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§14.2 #s 5, 8, 13, 16, 25, 29-32, 39, 40

#s 5-20 Find limit if it $\exists$, or show that it doesn't.

(5) $\lim_{(x,y)\to(1,2)} (5x^3 - x^2y^2) = 5(1)^3 - 1^2(2)^2 = \boxed{1 = \lim_{(x,y)\to(1,2)} f}$

(8) $\boxed{\lim_{(x,y)\to(1,0)} \ln\left(\frac{1+y^2}{x^2+xy}\right) = \ln\left(\frac{1}{1}\right) = 0}$

(13) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

Along $y = mx$: $\frac{xmx}{\sqrt{x^2+m^2x^2}} = \frac{mx^2}{\sqrt{(m^2+1)x^2}} = \frac{mx^2}{\sqrt{(m^2+1)}\,|x|}$

$= \frac{mx^2}{\sqrt{m^2+1}\,|x|} = \begin{cases} \frac{mx}{\sqrt{m^2+1}} & x \geq 0 \\ \frac{-mx}{\sqrt{m^2+1}} & x < 0 \end{cases} \xrightarrow{(x,y)\to(0,0)} \boxed{0 = \lim_{(x,y)\to(0,0)} f}$

(16) $\boxed{\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2(y)}{x^2+2y^2} = 0}$ See below

Along $y = mx$:

$\frac{x^2\sin^2(mx)}{x^2+2m^2x^2} = \frac{x^2\sin^2(mx)}{(2m^2+1)x^2} = \frac{\sin^2(mx)}{2m^2+1} \xrightarrow{x\to 0} 0$

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#s 25–6 Find $h(x,y) = g(f(x,y))$ & the set on which $h(x,y)$ is cont.

(25) $g(t) = t^2 + \sqrt{t}$ $\quad \underbrace{f(x,y) = 2x+3y-6}_{\mathcal{D} = \mathbb{R}\times\mathbb{R}}$

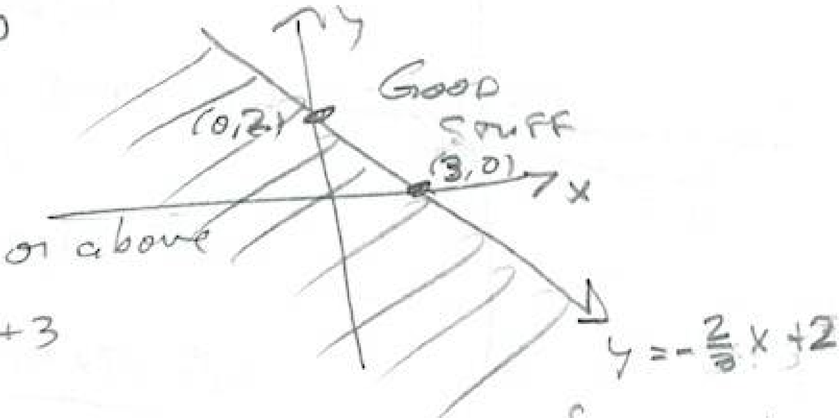
$h(x,y) = (2x+2y-6)^2 + \sqrt{2x+2y-6}$

Need $2x+3y-6 \geq 0$

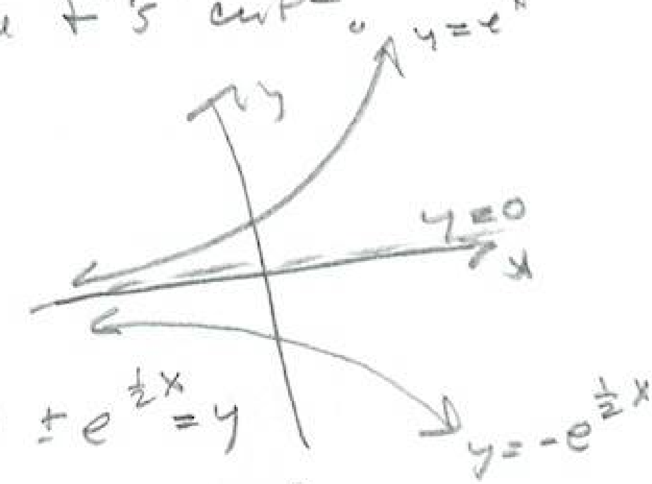
$3y \geq -2x+6$

$y \geq -\frac{2}{3}x + 3$

Everything on or above the line $y = -x+3$



#s 29–38 Determine where F's cont.



(29) $F(x,y) = \dfrac{\sin(xy)}{e^x - y^2}$

Need $e^x \neq y^2$, i.e.

$y \neq \pm (e^x)^{\frac{1}{2}} = \pm e^{\frac{1}{2}x}$

Cont: Everything not on the graph of $\pm e^{\frac{1}{2}x} = y$

(30) $F(x,y) = \dfrac{x-y}{1+x^2+y^2} \Longrightarrow$ Cont on $\mathbb{R}^2$.

(31) $F(x,y) = \arctan(x+\sqrt{y})$ cont on half-plane $y > 0$ ($y \geq 0$) $\Big\}$ Depends on how stringent we are about continuity on the boundary $y = 0$.

203 §14.2 #s 32, 39, 40

(32) $F(x,y) = e^{x^2y} + \sqrt{x+y^2}$

Need $x+y^2 \ge 0$

$x \ge -y^2$

Everything on or to the right of $x = -y^2$

#s 39–41 Use Polar coords to evaluate.

(39) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+y^3}{x^2+y^2} = \lim f(x,y)$

$$f(x,y) = \frac{(x+y)(x^2-xy+y^2)}{r^2} = \frac{(r\cos\theta + r\sin\theta)(r^2 - r^2\cos\theta\sin\theta)}{r^2}$$

$$= \frac{r(\cos\theta+\sin\theta)\,r^2(1-\cos\theta\sin\theta)}{r^2}$$

$$= r(\cos\theta+\sin\theta)(1-\cos\theta\sin\theta) \xrightarrow{r\to 0} \boxed{0 = \lim f}$$

(40) $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = \lim f$

$$f(r,\theta) = r^2\ln(r^2) = \frac{\ln(r^2)}{\frac{1}{r^2}} \xrightarrow{r\to 0^+} \frac{-\infty}{\infty}$$

$$\xrightarrow[L'H]{r\to 0^+} \frac{\frac{2r}{r^2}}{-2r^{-3}} = \frac{\frac{2}{r}}{\frac{-2}{r^3}} = -\frac{1}{r}\cdot\frac{r^3}{1} = -r^2 \xrightarrow{r\to 0^+} 0$$

$$\boxed{\lim f = 0}$$