \Rightarrow 2y = \lambda\left(\frac{\lambda y}{2}\right) = \frac{\lambda^2}{2} y \overset{SET}{\Rightarrow}$

$2y - \frac{\lambda^2}{2} y = y\left(2 - \frac{\lambda^2}{2}\right) = 0 \Rightarrow$

$\lambda^2 - 4 = 0 \Rightarrow \lambda = \pm 2 \Rightarrow$

$2x = \pm 2y \Rightarrow x = \pm y$ ALSO $x = \frac{1}{y} \Rightarrow$

$2y = \pm 2x \quad \frac{1}{y} = \pm y \Rightarrow$

$0 = \pm y - \frac{1}{y} = \frac{\pm y^2 - 1}{y} \Rightarrow y^2 - 1 = 0$

$\Rightarrow y = \pm 1 \Rightarrow x = \pm 1$

$(1,1), (1,-1), (-1,1), (-1,-1)$

203 §14.8 #s 3, 6, 10, 13, 14, 27, 29, 41, 45, 46

#3 cont'd

$f(\pm 1, 1) = 2$ Max

$f(\pm 1, -1) = 0$ Min

(6) $f(x,y) = e^{xy}$ st $x^3 + y^3 = 16$

$f_x = ye^{xy}$, $f_y = xe^{xy}$ $g_x = 3x^2$, $g_y = 3y^2$

Solving this analytically isn't easy!

$f_x = \lambda g_x \Rightarrow ye^{xy} = 3\lambda x^2$

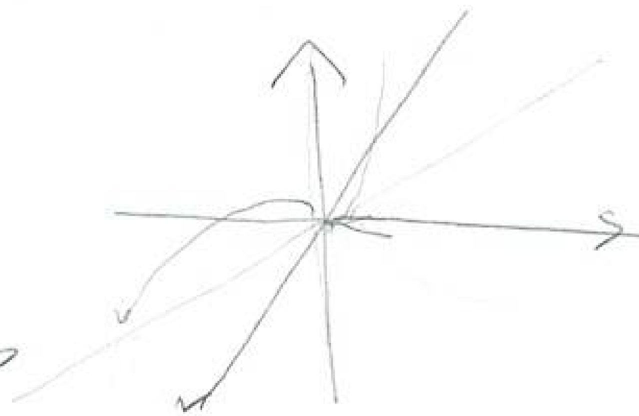
$- \quad f_y = \lambda g_y \Rightarrow xe^{xy} = 3\lambda y^2$

$(y-x)e^{xy} = (x^2 - y^2)(3\lambda)$

$-e^{xy} = 3\lambda(x-y)$

$yf_x = xye^{xy} = 3\lambda y x^2$

$-xf_y = -xye^{xy} = -3\lambda x y^2$



$= 0 \Rightarrow$

$\Rightarrow 3\lambda x^2 y - 3\lambda x y^2 = 0 \Rightarrow$

$3\lambda xy(x-y) = 0 \Rightarrow$

$x = 0, y = 0$, OR $x = y$

$x=0$: $f(0,y) = e^0 = 1$

$y=0$: $f(x,0) = e^0 = 1$

$x=y$: $e^{xy} = e^{x^2}$

Along the line $x=y$, i.e., the plane $x=y$, it becomes e^{x^2} s.t. $2x^3 = 16$

$\Rightarrow x^3 = 8$

$\Rightarrow x = 2$

$\Rightarrow y = 2$

$e^{xy} = e^4 \approx 54.5981500$ @ $(2,2)$

203 ~~5~~ 14, 8 ~~14 56~~ 10, 13, 14, 27, 29, 41, 45, 46

(10) $f(x,y,z) = x^2y^2z^2$, $x^2+y^2+z^2=1$

$f_x = 2xy^2z^2$, $f_y = 2x^2yz^2$, $f_z = 2x^2y^2z$

$f_x = 2xy^2z^2 = 2\lambda x = \lambda g_x$

$f_y = 2x^2yz^2 = 2\lambda y = \lambda g_y$

$f_z = 2x^2y^2z = 2\lambda z$

$x f_x = y f_y = z f_z = 2\lambda x^2 = 2\lambda y^2 = 2\lambda z^2$

so $x^2 = y^2 = z^2$

$x = y = z$

$\bar{r}_1 = \langle x, x, x \rangle t = \bar{u}_1 t$

$\bar{r}_2 = \langle x, -x, x \rangle t = \bar{u}_2 t$

$\bar{r}_3 = \langle x, -x, -x \rangle t = \bar{u}_3 t$

$\bar{r}_4 = \langle x, x, -x \rangle t = \bar{u}_4 t$

$x = t, y = t, z = t$

$x = t, y = -t, z = t$

$x = t, y = -t, z = -t$

$x = t, y = t, z = -t$

Also need

$x^2+y^2+z^2 = 1$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm \frac{\sqrt{3}}{3} = y = z.$

$\frac{\sqrt{3}}{3}\langle 1, 1, 1 \rangle$ | $\frac{\sqrt{3}}{3}\langle -1, 1, 1 \rangle$
$\frac{\sqrt{3}}{3}\langle 1, -1, 1 \rangle$ | $\frac{\sqrt{3}}{3}\langle -1, -1, 1 \rangle$
$\frac{\sqrt{3}}{3}\langle 1, -1, -1 \rangle$ | $\frac{\sqrt{3}}{3}\langle -1, -1, -1 \rangle$
$\frac{\sqrt{3}}{3}\langle 1, 1, -1 \rangle$ | $\frac{\sqrt{3}}{3}\langle -1, 1, -1 \rangle$

$f(x,y,z) = \left(\frac{\sqrt{3}}{3}\right)^6 = \frac{3^3}{3^6} = \boxed{\frac{1}{27}}$

min? Max?

Look @ Maple.

203 §14.8 #s 13, 14, 27, 29, 41, 45, 46

(13) $f(x,y,z,t) = x+y+z+t \; ; \; x^2+y^2+z^2+t^2=1$

$f_x = 1 = 2\lambda x$

$f_y = 1 = 2\lambda y$ $\quad x = y = z = t$

$f_z = 1 = 2\lambda z$ $\quad \Longrightarrow 4x^2 = 1$

$f_t = 1 = 2\lambda t$ $\quad \Longrightarrow x = \pm\frac{1}{2} = y, z, t$

$\boxed{f(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) = -2}$ MIN

$f(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) = -\frac{3}{2} = f(-\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2}) = f(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2})$
$= f(\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$

$f(-\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2}) = 0 = f(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2})$

$= f(\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2}) = f(\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2})$

$f(-\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}) = +\frac{1}{2} = f(\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}) = f(\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2})$

$= f(\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2})$

$\boxed{f(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}) = 2}$ MAX

(14) $f(x_1, x_2, \ldots, x_n) = \sum_{k=1}^{n} x_k$ s.t. $\sum_{k=1}^{n} x_k^2 = 1$

~~Max of~~

$n x_1^2 = 1 \Longrightarrow x = \pm\frac{\sqrt{n}}{n}$

Max of $\sum_{k=1}^{n} \frac{\sqrt{n}}{n} = \sqrt{n}$ @ $\bar{x} = (\frac{\sqrt{n}}{n}, \frac{\sqrt{n}}{n}, \ldots \frac{\sqrt{n}}{n})$

Min of $-\sum_{k=1}^{n} \frac{\sqrt{n}}{n} = -\sqrt{n}$ @ $\bar{x} = (-\frac{\sqrt{n}}{n}, -\frac{\sqrt{n}}{n}, \ldots -\frac{\sqrt{n}}{n})$

203 §14.8 #s 27, 29, 41, 45, 46

#s 27–39 Use Lagrange multipliers to solve 14.7 #s in

(27) 14.7 #39 Find distance from (2, 1, −1) to

$x+y-z=1 \Rightarrow z = x+y-1$ $\quad g(x,y) = x+y-1$

$d = \sqrt{(x-2)^2 + (y-1)^2 + (x+y-1+1)^2}$

$f(x,y) = (x-2)^2 + (y-1)^2 + (x-y)^2$

$f_x = 2(x-2) + 2(x-y) = \lambda$ $\quad 2x-4+2x-2y = 4x-2y-4 = \lambda$

$f_y = 2(y-1) - 2(x-y) = \lambda$ $\quad 2y-2-2x+2y = -2x+4y-2 = \lambda$

$\left[\begin{array}{cc|c} 4 & -2 & \lambda+4 \\ -2 & 4 & =\lambda+2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & -\frac{1}{2}\lambda - 1 \\ 2 & -1 & -\frac{1}{2}\lambda - 2 \end{array}\right]$ $\lambda + 2$, $-\frac{1}{2}\lambda - 2$

$\left[\begin{array}{cc|c} 1 & -2 & -\frac{1}{2}\lambda - 1 \\ 0 & 3 & \frac{1}{2}\lambda \end{array}\right]$ $\quad 3y = \frac{1}{2}\lambda$ $\quad y = \frac{1}{6}\lambda$

$x - 2(\frac{1}{6}\lambda) = -\frac{1}{2}\lambda - 1$

$x - \frac{1}{3}\lambda = -\frac{1}{2}\lambda - 1$

$6x - 2\lambda = -3\lambda - 6$

$6x = -\lambda - 6$

$x = \frac{-\lambda - 6}{6} = -\frac{\lambda}{6} - 1 = -y - 1$

$x = -y - 1$

$y = -x - 1$

$(x-1)^2 + (-x-2)^2 + (x-x$

$\sqrt{(x-1)^2 + (-x-2)^2 + 1^2}$

$\sqrt{(x-1)^2 + (-x-2)^2} + x^2+4x+4+1$

$= x^2 - 2x + 1 + x^2 + 4x + 4 + 1$

$= \sqrt{2x^2 + 2x + 6}$

$= \sqrt{2(x^2 + x + (\frac{1}{2})^2) - \frac{1}{2} + \frac{12}{2}}$

$= \sqrt{2(x^2+x+\ldots}$

$= \sqrt{2(x+\frac{1}{2})^2 + \frac{11}{2}}$

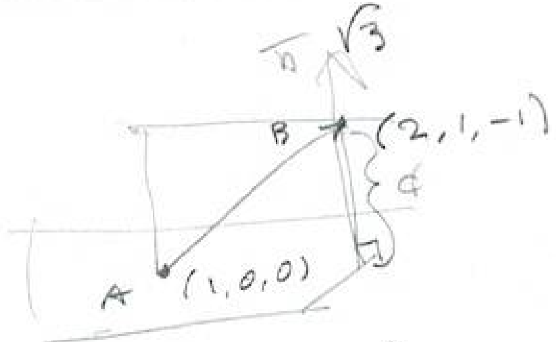
min @ $x = -\frac{1}{2}, y = -\frac{1}{2}$

of $\sqrt{\frac{11}{2}} = \frac{\sqrt{22}}{2}$

203 §14.8 #s 27, 29, 41, 45, 46

#27 CHECK BY FORMULA

$$\frac{|(1)(2)+(1)(1)+(-1)(-1)-1|}{} = \frac{3}{\sqrt{3}} = \sqrt{3}$$



$\bar{u} = \overrightarrow{AB} = \langle 1, 1, -1\rangle$

$$|\text{comp}_{\bar{n}}\bar{u}| = \frac{|\langle 1,1,-1\rangle \cdot \langle 1,1,-1\rangle|}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$ ✓

Lagrange Multipliers failed.

$d = \sqrt{(x-2)^2 + (y-1)^2 + (z+1)^2}$

$x + y - z = 1 \longrightarrow z = x + y - 1 = g(x,y)$

Hmmmm

$f = (x-2)^2 + (y-1)^2 + (z+1)^2$

$g = x + y - z$

$f_x = 2(x-2) = 2x-4 \overset{SET}{=} \lambda$ $\quad 2x - \lambda = 4$

$f_y = 2(y-1) = 2y-2 \overset{SET}{=} \lambda$ $\quad 2y - \lambda = 2$

$f_z = 2(z+1) = 2z+2 \overset{SET}{=} -\lambda$ $\quad 2z + \lambda = -2$

$x = \frac{1}{2}\lambda + 2$

$y = \frac{1}{2}\lambda + 1$

$z = -\frac{1}{2}\lambda - 2$

$$\left[\begin{array}{ccc|c|c} 2 & 0 & 0 & -1 & 4 \\ 0 & 2 & 0 & -1 & 2 \\ 0 & 0 & 2 & +1 & -2 \end{array}\right]$$

$-\frac{1}{2}\lambda - 2 = z = x + y - 1 = \lambda + 3 - 1 = \lambda + 2$

$-\lambda - 4 = 2\lambda + 4$

$-3\lambda = 00$

$\lambda = -00/3$

203 § 14.8 #s 27, 29, 41, 45, 46

#27 cont'd

$\lambda = -\frac{8}{3} \Rightarrow x = \frac{1}{2}\left(-\frac{8}{3}\right) + 2 = \frac{-4+6}{3} = \frac{2}{3} = x$

& $y = \frac{1}{2}\left(-\frac{8}{3}\right) + 1 = -\frac{4+3}{3} = \frac{1}{3} = y$

& $z = -\frac{1}{2}\left(-\frac{8}{3}\right) - 2 = \frac{+4-6}{3} = \frac{-2}{3} = z$

$\Rightarrow \sqrt{(x-2)^2 + (y-1)^2 + (z+1)^2}$

$= \sqrt{\left(\frac{2}{3}-2\right)^2 + \left(\frac{1}{3}-1\right)^2 + \left(-\frac{2}{3}+1\right)^2}$

$= \sqrt{\left(\frac{2-6}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$

$= \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{16+4+1}{9}}$

$= \sqrt{\frac{21}{3^2}} = \frac{\sqrt{21}}{3}$ Nope.

Found a mistake $z = -\frac{1}{2}\lambda - 2$

missed this

Fixed. Still wrong

§14.8 #27's Kicking my butt!

Distance from $(2, 1, -1)$ to $x+y-z=1$

d^2 = square of distance from B to P

$g(x,y,z) = x+y-z$

$= f(x,y,z) = (x-2)^2+(y-1)^2+(z+1)^2$

$f_x = 2(x-2) = 2x-4 = \lambda \implies 2x - \lambda = 4$

$f_y = 2(y-1) = 2y-2 = \lambda \implies 2y - \lambda = 2$

$f_z = 2(z+1) = -\lambda \implies 2z + \lambda = -2$

$2z+2 = -\lambda$

$x = \frac{\lambda+4}{2}, \; y = \frac{\lambda+2}{2}, \; z = \frac{-\lambda-2}{2}$

$x+y-z = \frac{\lambda+4}{2} + \frac{\lambda+2}{2} - \frac{-\lambda-2}{2} = 1$

$\implies \lambda+4+\lambda+2+\lambda+2 = 2$

$\implies 3\lambda+8 = 2 \to$ $\boxed{\lambda = -2}$!

$3\lambda = -6 \to$

$x = \frac{-2+4}{2} = 1, \; y = \frac{-2+2}{2} = 0, \; z = \frac{2-2}{2} = 0$

$\implies d = \sqrt{(1-2)^2+(0-1)^2+(0+1)^2} = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ FINALLY

203 §14.8 #s 29, 41, 45, 46

(29) #41 §14.7 Find points on the cone

$z^2 = x^2+y^2$ that are closest to the origin.

Minimize $\sqrt{x^2+y^2+z^2}$ s.t.

$x^2+y^2-z^2=0$

let $f(x,y,z) = x^2+y^2+z^2$

$g(x,y,z) = x^2+y^2-z^2$

Then $f_x = 2x = 2\lambda x$

$f_y = 2y = 2\lambda y$

$f_z = 2z = 2\lambda z$

$(2\lambda-2)z = 0$

$(2\lambda-2)y = 0$

$(2\lambda-2)z = 0$

$\lambda = 1$ OR $x=0$ OR $y=0$ OR $z=0$

$\lambda = 1 \Rightarrow$ TAUTOLOGY from $f_{x_i} = \lambda g_{x_i}$

$z = \pm\sqrt{x^2+y^2}$

minimum distance is @ origin, I think. Just from visual

$z^2 = x^2+y^2$

203 S' 14.8 #s ~~XX~~ 45, 46

(41) The plane $x+y+2z=2$ intersects the paraboloid $z=x^2+y^2$ in an ellipse. Find the points on this ellipse that are nearest & farthest from the origin.

$d=\sqrt{x^2+y^2+z^2}$

$f = x^2+y^2+z^2$ s.t. $x+y+2z=2$

$g(x,y,z)= x+y+2z$

$f_x=2x = \lambda$ $\quad x= \frac{\lambda}{2}=y$

$f_y = 2y=\lambda$ $\quad z=\lambda = 2x$

$f_z = 2z= 2\lambda$ $\quad (x,x,2x)$

Now $x + x + 2(2x) = 2$

$6x=2$

$x= \frac{1}{3} = y,\ z= \frac{2}{3}$

<u>WAIT</u> 2 constraints

$h(x,y,z) = x^2+y^2-z$

$f_x= 2x = \lambda + 2\mu x$

$f_y= 2y= \lambda + 2\mu y$

$f_z = 2z = 2\lambda - 2\mu z$

$(2-2\mu)x = \lambda$
$(2-2\mu)y = \lambda$ $\Big\}$ $x=y$

$(2+2\mu)z = 2\lambda$

$\lambda = (1+\mu)z$

203 §14.8 #s 41, 45, 46

4 cont'd

$(1+u)z = (2-2u)x$

Not seeing how to solve this w/o technology to solve the simultaneous eq'ns.

45

203 §14.8 #s 45, 46

(45) Find max of $\sqrt[n]{\prod_{k=1}^{n} x_k}$

given $x_k > 0 \ \forall k = 1, \ldots, n$

and $\sum_{k=1}^{n} x_k = C \in \mathbb{R}$ $\qquad g(\bar{x}) = \sum_{k=1}^{n} x_k$

$\sqrt[n]{*}$ is increasing. Minimize it by minimizing $f^n(\bar{x}) = \prod_{k=1}^{n} x_k$

$$f_{x_1} = \prod_{\substack{k=1 \\ k \neq 1}}^{n} x_k = \lambda$$

$\vdots$

$$f_{x_i} = \prod_{\substack{k=1 \\ k \neq i}}^{n} x_k = \lambda$$

$n = 2$: $\sqrt[2]{x_1 x_2}$

$$f(x_1, x_2) = x_1 x_2, \text{ s.t. } g(x_1, x_2) = x_1 + x_2$$

$$x_1 = x_2$$

$$f_{x_1} = x_2 = \lambda$$

$$f_{x_2} = x_1 = \lambda$$

$$x_1 = C - x_2 \Longrightarrow C - x_2 = x_2 = \lambda$$

$$2x_2 = C = x_2 + \lambda$$

$$x_1 = x_2 = \frac{C}{2}$$

$$\sqrt[2]{\frac{C^2}{2^2}} = \frac{C}{n}$$

Now, reasoning by this pattern

$\frac{C}{n} = \max$

$\& \ x_k = \frac{C}{n}$

$\sum x_k = n x_1$

$\frac{n x_1}{n} = x_1$

203 S'14.8 #s 45, 46

#45 cnt'd

Where $f(\bar{x})$ is max, we have

$\sqrt[n]{\prod_{k=1}^{n} x_k} = \frac{c}{\sqrt[n]{n}}$ and at that point,

we have $\frac{\sum x_k}{n} = \frac{\sum \frac{c}{\sqrt[n]{n}}}{n} = \frac{c}{\sqrt[n]{n}}$

and they are equal.

Otherwise, $\sqrt[n]{\prod_{k=1}^{n} x_k} \le \frac{c}{\sqrt[n]{n}}$, ~~[illegible]~~

and so $\frac{\sum x_k}{n} = \frac{c}{n} \ge c$

Not quite.

$f_{x_1} = x_2 x_3 = \lambda \Rightarrow x_3 = \frac{\lambda}{x_2}$

$f_{x_2} = x_1 x_3 = \lambda \Rightarrow x_1 \frac{\lambda}{x_2} = \lambda \Rightarrow x_1 \lambda = x_2 \lambda \Rightarrow x_1 = x_2$

$f_{x_3} = x_1 x_2 = \lambda$

So $\sum x_k = c \Rightarrow 3x_k = c \Rightarrow x_k = \frac{c}{n}$

$\sqrt[n]{\frac{c^n}{n^n}} = \frac{c}{n} =$ what we want

Now $\frac{\sum x_k}{n} = \frac{c}{n}$ and

so whatever c is, $\sqrt[n]{\prod_{k=1}^{n} x_k} \le \frac{c}{n} = \frac{\sum x_k}{n}$ ▨

Not convinced, yet, Steve.

203 § 14.8 #s 45, 46

(45) $f_{x_1} = x_2 x_3 x_4 = \lambda$

$f_{x_2} = x_1 x_3 x_4 = \lambda$

$f_{x_3} = x_1 x_2 x_4 = \lambda$

$f_{x_4} = x_1 x_2 x_3 = \lambda$

From this pattern, we DO get ✓

$x_1 = x_2 = \cdots = x_n$

How do we know this is a maximum s.t. constraint $\sum x_k = c$?

All we know is that we found where the partials of constraint and distance are parallel.

(46) (a) Maximize $\sum x_i y_i$ s.t. $\sum x_i^2 = 1$ and $\sum y_k^2 = 1$ $\quad g = \sum y_k^2 \quad h = \sum y_k^2$

~~$f_{x_1} = 2\lambda x_1 + \mu \cdot 0 =$~~

$f_{x_1} = y_1 = 2\lambda x_1 + \mu \cdot 0$

$f_{x_2} = y_2 = 2\lambda x_2$

$\vdots$

$f_{x_n} = y_n = 2\lambda x_n$

Then $\sum x_i y_i = \sum x_i (2\lambda x_i) = \sum 2\lambda x_i^2$

$= 2\lambda$

$x_1 = 2\mu y_1$

So same thing with x's & y's reversed & λ-term = 0

Alternately $\sum x_i y_i = 2\mu$

203 §14.8 #46

Cntd. We're on the boundary of the unit n-sphere.

No. $\sum x_i^2 = 1$ means that. You're on the boundary of a unit n-~~sphere~~ ball in $\mathbb{R}^n$

So does $\sum y_i^2 = 1$.

Question is, how many variables are we dealing with, exactly? 2n?

x_k, $k = 1, \ldots, n$

y_k, $k = 1, \ldots, n$

$x_1 y_1 + x_2 y_2 + x_3 y_3$

$\sum x_k y_k = \sum x_k (2\lambda x_k) = 2\lambda \sum x_k^2 = 2\lambda$

$x_1 = 2\mu y_1 = 2\mu(2\lambda x_1) = 4\mu\lambda x_1$ $\quad 4\lambda\mu = 1$

$\mu = \frac{1}{4\lambda}$

$x_2 = 4\mu\lambda$

$y_1 = 2\lambda x_1$ $\quad \lambda = \frac{1}{4\mu}$

$\vdots$

$x_n = 4\mu\lambda$

$y_1 = 2\lambda(2\mu y_1)$

$x_1 = x_2 = \ldots x_n$

$y_1 =$

$=$

Wait. We jus need to find λ or μ

$x_k = 2\mu y_k$

$\sum x_k y_k = \sum 2\mu y_k y_k = 2\mu$

$2\mu = 2\lambda$

$\mu = \lambda$

$\mu - \lambda = 0$

$\mu - \left(\frac{1}{4\mu}\right) = \frac{4\mu - 1}{4\mu} = 0 \Rightarrow \mu = \frac{1}{4}$

$\frac{1}{4\left(\frac{1}{4}\right)} = 1$

~~$\Rightarrow \lambda = \mu =$~~

$\frac{\partial}{\partial x_k}\left[\sum_{k=1}^{n} x_k y_k\right] = y_k = f_{x_k}$

$\frac{\partial}{\partial x_k}\left[\sum x_k^2\right] = 2x_k$

$\frac{\partial}{\partial y_k}\left[\sum y_k^2\right] = 2y_k$

Set $f_{x_k} = \lambda g_k + \mu h_k$

$\Rightarrow y_k = 2\lambda x_k$

Set $f_{y_k} = \lambda g_{y_k}$

$\Rightarrow x_k = 2\mu y_k$

$1 = \sum y_k^2 = \sum (2\lambda x_k)^2$

$= 4\lambda^2 \sum x_k^2 = 4\lambda^2$

$= 1 \Rightarrow \lambda^2 = \frac{1}{4} \Rightarrow$

$\lambda = \pm \frac{1}{2}$

$\lambda = \frac{1}{2} \Rightarrow \mu = \frac{1}{4\left(\frac{1}{2}\right)} = \frac{1}{2}$ ✓

203 5th M.8 #46 done properly

(46) Maximize $\sum_{i=1}^{n} x_i y_i$ s.t. $\sum x_i^2 = \sum y_i^2 = 1$

(a)

(b) Prove Cauchy – Schwarz

(1) $f_{x_k} = y_k = 2\lambda x_k$, $k = 1, \ldots, n$

(2) $f_{y_k} = x_k = 2\mu y_k$, $k = 1, \ldots, n$

By (1), $\sum y_k^2 = \sum (2\lambda x_k)^2 = 4\lambda^2 \sum x_k^2 = 4\lambda^2 = 1$

$\Rightarrow \lambda^2 = \frac{1}{4} \Rightarrow \lambda = \pm \frac{1}{2}$

$\Rightarrow y_k = \pm x_k$, $k = 1, \ldots, n$, from $y_k = 2(\pm \frac{1}{2}) x_k = \pm x_k$

Likewise, we obtain $\mu = \pm \frac{1}{2}$ by symmetry of argument.

Now if $y_k = x_k \; \forall k$, then $\sum x_i y_i = \sum x_k^2 = 1$

and if $y_k = -x_k \; \forall k$, " $\sum x_k(-x_k)$ $= -\sum x_k^2 = -1$. These give max & min values for $f(x_1, x_2, \ldots, x_n, y_1, \ldots, y_n)$

Otherwise, some are positive & some are negative & you get something between -1 & $+1$

(b) We show that $\sum a_i b_i \le \sqrt{\sum a_i^2} \sqrt{\sum b_i^2}$

Proof: Define $x_i = \dfrac{a_i}{\sqrt{\sum a_i^2}}$ & $y_i = \dfrac{b_i}{\sqrt{\sum b_i^2}}$

$\sum x_i y_i$

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cnt'd

Then $\sum x_i y_i$ satisfies $\sum x_i^2 = 1$, $\sum y_i^2 = 1$

$\Rightarrow \sum x_i y_i \le 1 \Rightarrow$

$$\sum \frac{a_i}{\sqrt{\sum a_k^2}} \cdot \frac{b_i}{\sqrt{\sum b_k^2}} \le 1 \Rightarrow$$

$$\boxed{\sum a_i b_i \le \sqrt{\sum a_k^2}\sqrt{\sum b_k^2}}$$

This is huge in analysis for proving nasty integrals converge.

Sometimes $\int \sqrt{f^2}\ \exists$, $\int g^2\ \exists$

When you're not sure about $\int_a^b f(x)g(x)\,dx$. Then

$$\left|\int f(x)g(x)\,dx\right| \le \sqrt{\int f(x)^2\,dx}\sqrt{\int g(x)^2\,dx}$$

This result ~~from~~ for integrals flows directly from $\int$ as the limit of $\sum$

This is a huge $\sum$ result for higher analysis