203 §14.3 #s 4, 13, 14, 21, 26, 29, 30, 50, 59, 78

10, 11, 15, 18, 47, 52-4, 56, 71, 82, 83

(4) wave height, h, is given as a function of v = wind velocity in knots and duration of that velocity, in hours. h is in feet.

(a) what's the meaning of $\partial h/\partial v$ & $\partial h/\partial t$

$\frac{\partial h}{\partial v}$ is rate of change in height wrt wind veloc. (at a fixed duration)

$\frac{\partial h}{\partial t}$ is rate of change in height wrt time (at a fixed v)

(b) Estimate $f_v(40,15)$ & $f_t(40,15)$

v \ t	10	15	20
30		16	
40	21	25	28
50		36	

$$\frac{h(40,15)-h(40,10)}{15-10} = \frac{25-21}{5} = \frac{4}{5}\ \text{ft/hr}$$

$$\frac{h(40,20)-h(40,15)}{20-15} = \frac{28-25}{5} = \frac{3}{5}\ \text{ft/hr}$$

$\frac{7}{10}$ ft/hr $= f_t(40,15)$

$$\frac{f(40,15)-f(30,15)}{10} = \frac{25-16}{10} = \frac{9}{10}\ \text{ft/knott}$$

$$\frac{f(50,15)-f(40,15)}{10} = \frac{36-25}{10} = \frac{11}{10}\ \text{ft/knott}$$

1 ft/knott $= f_v(40,15)$

203 §14.3 #s 10, 11, 13-15, 18, 21, 26, 29, 30, 47,

50, 52-4, 56, 59, 71, 78, 82, 83

(4) cnt'd
Estimate $\lim_{t \to \infty} \frac{dh}{dt}$

I'm not sure. Depends on wind velocity!
I'd guess it's zero, based on lack of
change between the last 2 or 3 columns.

(10) Use Figure to estimate $f_x(2,1)$ & $f_y(2,1)$

$f_y(2,1) \approx -2$, $f_x(2,1) \approx 3$

$\frac{f(2,2)-f(2,1)}{2-1}$ + $\frac{f(2,0)-f(2,1)}{0-1}$

(11) $f(x,y) = 16-4x^2-y^2$. Find $f_x(1,2)$, $f_y(1,2)$ & interpret these as slopes. Illustrate w/ sketch or computer plots

$f_x = -8x$, $f_y = -2y$

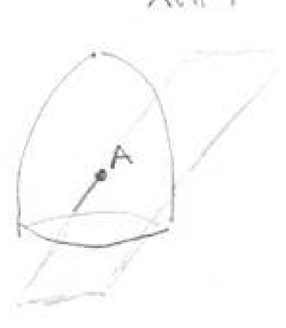
$f_x = -8x \longrightarrow \boxed{f_x(1,2) = -8}$

$f_y = -2y \longrightarrow \boxed{f_y(1,2) = -4}$

The slope of the trace $y=2$ @ $(1,2)$ is -8.

The slope of the trace $x=1$ @ $(1,2)$ is -4

A(1,2,8)



#s 13-4 Find f_x & f_y. graph f, f_x, f_y

(13) $f(x,y) = x^2+y^2+x^2y$

$f_x = 2x+2xy$

$f_y = 2y+x^2$

(14) $f(x,y) = xe^{-x^2-y^2}$ NA

203 §14.3 #s 15, 18, 21, 26, 29, 30, 47, 50, 52-4,
56, 59, 71, 78, 82, 83

#s 15-38 Find 1^{st} partials.

15 $f(x,y) = y^5 - 3xy \Longrightarrow$

$\boxed{f_x = -3y, \quad f_y = 5y^4 - 3x}$

18 $f(x,t) = \sqrt{x}\ \ln(t) \Longrightarrow$

$\boxed{f_x = \frac{1}{2}x^{-\frac{1}{2}}\ln(t), \quad f_y = \frac{\sqrt{x}}{t}}$

21 $f(x,y) = \frac{x-y}{x+y} = (x-y)(x+y)^{-1} \Longrightarrow$

$f_x = (x+y)^{-1} - (x+y)^{-2}(x-y) = \frac{(x+y) - (x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}$

$f_y = -(x+y)^{-1} - (x-y)(x+y)^{-2} = \frac{-(x+y) - (x-y)}{(x+y)^2} = \frac{-2x}{(x+y)^2}$

26 $f(x,t) = \arctan(x\sqrt{t})$

$\boxed{f_x = \frac{\sqrt{t}}{1+x^2t}, \quad f_t = \frac{\frac{1}{2}t^{-\frac{1}{2}}x}{1+x^2t}}$

29 $f(x,y,z) = xz - 5x^2y^3z^4$

$\boxed{f_x = z - 10xy^3z^4, \quad f_y = -15x^2y^2z^4, \quad f_z = x - 20x^2y^3z^3}$

203 § 14.3 #s 30, 47, 50, 52-4, 56, 59, 71, 78, 82, 83

30 $f(x,y,z) = x\sin(y-z)$

$$\boxed{f_x = \sin(y-z),\ f_y = x\cos(y-z),\ f_z = -x\cos(y-z)}$$

#s 45-8 Find z_x, z_y by implicit diff.

47 $x - z = \arctan(yz) \Rightarrow$

$$1 - z_x = \frac{y z_x}{1+y^2z^2} \Rightarrow$$

$$1+y^2z^2 - z_x(1+y^2z^2) = y z_x$$

$$\Rightarrow (y+1+y^2z^2) z_x = (y^2z^2+1)$$

$$\Rightarrow \boxed{z_x = \frac{\partial z}{\partial x} = \frac{(y^2z^2+1)}{y^2z^2+y+1}}$$

$$-z_y = \frac{z + y z_y}{1+z^2y^2} \Rightarrow$$

$$-z_y - z^2y^2 z_y = z + y z_y$$

$$(y+1+z^2y^2) z_y = -z$$

$$\boxed{z_y = \frac{-z}{z^2y^2+y+1}}$$

203 §14.3 #s 50, 52-4, 56, 59, 71, 78, 82, 83

52-3

49 (a) $\cancel{z = f(x) + g(y)} \Rightarrow$

$$\boxed{z_x = f'(x), \quad z_y = g'(y)}$$

(b) $z = f(x+y) \Rightarrow \boxed{z_x = f'(x+y) = z_y}$

50 (a) $z = f(x)g(y) \Rightarrow \boxed{\begin{aligned} z_x &= f'(x)g(y) \\ z_y &= f(x)g'(y) \end{aligned}}$

(b) $z = f(xy) \Rightarrow \boxed{z_x = \frac{\partial f}{\partial x} y, \quad z_y = \frac{\partial f}{\partial y} x}$

#s 51-6 ~~Fall~~ Find all 2nd partials

52 $f(x,y) = \sin^2(mx+ny)$

$f_x = 2m \sin(mx+ny)\cos(mx+ny)$

$f_{xx} = 2m^2\left[\cos^2(mx+ny) - \sin^2(mx+ny)\right] = 2m^2 \cos(2(mx+ny))$

$= f_{yx}$

$f_{xy} = 2mn\left[\cos^2(mx+ny) - \sin^2(mx+ny)\right] = 2mn \cos(2(mx+ny))$

$f_y = 2n \sin(mx+ny)\cos(mx+ny)$

$f_{yy} = 2n^2\left[\cos^2(mx+ny) - \sin^2(mx+ny)\right] = 2n^2 \cos(2(mx+ny))$

$f_{yx} = 2nm\left[\cos^2(mx+ny) - \sin^2(mx+ny)\right]$

203 §14.3 #s 53, 56, 59, 71, 78, 82, 83

53 $w = \sqrt{u^2+v^2} = (u^2+v^2)^{\frac{1}{2}}$

$w_u = \frac{1}{2}(u^2+v^2)^{-\frac{1}{2}}(2u) = u(u^2+v^2)^{-\frac{1}{2}}$

$w_{uu} = -\frac{1}{4}(u^2+v^2)^{-\frac{3}{2}}(4u^2) + (u^2+v^2)^{-\frac{1}{2}} = -\frac{4u^2+u^2+v^2}{(u^2+v^2)^{3/2}} = -\frac{2u^2+v^2}{(u^2+v^2)^{3/2}}$

$w_{uv} = -\frac{1}{4}(u^2+v^2)^{-\frac{3}{2}}(2v)(2u) = \frac{-uv}{(u^2+v^2)^{3/2}}$

$w_v = \frac{1}{2}(u^2+v^2)^{-\frac{1}{2}}(2v)$

$w_{vv} = -\frac{1}{4}(u^2+v^2)^{-\frac{1}{2}}(4v^2) + (u^2+v^2)^{-\frac{1}{2}} = \frac{-v^2+1}{\sqrt{u^2+v^2}}$

$w_{vu} = -\frac{1}{4}(u^2+v^2)^{-\frac{3}{2}}(2u)(2v) = \frac{-uv}{(u^2+v^2)^{3/2}}$

56 $v = e^{xe^y}$

$\boxed{v_x = e^y e^{xe^y}, \quad v_{xx} = e^{2y} e^{xe^y},} \quad v_{xy} =$

$\ln(v) = xe^y$

$\frac{v_y}{v} = xe^y \Longrightarrow v_y = v\,xe^y = e^{xe^y} xe^y = \boxed{xe^{xe^y+y} = v_y}$

$\frac{v_y}{x} = e^{xe^y+y}$

$\ln\left(\frac{v_y}{x}\right) = xe^y + y$

$\frac{\frac{v_{yy}}{x}}{\frac{v_y}{x}} = xe^y + 1 = \frac{v_{yy}}{v_y}$

$v_{yy} = v_y(e^y x + 1) = \boxed{(xe^{xe^y+y})(xe^y+1) = v_{yy}}$

203 §14.3 #s 59, 71, 78, 82, 83

57) Verify Clairaut

$u = x\sin(x+2y)$

$u_x = \sin(x+2y) + x\cos(x+2y)$

$u_{xy} = 2\cos(x+2y) - x\sin(x+2y)$

$u_y = 2x\cos(x+2y)$

$u_{yx} = 2\cos(x+2y) - 2x\sin(x+2y)$

59) $u = \ln\sqrt{x^2+y^2} = \ln\left((x^2+y^2)^{\frac{1}{2}}\right) = \frac{1}{2}\ln(x^2+y^2)$

$u_x = \frac{1}{2}\left(\frac{2x}{x^2+y^2}\right) = \frac{x}{x^2+y^2} = x(x^2+y^2)^{-1}$

$u_{xy} = -2xy(x^2+y^2)^{-2}$

$u_y = \frac{\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2y)}{(x^2+y^2)^{\frac{1}{2}}} = y(x^2+y^2)^{-1}$

$u_{yx} = -(x^2+y^2)^{-2}(2x)y = -2xy(x^2+y^2)^{-2}$

71) Verify $u = e^{-\alpha^2k^2t}\sin(kx)$ solves

Heat Conduction Eq'n $u_t = \alpha^2 u_{xx}$

$u_t = -\alpha^2k^2e^{-\alpha^2k^2t}\sin(kx)$

$u_x = e^{-\alpha^2k^2t}\cdot k\cos(kx)$

$u_{xx} = -k^2e^{-\alpha^2k^2t}\sin(kx)$

$\therefore\ u_t = \alpha^2 u_{xx}$

203 S' 14.3 #s 78, 82-3

(78) Show that Cobb-Douglas production function $P = bL^{\alpha}K^{\beta}$ satisfies

$$L\frac{\partial P}{\partial L} + K\frac{\partial P}{\partial K} = (\alpha+\beta)P$$

$$\frac{\partial P}{\partial L} = \alpha b L^{\alpha-1}K^{\beta} \longrightarrow$$

$$\frac{\partial P}{\partial K} = \beta b L^{\alpha}K^{\beta-1}$$

$$L\frac{\partial P}{\partial L} + K\frac{\partial P}{\partial K} = \alpha b L^{\alpha}K^{\beta} + \beta b L^{\alpha}K^{\beta}$$

$$= (\alpha+\beta)P \checkmark$$

(82) gas law for mass m of an ideal gas @ Temp T, press P, volume V is $PV = mRT$, where R = gas constant.

Show that $P_V V_T T_P = -1$

$$\frac{\partial P}{\partial V}V + P = 0 \Rightarrow \frac{\partial P}{\partial V} = -\frac{P}{V} = P_V$$

$$PV_T = mR \Rightarrow \frac{\partial V}{\partial T} = \frac{mR}{P} = V_T$$

$$V = mRT_P \Rightarrow T_P = \frac{V}{mR}$$

$$\Rightarrow P_V T_P V_T = -\frac{\cancel{P}}{\cancel{V}} \cdot \frac{\cancel{V}}{\cancel{mR}} \cdot \frac{\cancel{mR}}{\cancel{P}} = -1 \checkmark$$

203 §14.3 #83

83 For ideal gas of #83, show that

$T P_T V_T = mR$

P_T : $P_T V = mR \implies P_T = \frac{mR}{V}$

Using #82 :

$T P_T V_T = T \frac{mR}{V} \cdot \frac{mR}{P} = \frac{T(mR)^2}{PV} = \frac{T(mR)^2}{mRT} = mR$