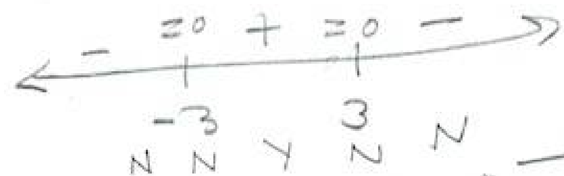
203 §13.1 #s 1, 4, 7, 8, 10, 13, 15, 18, 21-26, 27, 31, 33

① Find $\mathcal{D}(\bar r(t))$, where

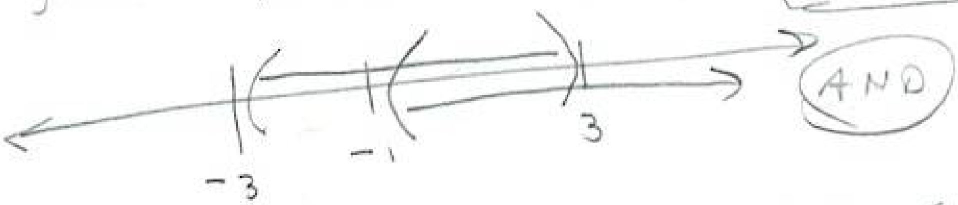
$$\bar r(t) = \langle \ln(t+1), \frac{t}{\sqrt{9-t^2}}, 2^t \rangle$$

Need $t+1>0$ & $9-t^2 \geq 0$ & $9-t^2 \neq 0$

$\Rightarrow t > -1$ & $(3-t)(3+t) > 0$



This gives $(-1, \infty) \cap (-3, 3) = \boxed{(-1, 3) = \mathcal{D}(\bar r)}$



④ Find the limit: $\lim_{t\to 1} \langle \frac{t^2-t}{t-1}, \sqrt{t+8}, \frac{\sin \pi t}{\ln t} \rangle = \lim_{t\to} \bar r(t)$

$$\frac{t^2-t}{t-1} = \frac{t(t-1)}{t-1} = t \xrightarrow{t\to1} 1$$

$$\sqrt{t+8} \xrightarrow{t\to1} \sqrt{9} = 3$$

$$\frac{\sin \pi t}{\ln t} \xrightarrow{t\to1} \frac{0}{0} \xrightarrow[\text{L'H}]{t\to1} \frac{\pi \cos \pi t}{\frac{1}{t}} = \pi t \cos \pi t \Rightarrow$$

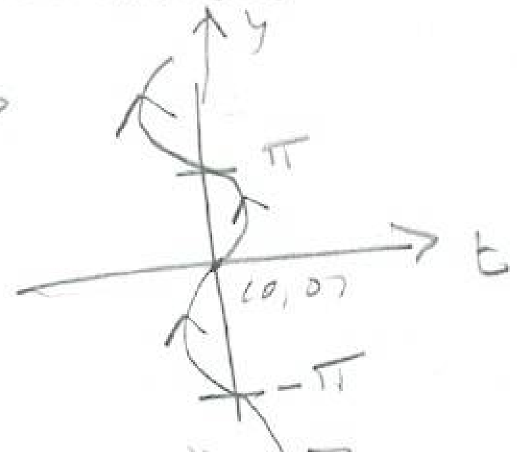
$$\xrightarrow{t\to1} \pi(1)(\cos\pi) = -\pi$$

$$\boxed{\lim_{t\to1} \bar r(t) = \langle 1, 3, -\pi \rangle}$$

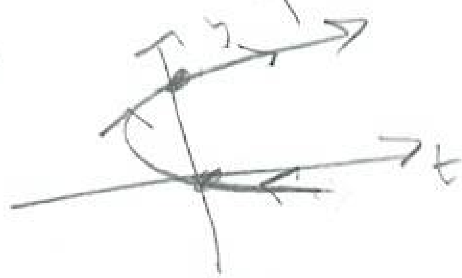
203 §13.1 #s 7, 8, 10, 13, 15, 18, 21–27, 31, 33

#s 7–14 sketch the curve w/ the given vector eq'n. Indicate the direction of increasing t with an arrow

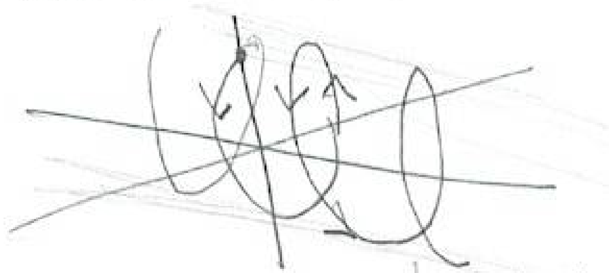
(7) $\bar{r}(t) = \langle \sin t, t \rangle$



(8) $\bar{r}(t) = \langle t^2 - t, t \rangle$



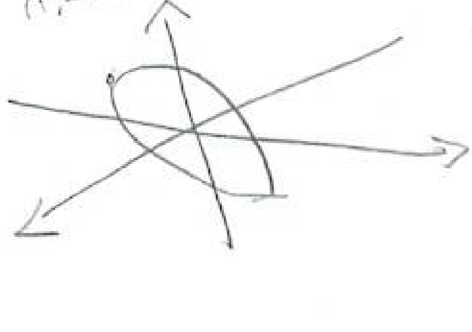
(10) $\bar{r}(t) = \langle \sin \pi t, t, \cos \pi t \rangle$



How to be anti-racist

(13) $\bar{r}(t) = \langle \cos t, -\cos t, \sin t \rangle$

Tilted circle?

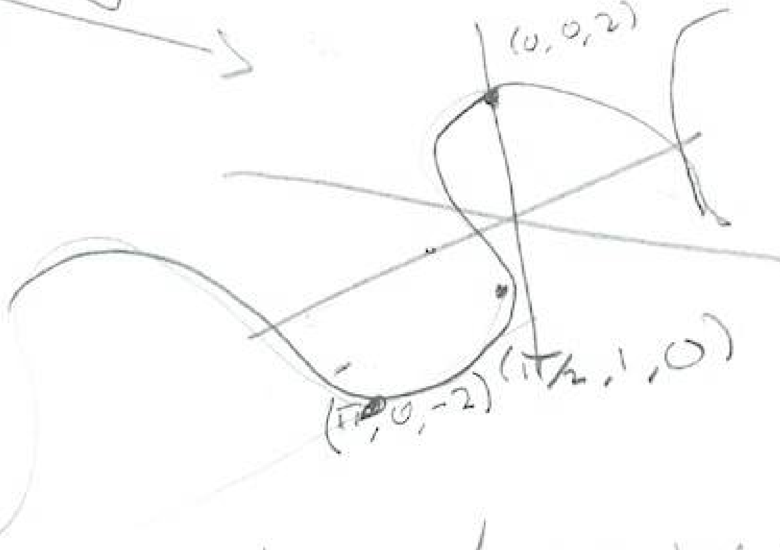


203 § 13.1 #s 15, 18, 21-27, 31, 33

(15) Use projections to help sketch the curve.

$\bar{r}(t) = \langle t, \sin t, 2\cos t \rangle$

struggling to sketch sort of a spiral.



(18) Find vector eq'n & parametric eq'ns for line segment from P(-1, 2, -2) to Q(-3, 5, 1)

$\bar{r}(t) = (1-t)\langle -1,2,-2\rangle + t\langle -3,5,1\rangle$

$= \langle -1,2,-2\rangle + t\langle 1,-2,2\rangle + t\langle -3,5,1\rangle$

$= \boxed{\langle -1,2,-2\rangle + t\langle -2,3,3\rangle}$

$\boxed{x = -2t-1,\ y = 3t+2,\ z = 3t-2}$

203 S'13 w1 #s 21-27, 31, 33

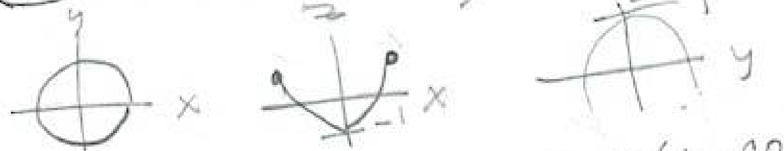
#s 21- 26 Matching

21 $\boxed{II}$ $\langle t\cos t, t, t\sin t\rangle$

22 $\boxed{IV}$ $\langle \cos t, \sin t, \frac{1}{t^2+1}\rangle$

23 $\langle t, \frac{1}{t^2+1}, t^2\rangle$ $\boxed{V}$

24 $\langle \cos t, \sin t, \cos 2t\rangle$



$\cos(2t) = 1-2\sin^2 t = 1-2(1-\cos^2 t) = 1-2+2\cos^2 t$

$= 2\cos^2 t - 1$

$\frac{\pi}{6}$ $\frac{1}{2}$ $1-2\sin^2(\frac{\pi}{6}) = 1-2(\frac{1}{4}) = \frac{1}{2}$

$= 2x^2-1$

$= 2(x^2-\frac{1}{2})$

$\frac{\pi}{3}\cdot\frac{1}{2}$ $1-2\sin^2(\frac{\pi}{3}) = 1-2(\frac{3}{4}) = -\frac{1}{2}$

$= 2(x-\frac{1}{\sqrt{2}})(x+\frac{1}{\sqrt{2}})$

$\frac{\pi}{2}$ -1, $1-2\sin^2\frac{\pi}{2} = -1$

$\cos 2t = 1-2\sin^2 t = 1-2y^2 = z$

$\boxed{V}$

$1-2y^2=0$

$y = \pm\frac{1}{\sqrt{2}}$

$\boxed{VI}$

25 $\langle \cos 8t, \sin^2 t, z=t\rangle$

$x = \sin^2 t = \frac{1-\cos 2t}{2} = \frac{1}{2} - \frac{1}{2}\cos(2t)$

26 $\langle \cos^2 t, \sin^2 t, t\rangle$

$x+y=1$

III

$\cos 8t = 1-2\sin^2(4t)$

$= 1-2(1-\cos^2(4t))$

$= 1-2+2\cos^2(2t))$

$= -1+2(1-2\sin^2(2t)$ $+4\cos^2(2t)$

$= -1+2-4\sin^2(2t)$

$= 1-4(1-\cos^2(2t)) = 1-4$

$= 4\cos^2(2t) - 3 = x$

$y = \frac{1}{2} - \frac{1}{2}\cos(2t)$

$y - \frac{1}{2} = -\frac{1}{2}\cos(2t)$

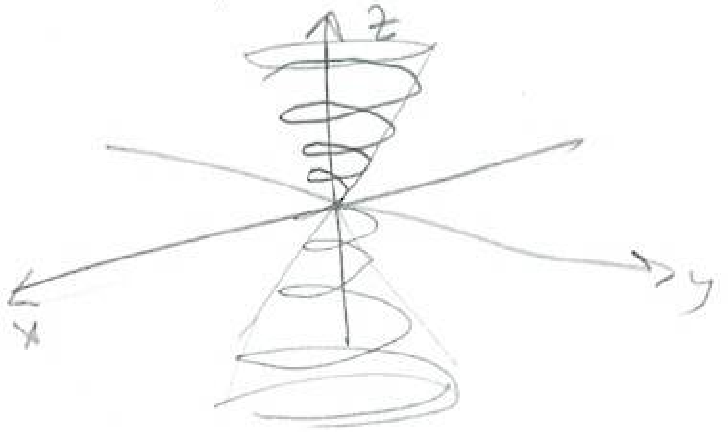
$-2y+1 = \cos(2t)$

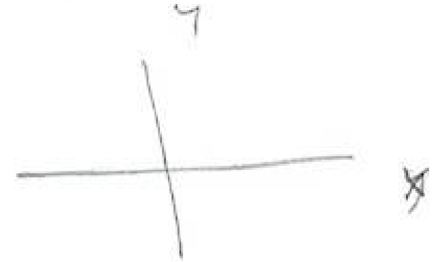
so $x = \frac{1}{2} - \frac{1}{2}(-2y+1) = -\frac{1}{2} + y - \frac{1}{2} = y$

203 §13.1 #s 27, 31, 33

(27) Show that $< t\cos t, t\sin t, t >$ lies on the cone $z^2 = x^2 + y^2$ & use this to help sketch.

$x^2 + y^2 = t^2 = z^2$, b/c $\sin^2 t + \cos^2 t = 1$





(31) At what points does $< t, 0, 2t - t^2 >$ intersect the paraboloid $z = x^2 + y^2$

$y = 0 \Rightarrow z = x^2$

The intersection is the curve

$\vec{r}(t) = < t, 0, t^2 >$?

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

$y = 0$

$z = 0, 1$

2 points

$\boxed{(0,0,0), (1,0,1)}$

203 §13.1 # 33

33 Use a computer to graph

$\vec{r}(t) = \langle \cos(t)\sin(2t), \sin(t)\sin(2t), \cos(2t) \rangle$