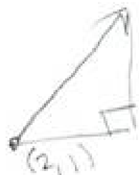
203 §14.6 #s 4, 8, 9, 16, 19, 24, 34, 40

(4) Find $D_{\bar{u}} f$ @ $(2,1)$ for $f(x,y) = x^2y^3 - y^4$, $\theta = \frac{\pi}{4}$



$$\bar{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$(\nabla f)\Big|_{(x,y)=(2,1)} \cdot \bar{u} = \left\langle 2xy^3, 3x^2y^2 - 4y^3 \right\rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle \Bigg|_{(x,y)=(2,1)}$$

$$= \left\langle 2(2)(1)^3, 3(2)^2 - 4(1)^3 \right\rangle \cdot \bar{u} = \langle 4, 8 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{4}{\sqrt{2}} + \frac{8}{\sqrt{2}} = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = \boxed{6\sqrt{2}}$$

#s 7-10 (a) ∇f (b) ∇f @ P / (c) Find $D_{\bar{u}} f$ @ P.

(8) $f(x,y) = \frac{y^2}{x}$, $P(1,2)$, $\bar{u} = \frac{1}{3}\langle 2, \sqrt{5} \rangle$

(a) $\boxed{\nabla f = \left\langle -\frac{y^2}{x^2}, \frac{2y}{x} \right\rangle}$

(b) $\nabla f(1,2) = \left\langle -\frac{2^2}{1^2}, \frac{2(2)}{1} \right\rangle = \boxed{\langle -4, 4 \rangle = \nabla f(1,2)}$

(c) $(\nabla f(1,2)) \cdot \bar{u} = \langle -4, 4 \rangle \cdot \frac{1}{3}\langle 2, \sqrt{5} \rangle = \boxed{-\frac{8}{3} + \frac{4\sqrt{5}}{3}}$

(9) $f(x,y,z) = xe^{2yz}$, $P(3,0,2)$, $\bar{u} = \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$

(a) $\boxed{\nabla f = \left\langle e^{2yz}, 2xze^{2yz}, 2xye^{2yz} \right\rangle}$

(b) $\boxed{\nabla f(3,0,2) = \langle 1, 12, 0 \rangle}$

(c) $D_{\bar{u}} f(3,0,2) = \langle 1, 12, 0 \rangle \cdot \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle = \frac{2}{3} - \frac{24}{3} = \boxed{-\frac{22}{3} = D_u f(3,0,2)}$

203 §14.6 #s 16, 19, 24, 34, 40

(16) Find $D_{\bar{v}} f(x_0, y_0, z_0)$ for

$f(x,y,z) = \sqrt{xyz}$, $\bar{v} = \langle -1,-2,2\rangle$, $(x_0,y_0,z_0) = (3,2,6)$

$\nabla f : \left\langle \frac{yz}{2\sqrt{xyz}}, \frac{xz}{2\sqrt{xyz}}, \frac{xy}{2\sqrt{xyz}} \right\rangle$

$\nabla f(3,2,6) = \left\langle \frac{2(6)}{2\sqrt{3(2)(6)}}, \frac{3(6)}{2(6)}, \frac{3(2)}{2(6)} \right\rangle = \langle 1, \frac{3}{2}, \frac{1}{2}\rangle$

$\Rightarrow D_{\bar{v}} f(3,2,6) = \frac{1}{2}\langle 2,3,1\rangle \cdot \frac{\langle -1,-2,2\rangle}{\sqrt{1+4+4}}$

$= \frac{1}{2}\langle 2,3,1\rangle \cdot \frac{1}{3}\langle -1,-2,2\rangle = \frac{1}{6}(-2-6+2) \boxed{= -1 = D_{\bar{v}} f(3,2,6)}$

(19) Find directional derivative of $f(x,y) = \sqrt{xy}$ @ $P(2,8)$ in the direction of $Q(5,4)$.

$\bar{v} = \langle 5-2, 4-8\rangle = \langle 3,-4\rangle$

$\|\bar{v}\| = \sqrt{9+16} = \sqrt{25} = 5$

Let $\bar{u} = \frac{\bar{v}}{\|\bar{v}\|} = \frac{1}{5}\langle 3,-4\rangle = \bar{u}$

$\nabla f = \left\langle \frac{y}{2\sqrt{xy}}, \frac{x}{2\sqrt{xy}} \right\rangle$, $\nabla f(2,8) = \frac{1}{2}\left\langle \frac{8}{\sqrt{16}}, \frac{2}{4}\right\rangle = \langle 1, \frac{1}{4}\rangle$

$\Rightarrow (\nabla f(2,8)) \cdot \bar{u} = \frac{1}{10}\langle 2, \frac{1}{2}\rangle \cdot \langle 3,-4\rangle = \frac{1}{10}(6-2)$

$= \boxed{\frac{2}{5} = D_{\bar{u}} f(2,8)}$

203 §14.6 24, 34, 40

(24) Find max rate of change ($\|\nabla f(1,1,-1)\|$)
and its direction ($\nabla f(1,1,-1)$)

for $f(x,y,z) = \dfrac{x+y}{z}$

$$\nabla f = \left\langle \frac{1}{z}, \frac{1}{z}, -\frac{(x+y)}{z^2} \right\rangle$$

$$\nabla f(1,1,-1) = \left\langle \frac{1}{-1}, -1, -\frac{(1+1)}{(-1)^2} \right\rangle = \langle -1,-1,-2\rangle \Longrightarrow$$

Direction is $\langle -1,-1,-2\rangle = \nabla f(1,1,-1)$
& max rate of change is $\sqrt{1^2+1^2+2^2} = \sqrt{6}$

(34) Shape of a hill is elliptic paraboloid
$z = 1000 - .005x^2 - .01y^2$, where x, y, and z are measured
in meters and you're standing at a point
w/ coordinates $(60, 40, 966)$. The positive x-axis
points East & the positive y-axis is North. $\vec{u} = \langle 0,1\rangle$

(a) If you walk due south, will you ascend or descend? AT WHAT RATE?
$\vec{v} = \langle 0,-1\rangle$

(b) If you walk Northwest, " " " " ? AND AT WHAT RATE?

(c) In which direction is the slope largest?
What's the rate of ascent in that direction?
At what angle above the horizontal does the
path begin?

203 §14.6 #s 24, 34, 40

24 cont'd

(a) We find $D_{\bar{u}} f(60,40,966)$, where $\bar{u} = \langle 0,1 \rangle$ NORTH!

$f_x = -.010x \quad f_x(60,40) = -.01(60) = -.6$

$f_y = -.02y \quad f_y(60,40) = -.02(40) = -.8$

Then $D_{\bar{u}} f(60,40) = \langle -.6, -.8 \rangle \cdot \langle 0,1 \rangle = \boxed{-.8 = D_u f(60,40)}$

Descending

$-.8$ is the rate

(b) We find $D_{\bar{v}} f(60,40)$, where $\bar{v} = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ Northwest!

$\langle -.6, -.8 \rangle \cdot \frac{1}{\sqrt{2}} \langle -1, 1 \rangle = \frac{1}{\sqrt{2}}(.6 - .8) = \boxed{\frac{-.2}{\sqrt{2}} \approx -.1414213562}$

$+0.8$ is the rate

(c) Slope is largest in the direction $\langle -.6, -.8 \rangle$ and the rate is $\sqrt{.6^2 + .8^2}$

$= \sqrt{.36 + .64} = \boxed{1 = \text{Max rate}}$

The angle above the horizontal?

Increase at a rate of 1 ft per foot in direction of max rate. This gives an angle of $\boxed{\theta = \frac{\pi}{4}}$ from $\arctan(1) = \frac{\pi}{4}$

#s 39–44 Find tangent plane (a) & normal line (b)

(a) specified point

(40) $y = x^2 - z^2$ @ $(4,7,3)$

$0 = 2x - 2z z_x \Rightarrow z_x = \frac{2x}{2z} \underset{=}{x=4, z=3} \frac{8}{6} = \frac{4}{3}$

$1 = -2z z_y \Rightarrow z_y = \frac{-1}{2z} \underset{=}{z=3} -\frac{1}{6}$

203 §14.6 #40

#40 cnt'd

$\frac{4}{3}x - \frac{16}{3} - \frac{1}{6}y + \frac{7}{6} + 3 = z$

$8x - 32 - y + 7 + 18 = 6z$

So $\nabla z(4,7) = \langle \frac{4}{3}, -\frac{1}{6} \rangle$

& Plane is $\boxed{\frac{4}{3}(x-4) - \frac{1}{6}(y-7) + 3} = L(x,y)$

(b) $\bar{n} = \langle \frac{4}{3}, -\frac{1}{6}, -1 \rangle$ is direction vector for Line

$\langle 4, 7, 3 \rangle = \bar{r}_0$ Then

$\langle 8, -1, -6 \rangle$

$\bar{r} = \bar{r}_0 + t \langle \frac{4}{3}, -\frac{1}{6}, -1 \rangle$

or $\boxed{\bar{r} = \langle 4, 7, 3 \rangle + t \langle 8, -1, -6 \rangle}$

$\boxed{x = 8t + 4, \; y = -t + 7, \; z = -6t + 3}$

$8x - y - 6z = 7$