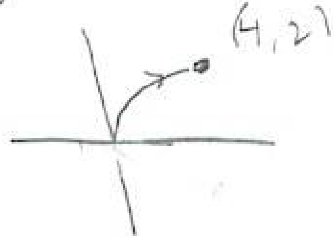
203 §13.2 #s 2, 3, 7, 10, 11, 14, 19, 20, 23, 32, 37, 41, 53

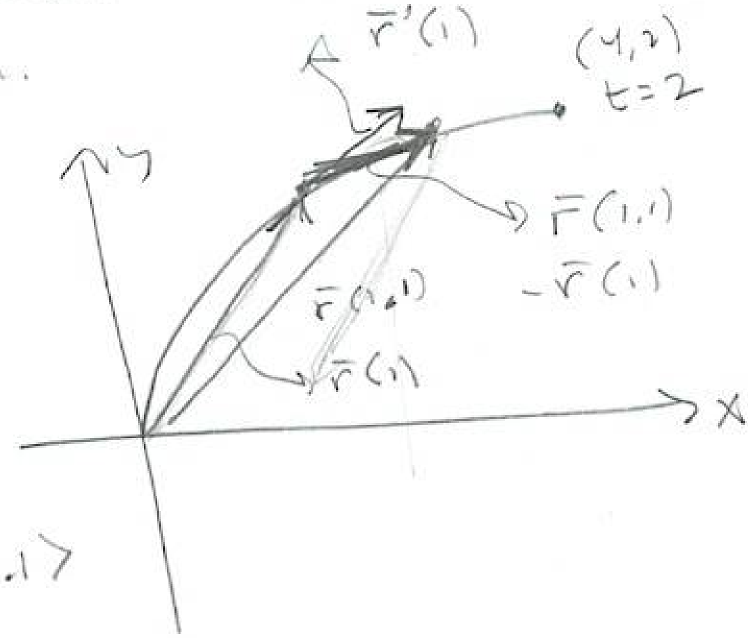
(2) (a) Make a large sketch of $\bar{r}(t) = \langle t^2, t \rangle$, $0 \le t \le 2$ & draw $\bar{r}(1), \bar{r}(1.1), \bar{r}(1.1) - r(1)$

(b) Draw $\bar{r}\,'(1)$ starting @ $(1,1)$ & compare it to

$$\frac{\bar{r}(1.1) - \bar{r}(1)}{.1}$$

Explain why these vectors are so similar in length & direction.





$\bar{r}(1) = \langle 1, 1 \rangle$

$\bar{r}(1.1) = \langle 1.21, 1.1 \rangle$

$\bar{r}(1.1) - \bar{r}(1) = \langle .21, .1 \rangle$

$\bar{r}\,'(1) = \langle 2, 1 \rangle$

$$\frac{\bar{r}(1.1) - \bar{r}(1)}{.1} = \frac{\langle .21, .1 \rangle}{.1} = \langle 2.1, 1 \rangle$$

$$\frac{\bar{r}(1.1) - \bar{r}(1)}{.1} \approx \bar{r}\,'(1) = \lim_{h \to 0} \frac{\bar{r}(1+h) - \bar{r}(1)}{h}$$

203 §13.2 #s 3, 7, 10, 11, 14, 19, 20, 23, 32, 37, 41, 53

#s 3–8 (a) sketch

(b) Find $\bar{r}'(t)$

(c) sketch $\bar{r}(t_0)$, $\bar{r}'(t_0)$

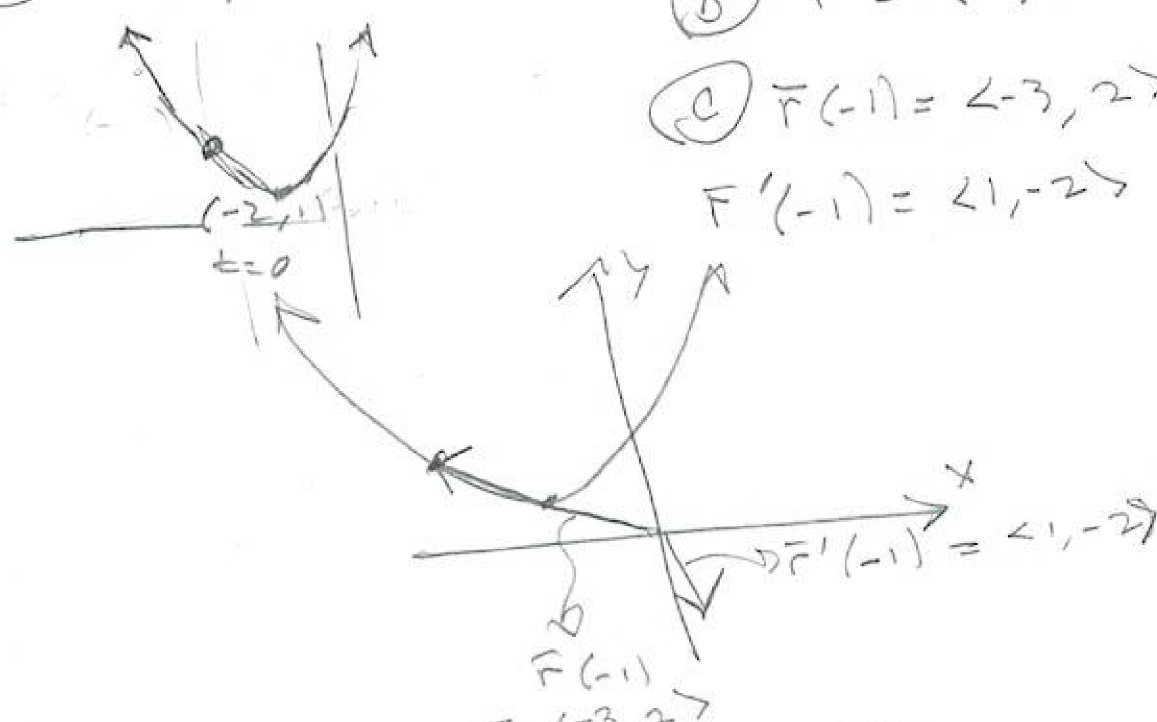
(3) $\bar{r}(t) = \langle t-2,\ t^2+1 \rangle$, $t_0 = -1$

(a) $= \langle x,\ (x+2)^2+1 \rangle$

(b) $\bar{r}' = \langle 1, 2t \rangle$

(c) $\bar{r}(-1) = \langle -3, 2 \rangle$

$\bar{r}'(-1) = \langle 1, -2 \rangle$



(7) $\bar{r}(t) = \langle 4\sin t, -2\cos t \rangle$, $t_0 = \frac{3\pi}{4}$

$\frac{x}{4} = \sin t$, $\frac{y}{-2} = \cos t$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$ Ellipse

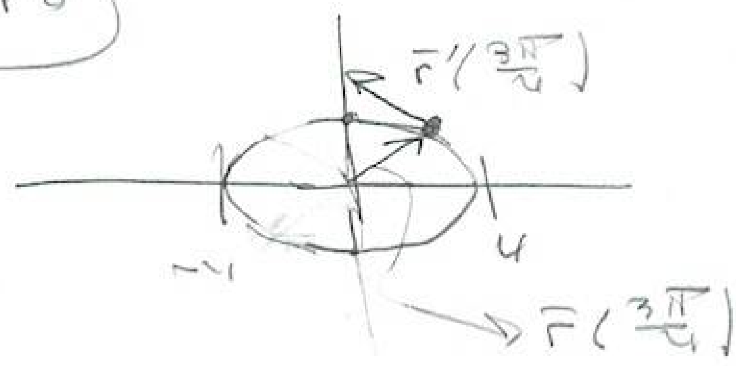
$\bar{r}\left(\frac{3\pi}{4}\right) = \langle 4\left(\frac{1}{\sqrt{2}}\right), -2\left(-\frac{1}{\sqrt{2}}\right) \rangle = \langle \frac{4}{\sqrt{2}}, \frac{2}{\sqrt{2}} \rangle$

$\bar{r}' = \langle 4\cos t, 2\sin t \rangle$

$\bar{r}'\left(\frac{3\pi}{4}\right) = \langle \frac{-4}{\sqrt{2}}, \frac{2}{\sqrt{2}} \rangle$

203 §13.2 #s 7, 10, 11, 14, 19, 20, 23, 32, 37, 41, 53

7 cont'd



#s 9–16 Find $\bar{r}'$.

(10) $\bar{r}(t) = \langle \sqrt{t-2}, 3, \frac{1}{t^2} \rangle$

$\Rightarrow \bar{r}' = \langle \frac{1}{2\sqrt{t-2}}, 0, \frac{-2}{t^3} \rangle$

(11) $\bar{r}(t) = \langle t^2, \cos(t^2), \sin^2(t) \rangle \longrightarrow$

$\boxed{\bar{r}' = \langle 2t, -2t\sin(t^2), 2\sin(t)\cos(t) \rangle}$

(14) $\bar{r}(t) = \langle \sin^2(at), te^{bt}, \cos^2(ct) \rangle$

$\Rightarrow \boxed{\bar{r}' = \langle 2a\sin(at)\cos(at), e^{bt} + bte^{bt}, 2c\cos(ct)\sin(ct) \rangle}$

#s 17–20 Find unit tangent vector $\bar{T}(t_0)$.

(19) $\bar{r}(t) = \langle \cos t, 3t, 2\sin(2t) \rangle, t_0 = 0$

$\bar{r}' = \langle -\sin t, 3, 4\cos(2t) \rangle$

$\|\bar{r}'\| = \sqrt{\sin^2 t + 9 + 16\cos^2(2t)}$

$\bar{T}(0) = \frac{\langle 0, 3, 4 \rangle}{\sqrt{9+16}} = \boxed{\frac{1}{5}\langle 0, 3, 4 \rangle}$

203 § 13.2 #s 20, 23, 32, 37, 41, 53

20 $\bar{r} = \langle \sin^2 t, \cos^2 t, \tan^2 t \rangle$, $t_0 = \frac{\pi}{4}$

$\bar{r}' = \langle 2\sin t \cos t, -2\cos t \sin t, 2\tan t \sec^2 t \rangle$

$$\bar{T}\left(\frac{\pi}{4}\right) = \frac{\langle 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}, -2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}, 2(1)\frac{\sqrt{2}^2}{1^2}\rangle}{\sim}$$

$$= \frac{\langle 1,-1,4\rangle}{\sqrt{1+1+16}} = \frac{1}{\sqrt{18}}\langle 1,-1,4\rangle = \boxed{\frac{1}{3\sqrt{2}}\langle 1,-1,4\rangle}$$

#s 23-26

23 Find parametric eq'ns of tangent line to the curve @ the specified point.

$x = t^2+1, y = 4\sqrt{t}, z = e^{t^2-t}$, $(2,4,1) \rightsquigarrow t = 1$

$\bar{r}' = \langle 2t, \frac{4}{2\sqrt{t}}, (2t-1)e^{t^2-t} \rangle$

$\bar{r}'(1) = \langle 2, 2, 1\rangle$

So Tangent Line is $\bar{r} = \langle 2,4,1\rangle + t\langle 2,2,1\rangle$

$\Rightarrow \boxed{x = 2t+2, y = 2t+4, z = t+1}$

203 §13.2 #s 32, 37, 41, 53

(32) (a) Find pt of intersection of tangent lines to $\bar{r} = \langle \sin \pi t, 2\sin \pi t, \cos \pi t \rangle$ @ $t = 0, 0.5$

$\bar{r}(0) = \langle 0, 0, 1 \rangle$

$\bar{r}(\frac{1}{2}) = \langle \sin \frac{\pi}{2}, 2\sin \frac{\pi}{2}, \cos \frac{\pi}{2} \rangle = \langle 1, 2, 0 \rangle$

$\bar{r}'(t) = \langle \pi \cos \pi t, 2\pi \cos \pi t, -\pi \sin \pi t \rangle$

$\bar{r}'(0) = \langle \pi, 2\pi, 0 \rangle$

$\bar{r}'(\frac{1}{2}) = \langle \pi \cos \frac{\pi}{2}, 2\pi \cos \frac{\pi}{2}, -\pi \rangle = \langle 0, 0, -\pi \rangle$

L_1: $\langle 0, 0, 1 \rangle + t \langle \pi, 2\pi, 0 \rangle = \langle \pi t, 2\pi t, 1 \rangle$

L_2: $\langle 1, 2, 0 \rangle + s \langle 0, 0, -\pi \rangle = \langle 1, 2, -\pi s \rangle$

$\pi t = 1$ $\quad t = \frac{1}{\pi}$

$2\pi t = 2$ $\quad s = -\frac{1}{\pi}$

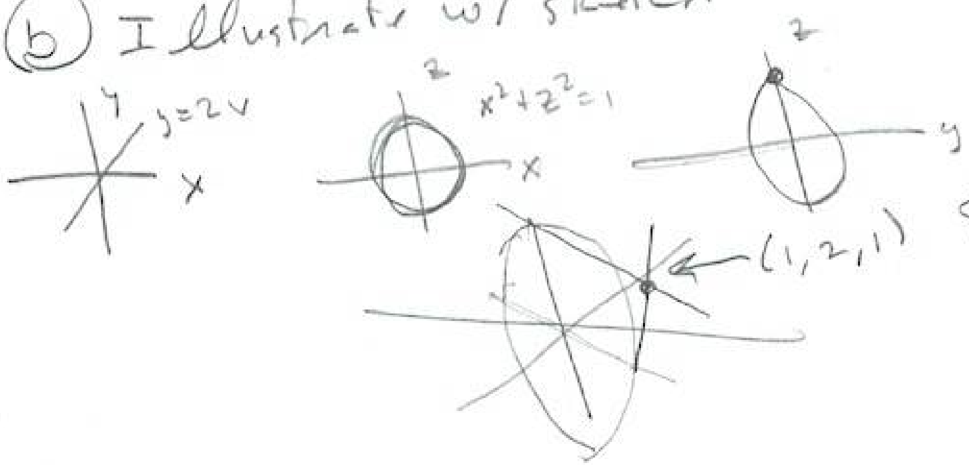
$1 = -\pi s$

Check: L_1: $\langle \pi \cdot \frac{1}{\pi}, 2\pi \cdot \frac{1}{\pi}, 1 \rangle = \langle 1, 2, 1 \rangle$ ✓

L_2: $\langle 1, 2, -\pi \cdot -\frac{1}{\pi} \rangle = \langle 1, 2, 1 \rangle$ ✓

Point of intersection = $\boxed{(1, 2, 1)}$

(b) Illustrate w/ sketch



sort of.

203 §13.2 #s 37, 41, 53

37 Evaluate $\displaystyle\int_0^1 \left\langle \frac{1}{t+1}, \frac{1}{t^2+1}, \frac{1}{2}\frac{2t}{t^2+1} \right\rangle dt$

$$= \left\langle \ln(t+1), \arctan(t), \tfrac{1}{2}\ln(t^2+1) \right\rangle \Big|_0^1$$

$$= \left\langle \ln(2)-\ln(1), \arctan(1)-\arctan(0), \tfrac{1}{2}\ln(2)-\tfrac{1}{2}\ln(1) \right\rangle$$

$$= \boxed{\left\langle \ln(2), \tfrac{\pi}{4}, \tfrac{1}{2}\ln(2) \right\rangle}$$

41 Find $\bar{r}(t)$ if $\bar{r}'(t) = \langle 2t, 3t^2, \sqrt{t} \rangle$ &

$\bar{r}(1) = \langle 1, 1, 0 \rangle$

$$\boxed{\bar{r}(t) = \left\langle t^2 + c_1, t^3 + c_2, \tfrac{2}{3}t^{\frac{3}{2}} + c_3 \right\rangle}$$

$$\bar{r}(1) = \left\langle 1 + c_1, 1 + c_2, \tfrac{2}{3} + c_3 \right\rangle = \langle 1, 1, 0 \rangle \Longrightarrow$$

$$\boxed{c_1 = c_2 = 0, \; c_3 = -\tfrac{2}{3}}$$

$$\boxed{\bar{r}(t) = \left\langle t^2, t^3, \tfrac{2}{3}t^{3/2} - \tfrac{2}{3} \right\rangle}$$

203 § 13.2 #53

53 Show that if $\bar{r}''$ $\exists$, then

$$[\bar{r} \times \bar{r}']' = \bar{r} \times \bar{r}''$$

By Thm, $(\bar{r} \times \bar{r}')' = \bar{r}' \times \bar{r}' + \bar{r} \times \bar{r}''$

$= 0 + \bar{r} \times \bar{r}''$, since $\bar{a} \times \bar{a} = 0 \ \forall \bar{a} \in V^3$

$= \bar{r} \times \bar{r}''$ $\blacksquare$

$\langle r_1, r_2, r_3 \rangle, r_1, r_2$

$\langle r_1', r_2', r_3' \rangle, r_1', r_2'$

$\bar{r} \times \bar{r}' = \langle r_2 r_3' - r_2' r_3, r_3 r_1' - r_3' r_1, r_1 r_2' - r_1' r_2 \rangle$

$\bar{r} \times \bar{r}'' = \langle r_2 r_3'' - r_2'' r_3, r_3 r_1'' - r_3'' r_1, r_1 r_2'' - r_1'' r_2 \rangle$

$$\begin{aligned}(\bar{r} \times \bar{r}')' = \langle & r_2' r_3' + r_2 r_3'' - r_2'' r_3 - r_2' r_3', \\ & r_3' r_1' + r_3 r_1'' - r_3'' r_1 - r_3' r_1', \\ & r_1' r_2' + r_1 r_2'' - r_1'' r_2 - r_1' r_2' \rangle\end{aligned}$$

$= \langle r_2 r_3'' - r_2'' r_3, r_3 r_1'' - r_3'' r_1, r_1 r_2'' - r_1'' r_2 \rangle$

$= \bar{r} \times \bar{r}''$ $\blacksquare$