203 §14.5 #s 4, 7, 21, 25, 28, 32, 45, 48

(4) Use Chain Rule to find $\frac{dz}{dt}$

$z = \arctan\left(\frac{y}{x}\right)$, $x = e^t$, $y = 1 - e^{-t}$ $\qquad \frac{1-e^{-t}}{e^t} = e^{-t} - e^{-2t}$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

ugh!

$$= \frac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2}\cdot e^t + \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2}\cdot e^{-t} = -\frac{y e^t}{x^2\left(1+\left(\frac{y}{x}\right)^2\right)} + \frac{e^{-t}}{x\left(1+\left(\frac{y}{x}\right)^2\right)}$$

$$= \frac{-(1-e^{-t})e^t}{e^{2t}\left(1+\left(\frac{1-e^{-t}}{e^t}\right)^2\right)} + \frac{e^{-t}}{e^t\left(1+\left(\frac{1-e^{-t}}{e^t}\right)^2\right)} = \cancel{\frac{e^{2t} + e^{-2t} - 2e^{-t} + }{e^{2t}}}$$

#s 7-12 Find $z_s$, $z_t$

(7) $z = x^2y^3$, $x = s\cos t$, $y = s\sin t$

$$z_s = z_x x_s + z_y y_s$$

$$= 2xy^3\cos t + 3x^2y^2 s\sin t$$

$$= \boxed{2s^4\cos^2 t\sin^3 t + 3s^4\cos^2 t\sin^4 t} = z_s$$

$$z_t = 2xy^3(-s\sin t) + 3x^2y^2(s\cos t)$$

$$= 2s(\cos t)(s^3\sin^3 t)(-s\sin t) + 3(s^2\cos^2 t)(s^3\sin^3 t)\cdot s\cos t$$

$$= \boxed{-2s^5\cos t\sin^4 t + s^6\cos^3 t\sin^3 t}$$

203 §14.5 #s ~~24~~, 25, 28, 33, 45, 48

#s 21-26 Find indicated partials.

25 ~~$u = x^2 + xy^3$, $x = uv^2 + w^3$, $y = u + ve^w$~~

$u = x^2 + yz$, $x = pr\cos\theta$, $y = pr\sin\theta$, $z = p + r$

Find $u_p, u_r, u_\theta$ @ $(p, r, \theta) = (2, 3, 0)$

$u_p = u_x x_p + u_y y_p + u_z z_p$

$= 2x\, r\cos\theta + z\, r\sin\theta + y$

$u_p(2,3,0) = 2(6)(3)(1) + 2(3)(0) + 0 = \boxed{36 = u_p(2,3,0)}$

$x(2,3,0) = 2(3)(1) = 6$ $\quad$ $x(2,3,0) = (2)(3)\cos(0) = 6$

$y(2,3,0) = 2(3)(0) = 0$ $\quad$ $y(2,3,0) = 2(3)(0) = 0$

$z(2,3,0) = 2$ $\quad$ $z(2,3,0) = 2 + 3 = 5$

$u_r = u_x x_r + u_y y_r + u_z z_r$

$= 2x\, p\cos\theta + z\, r\sin\theta + y$

$u_r(2,3,0) = 2(6)(2)(1) + 2(3)(0) + 0 = \boxed{24 = u_r(2,3,0)}$

$u_\theta = u_x x_\theta + u_y y_\theta + u_z z_\theta$

$= 2x(-pr\sin\theta) + z\, pr\cos\theta + y(0) = u_\theta$

$\boxed{u_\theta(2,3,0) = 2(6)(-2)(3)(\sin(0)) + 5(2)(3)(1) = 30}$

203 §14.5 #s 28, 32, 45, 48

(28) Use $\boxed{6}$ $\frac{dy}{dx} = \frac{-F_x}{F_y}$ to find $\frac{dy}{dx}$ for

$y^5 + x^2y^3 = 1 + ye^{x^2} \Longrightarrow$

$F = y^5 + x^2y^3 - 1 - ye^{x^2} \Longrightarrow$

$F_x = 2xy^3 - 2xye^{x^2}$, $F_y = 5y^4 + 3x^2y^2 - e^{x^2} \Longrightarrow$

$$\boxed{\frac{dy}{dx} = -\frac{2xy^3 - 2xye^{x^2}}{5y^4 + 3x^2y^2 - e^{x^2}}}$$

(32) Use Eq'n $\boxed{7}$ to find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$

$xyz = \cos(x+y+z) \Longrightarrow$

$F = xyz - \cos(x+y+z) \Longrightarrow$

$F_x = yz + \sin(x+y+z)$, $F_y = xz + \sin(x+y+z)$

& $F_z = xy + \sin(x+y+z) \Longrightarrow$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{yz + \sin(x+y+z)}{xy + \sin(x+y+z)} \text{ \& } \frac{\partial z}{\partial y} = -\frac{xz + \sin(x+y+z)}{xy + \sin(x+y+z)}}$$

203 §14.5 #s 45, 48

#s 45 – 8 Assume all in sight are difbl

(45) $z = f(x, y)$, where $x = r\cos\theta$, $y = r\sin\theta$.

(a) Find $\frac{\partial z}{\partial r}$ & $\frac{\partial z}{\partial r}$ & show that

(b) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

(a) $\boxed{\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}}$

$\boxed{= z_x\cos\theta + z_y\sin\theta}$ &

$\frac{\partial z}{\partial \theta} = z_x x_\theta + z_y y_\theta = \boxed{-z_x r\sin\theta + z_y r\cos\theta = z_\theta}$

(b) $(z_r)^2 + \frac{1}{r^2}(z_\theta)^2 = (z_x\cos\theta + z_y\sin\theta)^2$

$+ \frac{1}{r^2}(-z_x r\sin\theta + z_y r\cos\theta)^2$

$= z_x^2\cos^2\theta + 2z_x z_y\sin\theta\cos\theta + z_y^2\sin^2\theta$

$+ \frac{1}{r^2}\left[z_x^2 r^2\sin^2\theta - 2z_x z_y r^2\sin\theta\cos\theta + z_y^2 r^2\cos^2\theta\right]$

$= z_x^2(\cos^2\theta + \sin^2\theta) + z_y^2(\sin^2\theta + \cos^2\theta)$

$= z_x^2 + z_y^2$ ✓ ▨

203 § 14.5 ~~#48~~

48 If $z = f(x,y)$, $x = s+t$ & $y = s-t$, show

that $(z_x)^2 - (z_y)^2 = z_s z_t$

$(z_x)^2 - (z_y)^2 = ?$

$z_s z_t = (z_x x_s + z_y y_s)(z_x x_t + z_y y_t)$

$= (z_x + z_y)(z_x - z_y) = (z_x)^2 - (z_y)^2$ ∎