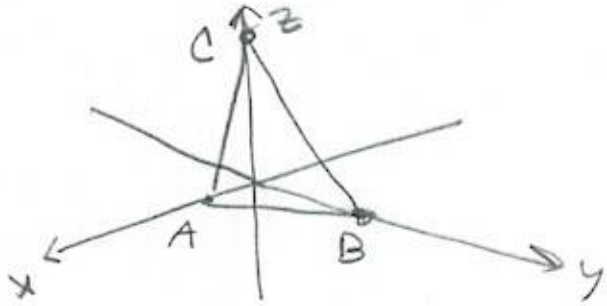


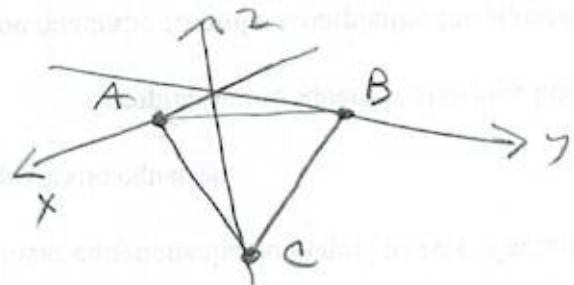
(41) Use intercepts to sketch $2x + 5y + z = 10$

$A(5, 0, 0), B(0, 2, 0), C(0, 0, 10)$



(44) $6x + 5y - 3z = 15$

$A(\frac{5}{2}, 0, 0), B(0, 3, 0), C(0, 0, -5)$



(48) where does line thru $\langle -3, 1, 0 \rangle$ & $\langle -1, 5, 6 \rangle$ intersect $2x + y - z = -2$?

$$\begin{aligned} \downarrow &: (1-t)\langle -3, 1, 0 \rangle + t\langle -1, 5, 6 \rangle \\ &= \langle -3, 1, 0 \rangle + \langle 3t, -t, 0 \rangle + \langle -t, 5t, 6t \rangle \\ &= \vec{r}(t) = \langle 2t, 4t, 6t \rangle + \langle -3, 1, 0 \rangle \\ &= \langle 2t-3, 4t+1, 6t \rangle \end{aligned}$$

$$\begin{aligned} 2(2t-3) + 4t+1 - 6t &= -2 \\ 4t-6-2t+1 &= 2t-5 = -2 \Rightarrow \\ 2t &= 3 \Rightarrow t = \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \langle 2(\frac{3}{2})-3, 4(\frac{3}{2})+1, 6(\frac{3}{2}) \rangle &\rightarrow \boxed{\langle 0, 7, 9 \rangle} \\ = \langle 3-3, 6+1, 9 \rangle &= \langle 0, 7, 9 \rangle \end{aligned}$$

203 of 125 II #s 49-51, 53, 58, 59, 61, 63-65, 67-9, 71, 73, 75, 76

(49) Find direction #s for line of intersection of $x+y+z=1$ & $x+z=0$

$$\begin{bmatrix} 1 & 0 & 1 & | & 0 \\ 1 & 1 & 1 & | & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & | & 0 \\ 0 & -1 & 0 & | & -1 \end{bmatrix}$$

$$y=1, x+z=-1$$

$$\langle x, y, z \rangle = \langle -t, 1, t \rangle \Rightarrow$$

Direction #s are $\langle -1, 0, 1 \rangle$

for line of intersection \rightarrow

Direction #s are $\boxed{\langle -1, 0, 1 \rangle}$ OR $\langle 1, 0, -1 \rangle$

(50) Find cosine of the angle between the planes $x+y+z=0$ & $x+2y+3z=1$

$$\vec{n}_1 = \langle 1, 1, 1 \rangle, \vec{n}_2 = \langle 1, 2, 3 \rangle$$

$$\cos \theta = \frac{1+2+3}{\sqrt{3}\sqrt{14}} = \frac{6}{\sqrt{42}} \text{ OR}$$

$$\boxed{\frac{\sqrt{42}}{7} = \cos \theta}$$

Angle between their normals = angle between the planes

(51) \parallel , \perp , neither? If neither, find angle between them, to one decimal.

$$x+4y-3z=1, -3x+6y+7z=0$$

$$\vec{n}_1 = \langle 1, 4, -3 \rangle, \vec{n}_2 = \langle -3, 6, 7 \rangle$$

Not multiples \Rightarrow not \parallel

$$\vec{n}_1 \cdot \vec{n}_2 = -3+24-21=0 \Rightarrow$$

$$\boxed{\perp}$$

Same instructions #s 51-56

203 S12.5R #s 53, 58, 59, 61, 63-65, 67-9, 71, 73, 75, 96

(53) $x+2y-z=2, 2x-2y+z=1$

$\vec{n}_1 = \langle 1, 2, -1 \rangle, \vec{n}_2 = \langle 2, -2, 1 \rangle$

Not ||, by inspection.

$\vec{n}_1 \cdot \vec{n}_2 = 2 - 4 - 1 = -3 \neq 0 \Rightarrow$ ~~not~~

$\cos \Theta = \frac{-3}{\sqrt{1+4+1}\sqrt{4+4+1}} = \frac{-3}{\sqrt{6}\sqrt{9}} = \frac{-3}{3\sqrt{6}} \Rightarrow$

$\Theta = \arccos\left(-\frac{3}{3\sqrt{6}}\right) \approx 65.90515745^\circ \approx \boxed{65.9^\circ} \approx \Theta$

≈ 1.150261992
 ≈ 1.2 radians

(58) (a) Find parametric eqns for line of intersection of the planes & (b) Find \angle between planes

$3x - 2y + z = 1, 2x + y - 3z = 3$

$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 3 \\ 3 & -2 & 1 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -3 & 3 \\ 0 & -7 & 11 & -7 \end{array} \right]$

$-7y + 11z = -7 \Rightarrow y - \frac{11}{7}z = 1 \Rightarrow y = \frac{11}{7}z + 1$

$2x + \frac{11}{7}z + 1 - 3z = 3$

$2x - \frac{10}{7}z = 2$

$2x = \frac{10}{7}z + 2$

$x = \frac{5}{7}z + 1$

Check: $(6, 12, 7)$

$3(6) - 2(12) + 7 = 1 \checkmark$

$2(6) + 12 - 3(7) = 3 \checkmark$

(b) $\cos \Theta = \frac{\vec{n}_0 \cdot \vec{n}_1}{\|\vec{n}_0\| \|\vec{n}_1\|} = \frac{6 - 2 - 3}{\sqrt{14}\sqrt{14}} = \frac{1}{14}$

$\Rightarrow \Theta \approx 85.90395624^\circ$

$\approx \boxed{85.9^\circ}$

203 S12.5 II #9, 59, 61, 63-5, 67-9, 71, 73, 75, 76

(59) $5x - 2y - 2z = 1, 4x + y + z = 6$

Find symmetric eqns of intersection

$$\left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 4 & 1 & 1 & 6 \end{array} \right] \sim \left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 0 & 13 & 13 & 26 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 0 & 1 & 1 & 2 \end{array} \right] \quad \begin{array}{l} 5x - 2y - 2z = 1 \\ y + z = 2 \\ y = 2 - z \end{array}$$

$$5x - 2(2 - z) - 2z = 1$$

$$5x - 4 + 2z - 2z = 1$$

$$5x = 5$$

$$x = 1$$

$$(1, 2 - t, t)$$

$$x = 1, y = 2 - t, z = t$$

$$\boxed{x = 1, 2 - y = z}$$

(61) Find eq'n for plane consisting of all points equidistant from $(1, 0, -2)$ & $(3, 4, 0)$.
 Direction vector for line segment between the two points is \vec{n} . midpoint is pt on plane.

$$\vec{n} = \langle 3-1, 4-0, 0-(-2) \rangle = \langle 2, 4, 2 \rangle = \vec{n}$$

$$\text{midpt: } \left(\frac{1+3}{2}, \frac{0+4}{2}, \frac{-2+0}{2} \right) = (2, 2, -1)$$

$$\text{So } 2(x-2) + 4(y-2) + 2(z+1) = 0$$

$$2x - 4 + 4y - 8 + 2z + 2 = 0$$

$$\boxed{2x + 4y + 2z = 10}$$

$$\text{OR } \boxed{x + 2y + z = 5}$$

203 #s 63-5, 67-9, 71, 73, 75, 76

(63) Find eqn of plane with x-int a, y-int b, z-int c

$$A(a, 0, 0), B(0, b, 0), C(0, 0, c)$$

$$\vec{u} = \vec{AB} = \langle -a, b, 0 \rangle, \vec{v} = \langle -a, 0, c \rangle$$

$$\vec{u} = \langle -a, b, 0 \rangle, -a, b$$

$$\times \vec{v} = \langle -a, 0, c \rangle, -a, 0$$

$$\vec{n} = \langle bc, ac, ab \rangle \rightarrow$$

$$bc(x-a) + acy + abz = 0$$

$$bcx - abc + acy + abz = 0$$

$$bcx + acy + abz = abc$$

(64) (a) Find pt (a) which the lines intersect?

$$\vec{r}_1 = \langle t+1, -t+1, 2t \rangle$$

$$\vec{r}_2 = \langle -s+2, s, 2 \rangle$$

$$t+1 = -s+2$$

$$-t+1 = s$$

$$2t = 2$$

$$t = 1$$

$$2 = -s+2$$

$$s = 0$$

$$\vec{r}_1(1) = \langle 2, 0, 2 \rangle$$

$$\vec{r}_2(0) = \langle 2, 0, 2 \rangle \checkmark$$

A(2, 0, 2) is intersection

(b) Eqn of plane containing them.

$$\vec{u} = \langle 1, -1, 2 \rangle, 1, -1$$

$$\vec{v} = \langle -1, 1, 0 \rangle, -1, 1$$

$$\vec{n} = \langle -2, -2, 0 \rangle$$

$$2x + 2y = 4$$

203 S/2.5 II * 65, 67-9, 71, 73, 75, 76

(65) Find parametric eqns for the line thru $A(0,1,2)$ that is \parallel to plane P and \perp to the line L_1 , where

$$L_1: \vec{r}(t) = \langle t+1, -t+1, 2t \rangle = \vec{r}_0 + t\vec{u}$$

$$P: x+y+z=2, \vec{n} = \langle 1, 1, 1 \rangle \perp \text{ to } P.$$

$$\vec{u} = \langle 1, -1, 2 \rangle \text{ is direction vector for } L_1.$$

$\vec{r}_0 = \langle 1, 1, 0 \rangle \in L_1 \cap P$ is convenient, since

$$1+1+0=2 \Rightarrow B(1,1,0) \in P.$$

\vec{n} is perpendicular to $L_2 =$ line we want. Let \vec{v} be direction vector for L_2

$$\text{Then } \vec{n} \cdot \vec{v} = \langle 1, 1, 1 \rangle \cdot \langle v_1, v_2, v_3 \rangle = 0$$

$$v_1 + v_2 + v_3 = 0$$

$$\text{Also } \vec{u} \cdot \vec{v} = \langle 1, -1, 2 \rangle \cdot \langle v_1, v_2, v_3 \rangle = 0$$

$$= v_1 - v_2 + 2v_3 = 0 \Rightarrow$$

$$v_1 + v_2 + v_3 = v_1 - v_2 + 2v_3 \Rightarrow$$

$$v_2 + v_3 = -v_2 + 2v_3 \Rightarrow$$

$$2v_2 = v_3 \Rightarrow$$

$$v_2 = \frac{1}{2}v_3 \Rightarrow$$

$$\text{So, } v_1 + v_2 + v_3 = 0 \Rightarrow$$

$$v_1 + \frac{1}{2}v_3 = 0$$

$$\Rightarrow v_1 = -\frac{1}{2}v_3$$

$$\text{So } \vec{v} = \langle v_1, \frac{1}{2}v_3, v_3 \rangle = \vec{v}$$

$$\text{So } \vec{v} = \langle -\frac{1}{2}v_3, \frac{1}{2}v_3, v_3 \rangle$$

$$\vec{v} = \langle 1, -1, 2 \rangle$$

203 S 12.5 IV #s 65, 67-9, 71, 73, 75, 76

#65 cont'd -

Now Build eq'n for L_2

$\vec{w}_0 = \langle 0, 1, 2 \rangle$ From $A(0, 1, 2) \in L_2$.

So L_2 is $\vec{w} = \vec{w}_0 + t\vec{v}$

$$= \langle 0, 1, 2 \rangle + t \langle 1, -1, 2 \rangle$$

$$= \langle t, -t+1, 2t+2 \rangle$$

$$\boxed{x = t, y = -t+1, z = 2t+2}$$

(67) which of the following planes are \parallel ?
Are any identical?

$$P_1: 3x + 6y - 3z = 6$$

$$x + 2y - z = 2$$

$$P_2: 4x - 12y + 8z = 5$$

$$x - 3y + 2z = \frac{5}{4}$$

$$P_3: 9y = 1 + 3x + 6z$$

$$-6z - 3x + 9y = 1$$

$$-3x + 9y - 6z = 1$$

$$P_4: z = x + 2y - 2$$

$$-x - 2y + z = -2$$

$$x + 2y - z = 2$$

$$x - 3y + 2z = -\frac{1}{3}$$

$$\boxed{P_1 = P_4, P_2 \parallel P_3}$$

203 § 12.5 #s 68-9, 71, 73, 75, 76

68) Which of the following 4 lines are \parallel ?
Are any identical?

$\mathcal{L}_1: x=1+6t, y=1-3t, z=12t+5$

$\vec{u}_1 = \langle 6, -3, 12 \rangle, \vec{r}_{01} = \langle 1, 1, 5 \rangle$

$\frac{1}{3}\vec{u}_1 = \langle 2, -1, 4 \rangle$

$\mathcal{L}_2: x=1+2t, y=t, z=1+4t$

$\vec{u}_2 = \langle 2, 1, 4 \rangle, \vec{r}_{02} = \langle 1, 0, 1 \rangle$

$\mathcal{L}_3: t = 2x - 2 = 4 - 4y = z + 1$

$\vec{u}_3 = \langle \frac{1}{2}, -\frac{1}{4}, -1 \rangle \Rightarrow 4\vec{u}_3 = \langle 2, -1, -4 \rangle$

~~\vec{r}_{03}~~

$x = \frac{t+2}{2} = \frac{1}{2}t + 1$

$y = \frac{t-4}{-4} = -\frac{1}{4}t + 1$

$\vec{r}_{03} = \langle 1, 1, -1 \rangle$

$z = t - 1$

$\mathcal{L}_4: \vec{r} = \langle 3, 1, 5 \rangle + t \langle 4, 2, 8 \rangle$

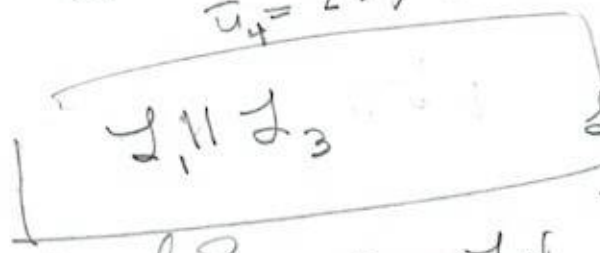
$\vec{r}_{04} = \langle 3, 1, 5 \rangle$

$\vec{u}_4 = \langle 4, 2, 8 \rangle$

$x = 4t + 3$

$y = -2t + 1$

$z = 8t + 5$



$\mathcal{L}_4 \langle 3, 1, 5 \rangle$

$\mathcal{L}_2 \langle 1, 0, 1 \rangle$

$\langle 2, 1, 4 \rangle$

Any identical? $\mathcal{L}_2 = \mathcal{L}_4$

203 $\int 12.5 \text{ II} \#5 68-9, 71, 73, 75, 76$

#68 cont'd

$\mathcal{J}_1, \mathcal{J}_3$ $6t + 1 = \frac{s+2}{2} \rightarrow 12t + 2 = s + 2$

$t = -\frac{1}{4}s + 1$ $\leftarrow 12t = s$
 $\rightarrow t = -\frac{1}{4} \cdot 12t + 1$

$12t + 5 = 5 - 1$
 $12(\frac{1}{4}) + 5 = 8$
 $= 3 - 1 = 2?$

$t = -3t + 1$

$4t = 1$

$t = \frac{1}{4} \Rightarrow s = 3$

~~$\mathcal{J}_1 \neq \mathcal{J}_3$~~

$\mathcal{J}_1, \mathcal{J}_4$ $6t + 1 = 4s + 3 \rightarrow 6t = 4s + 2$
 $t = \frac{4s+2}{6} = \frac{2s+1}{3}$

$1 - 3t = -2s + 1$

$12t + 5 = 8s + 5$

$1 - 3\left(\frac{2s+1}{3}\right) = -2s + 1$

~~$\mathcal{J}_1 \neq \mathcal{J}_4$~~

$1 - 2s - 1 = -2s + 1$

$-2s = -2s + 1$

$0 = 1$

$\mathcal{J}_3, \mathcal{J}_4$ $\frac{t+2}{2} = 4s + 3$

$\frac{t-4}{2} = -2s + 1$

$t - 1 = 8s + 5$

By:

$t = 8s + 6$

$\frac{8s+6+2}{2} = 4s+3$

$8s+8 = 8s+6$

$0 = -2$

So $\mathcal{J}_3 \neq \mathcal{J}_4$

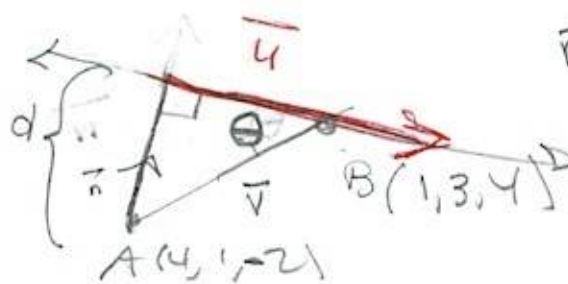
~~None identical~~

203 S12.5 #s 69, 71, 73, 75, 76

(69) Use formula in 12.4.45 to find distance from point to line

Let $d =$ distance desired.

(69) $A(4, 1, -2)$ $x = 1 + t, y = 3 - 2t, z = 4 - 3t$



$\vec{r} = \vec{r}_0 + t\vec{u}$, where
 $\vec{r}_0 = \langle 1, 3, 4 \rangle$, $\vec{u} = \langle 1, -2, -3 \rangle$

$\vec{v} = \vec{AB} = \langle -3, 2, 6 \rangle$ $\|\vec{u}\| \|\vec{v}\| \sin \theta = \|\vec{u} \times \vec{v}\|$

$\frac{d}{\|\vec{v}\|} = \sin \theta \Rightarrow \sin \theta = \frac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\| \|\vec{v}\|}$

$\Rightarrow d = \|\vec{v}\| \sin \theta = \|\vec{v}\| \frac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\| \|\vec{v}\|}$

Scratch $= \frac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\|}$

$\vec{u} = \langle 1, -2, -3 \rangle, -2$

$\vec{v} = \langle -3, 2, 6 \rangle, -3, 2$

$\vec{u} \times \vec{v} = \langle -6, 3, -4 \rangle$

$$\begin{array}{r} 2 \overline{) 854} \\ 7 \overline{) 427} \\ 61 \end{array}$$

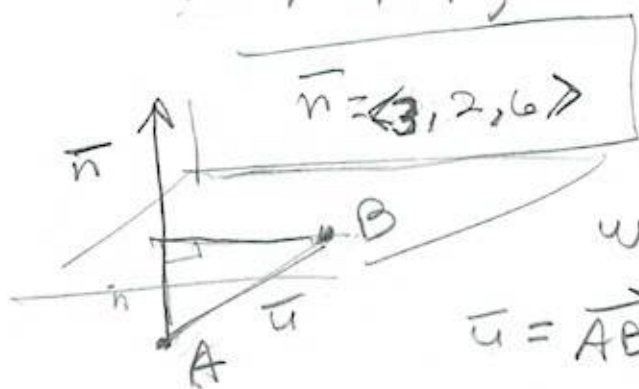
$$\begin{aligned} &= \frac{\|\langle -6, 3, -4 \rangle\|}{\|\langle 1, -2, -3 \rangle\|} \\ &= \frac{\sqrt{36 + 9 + 16}}{\sqrt{1 + 4 + 9}} \\ &= \frac{\sqrt{61}}{\sqrt{14}} = \frac{\sqrt{61} \sqrt{14}}{14} \\ &= \frac{\sqrt{854}}{14} = d \end{aligned}$$

$$\begin{array}{r} 61 \\ 14 \overline{) 854} \\ 244 \\ \hline 610 \\ 854 \end{array}$$

203 S125 U #5 71, 73, 75, 76

(71) Find distance from pt to plane

$A(1, -2, 4)$, $P: 3x + 2y + 6z = 5$



$(\frac{5}{3}, 0, 0) = B \in P$

want $|\text{comp}_{\bar{n}} \bar{u}|$

$\bar{u} = \overrightarrow{AB} = \langle \frac{2}{3}, 2, -4 \rangle$

~~Directions all we need, so No~~
 ~~$\bar{v} = 3\bar{u} = \langle 2, 6, -12 \rangle$ will suffice.~~

So $|\text{comp}_{\bar{n}} \bar{v}| = \frac{|\bar{n} \cdot \bar{v}|}{\|\bar{n}\|} = \frac{|\langle 3, 2, 6 \rangle \cdot \langle \frac{2}{3}, 2, -4 \rangle|}{\sqrt{3^2 + 2^2 + 6^2}}$

2 | 184
 2 | 92
 2 | 46
 23

$\frac{72}{54} = \frac{2 + 4 - 24}{\sqrt{9 + 4 + 36}}$

$= \frac{|-18|}{\sqrt{49}} = \frac{18}{7} = d$

$= \frac{54}{7} = d$

$\bar{n} = \langle 2, -3, 1 \rangle$

(73) Find distance between P_1 & P_2

$P_1: 2x - 3y + z = 4$

$P_2: 4x - 6y + 2z = 3$

$P_1: (2, 0, 0) = A$

$P_2: (0, -\frac{1}{2}, 0) = B$

$\overrightarrow{BA} = \langle 2, \frac{1}{2}, 0 \rangle$

want $|\text{comp}_{\bar{n}} \bar{u}| = \frac{|\bar{n} \cdot \bar{u}|}{\|\bar{n}\|}$

$= \frac{|\langle 2, -3, 1 \rangle \cdot \langle 2, \frac{1}{2}, 0 \rangle|}{\sqrt{4 + 9 + 1}} = \frac{4 - \frac{3}{2}}{\sqrt{14}} = \frac{5}{2\sqrt{14}} = \frac{5\sqrt{14}}{28}$

(75) Show that the distance between parallel planes $ax+by+cz=d_1$ and $ax+by+cz=d_2$ is given by

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

As before, we want $|\text{comp}_{\vec{n}} \vec{u}|$, where $\vec{n} = \langle a, b, c \rangle$ & \vec{u} is vector from a point on P_1 to a pt on P_2
use x -int?

$$\text{Let } A(x_0, y_0, z_0) \in P_1$$

$$B(x_1, y_1, z_1) \in P_2$$

$$\text{Then } \vec{u} = \vec{AB} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle,$$

$$\text{and so } \frac{|\vec{n} \cdot \vec{u}|}{\|\vec{n}\|} = \frac{|a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|ax_1 + bx_1 + cx_1 - (ax_0 + by_0 + cz_0)|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

(76) Find eqns of the planes that are parallel to P , $x+2y-2z=1$ and 2 units away from it.
 we need to move 2 units away in the direction of $\pm \bar{n} = \langle 1, 2, -2 \rangle$

Let $A = (1, 0, 0) \in P$. $\underbrace{\langle 1, 0, 0 \rangle}_{\text{position vector}}$, 3

2 units in \bar{n} direction

$\langle 1, 0, 0 \rangle \pm 2 \frac{\bar{n}}{\|\bar{n}\|}$ gives position vector

for a pt on each plane. $\circ \circ$

$$\bar{n} = \langle 1, 2, -2 \rangle, \quad \|\bar{n}\| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$\text{so } \bar{u}_1 = \bar{u}_0 + 2 \frac{\bar{n}}{\|\bar{n}\|} =$$

$$= \langle 1, 0, 0 \rangle + 2 \frac{\langle 1, 2, -2 \rangle}{3}$$

$$= \langle 1 + \frac{2}{3}, \frac{4}{3}, -\frac{4}{3} \rangle = \langle \frac{5}{3}, \frac{4}{3}, -\frac{4}{3} \rangle \quad \left(\frac{5}{3}, \frac{4}{3}, -\frac{4}{3} \right)$$

$$\text{or } \bar{u}_2 = \langle 1, 0, 0 \rangle - \frac{2}{3} \langle 1, 2, -2 \rangle$$

$$= \langle \frac{1}{3}, -\frac{4}{3}, \frac{4}{3} \rangle \quad \left(\frac{1}{3}, -\frac{4}{3}, \frac{4}{3} \right)$$

$$P_1: \bar{n} \cdot \left(x - \frac{5}{3} \right) + 2 \left(y - \frac{4}{3} \right) - 2 \left(z + \frac{4}{3} \right) = 0$$

$$x - \frac{5}{3} + 2y - \frac{8}{3} - 2z - \frac{8}{3} = 0$$

$$\boxed{x + 2y - 2z = 7}$$

203 8'12.5 II # 76

$$P_2: (x - \frac{1}{3}) + 2(y + \frac{4}{3}) - 2(z - \frac{4}{3}) = 0$$

$$x - \frac{1}{3} + 2y + \frac{8}{3} - 2z + \frac{8}{3} = 0$$

$$\boxed{x + 2y - 2z = -5}$$

#71 using formula

$$\frac{|ax + by + cz + d|}{\sqrt{a^2 + b^2 + c^2}}$$

S12.5 II
#71

$$= \frac{|3(1) + 2(-2) + 6(4) - 5|}{\sqrt{3^2 + 2^2 + 6^2}}$$

$$= \frac{|3 - 4 + 24 - 5|}{\sqrt{9 + 4 + 36}} = \frac{18}{\sqrt{49}} = \boxed{\frac{18}{7}}$$