[redacted]

203 §14.4 #s 4, 10, 12, 20, 24, 26, 32, 36,

(4) Tangent Plane to $z = y \ln x$ @ $(1, 4, 0)$

$f_x = \frac{y}{x}$, $f_y = \ln x$

$f_x(1,4) = \frac{4}{1}$, $f_y(1,4) = 0$

$z - 0 = f_x(x-1) + f_y(y-4)$

$$\boxed{z = 4(x-1)}$$

(10) Graph $f(x,y) = e^{-\frac{xy}{10}}(\sqrt{x} + \sqrt{y} + \sqrt{xy})$, $(1, 1, 3e^{-.1})$

and the tangent plane. $f_x = -\frac{y}{10}e^{-\frac{xy}{10}}(\sqrt{x}+\sqrt{y}+$

$+ e^{-\frac{xy}{10}}\left(\frac{1}{2\sqrt{x}} + \frac{y}{2\sqrt{xy}}\right)$

See Maple.

$f_y = -\frac{x}{10}e^{-\frac{xy}{10}}(\sqrt{x}+\sqrt{y}+\sqrt{xy}) + e^{-\frac{xy}{10}}\left(\frac{1}{2\sqrt{y}} + \frac{x}{2\sqrt{xy}}\right)$

(12) Explain why the function is difb @ $(1,1)$ & Find its linearization @ $(1,1)$.

$f(x,y) = x^3y^4$

$\boxed{f(1,1) = 1}$

$f_x = 3x^2y^4 \Rightarrow f_x(1,1) = 3$

$f_y = 4x^3y^3 \Rightarrow f_y(1,1) = 4$

$$\rightarrow \boxed{L(x,y) = 3(x-1) + 4(y-1) + 1}$$

203 §14.4 #s 20, 24, 26, 32, 36

(20) Use $L(x,y)$ @ $(7,2)$ to ~~f~~ estimate

$f(6.9, 2.06)$ for $f(x,y) = \ln(x-3y)$

$f_x = \frac{1}{x-3y} \Rightarrow f_x(7,2) = \frac{1}{7-6} = 1$

$f_y = \frac{-3}{x-3y} \Rightarrow f_y(7,2) = \frac{-3}{7-6} = -3$

$f(7,2) = \ln(7-3(2)) = 0$

$L(x,y) = (x-7) - 3(y-2)$

$L(6.9, 2.06) = -.1 - 3(.06) = -.1 - .18 = \boxed{-.28 \approx f(6.9, 2.06)}$

(24) W = wind chill index $= W(T,v)$

$= f(T,v)$, where T = Temp & v = wind speed.

Use table to find linear approx to f

near $-15°C$ & $v = 50 \frac{km}{hr}$. Then estimate

$f(-17, 55)$

T \ v	40	50	60
-10		-22	
-15	-27	-29	-30
-20		-35	

$f_T: \frac{-29-(-22)}{-15-(-10)} = \frac{-7}{-5} = \frac{7}{5}, \quad \frac{-35-(-29)}{-20-(-15)} = \frac{-6}{-5} = \frac{6}{5}$

$\frac{6+7}{10} = \frac{13}{10}$

$f_v: \frac{-30-(-29)}{10} = \frac{-1}{10}, \quad \frac{-29+27}{10} = -\frac{1}{5}$

$\frac{-1-2}{20} = -\frac{3}{20}$

$L(T,v) = \frac{11}{10}(\cancel{T}+15) - \frac{3}{20}(v-50) - 29$

$L(-17, 55) = \frac{13}{10}(-2) - \frac{3}{20}(5) = -\frac{13}{5} - \frac{3}{4} = \frac{-52-15}{20} = \frac{-67}{20} =$

-29

$-29 - \frac{67}{20} = \boxed{-32.35}$

203 §14.4 #s 26, 32, 36

(26) Find the differential of the function.

$v = y\cos(xy)$

$v_x = -y^2\sin(xy)$, $v_y = -xy\sin(xy) + \cos(xy)$

$$\boxed{dV = -y^2\sin(xy)\,dx + (-xy\sin(xy) + \cos(xy))\,dy}$$

(32) If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $(2.96, -.95)$, compare Δz & dz

$f_x = 2x - y$, $f_y = -x + 6y$

$dz = (2x - y)\,dx + (-x + 6y)\,dy$

$$dz\Big|_{\substack{x=3,\ \Delta x = -.04 \\ y=-1,\ \Delta y = .05}} = (2(3)+1)(-.04) + (-3-6)(.05)$$

$$= -7(.04) - 9(.05) = -.28 - .45$$

$$= \boxed{-.73} = dz$$

$f(3,-1) = 3^2 - 3(-1) + 3(-1)^2 = 9 + 3 + 3 = 15$

$f(2.96, -.95) = (2.96)^2 - 2.96(-.95) + 3(-.95)^2$

$= 14.2811$

$$\Delta z = z_1 - z_0 = 14.2811 - 15 = \boxed{-.7189 = \Delta z}$$

203 §14.4 #s ~~32~~, 36

36 Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high & 4 cm in diameter if the metal in the top & bottom is .1 cm & the metal in the sides is 0.05 cm.

$$V = \pi r^2 h$$

$$\frac{\partial V}{\partial r} = 2\pi r h \quad , \quad \frac{\partial V}{\partial h} = \pi r^2$$

$dr = 0.05$ cm $\quad dh = 2(.1) = .2$ cm

$$dV\Big|_{\substack{r=2,\, dr=.05 \\ h=10,\, dh=.2}} = 2\pi r h\,dr + \pi r^2 dh \Big| \text{ same}$$

$$= 2\pi(2)(10)(.05) + \pi(2)^2(.2)$$

$$= \pi\left[40(.05) + 4(.2)\right] = \pi\left[2 + .8\right] \boxed{= 2.8\pi \text{ cm}^3}$$

$\approx 8.7964594 3$ cm^3