203 S125 II #s75,76

(75) Show that the distance between parallel planes $ax+by+cz=d_1$ & $ax+by+cz=d_2$ is given by

$$\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}.$$

As before, we want $|\text{comp}_{\bar{n}}\,\bar{u}|$, where $\bar{n}=\langle a,b,c\rangle$ & $\bar{u}$ is vector ~~from~~ a point on P_1 to a pt on P_2

use x-int?

Let $A(x_0,y_0,z_0)\in P_1$

$B(x_1,y_1,z_1)\in P_2$

Then $\bar{u}=\overrightarrow{AB}=\langle x_1-x_0,y_1-y_0,z_1-z_0\rangle$,

and so $$\frac{|\bar{n}\cdot\bar{u}|}{\|\bar{n}\|}=\frac{|a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$=\frac{|ax_1+bx_1+cx_1-(ax_0+by_0+cz_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$=\frac{|d_2-d_1|}{\sqrt{a^2+b^2+c^2}}=\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

203 §12.5 II #76

(76) Find eq'ns of the planes that are parallel to P: $x+2y-2z=1$ & 2 units away from it.

we need to move 2 units away in the direction of $\pm\ \bar{n} = \langle 1,2,-2\rangle$

Let $A = (1,0,0) \in P$. $\boxed{\bar{u}_0 = \langle 1,0,0\rangle}$ .3 position vector

2 units in $\bar{n}$ direction gives position vector $\langle 1,0,0\rangle \pm 2\frac{\bar{n}}{\|\bar{n}\|}$ for a pt on each plane.

$\bar{n} = \langle 1,2,-2\rangle$ _ $\|\bar{n}\| = \sqrt{1+4+4} = \sqrt{9} = 3$

So $\bar{u}_1 = \bar{u}_0 + 2\frac{\bar{n}}{\|\bar{n}\|} =$

$= \langle 1,0,0\rangle + 2\frac{\langle 1,2,-2\rangle}{3}$

$= \langle 1+\frac{2}{3}, \frac{4}{3}, -\frac{4}{3}\rangle = \langle \frac{5}{3}, \frac{4}{3}, -\frac{4}{3}\rangle \leftrightarrow (\frac{5}{3}, \frac{4}{3}, -\frac{4}{3})$

& $\bar{u}_2 = \langle 1,0,0\rangle - \frac{2}{3}\langle 1,2,-2\rangle$

$= \langle \frac{1}{3}, -\frac{4}{3}, \frac{4}{3}\rangle \leftrightarrow (\frac{1}{3}, -\frac{4}{3}, \frac{4}{3})$

P_1: $(x-\frac{5}{3}) + 2(y-\frac{4}{3}) - 2(z+\frac{4}{3}) = 0$

$x - \frac{5}{3} + 2y - \frac{8}{3} - 2z - \frac{8}{3} = 0$

$\boxed{x+2y-2z = 7}$

203 §12.5 II #76

P_2: $(x-\frac{1}{3})+2(y+\frac{4}{3})-2(z-\frac{4}{3})=0$

$x-\frac{1}{3}+2y+\frac{8}{3}-2z+\frac{8}{3}=0$

$$\boxed{x+2y-2z=-5}$$

#71 using formula

S12.5 II
#71

$$\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|3(1)+2(-2)+6(4)-5|}{\sqrt{3^2+2^2+6^2}}$$

$$= \frac{|3-4+24-5|}{\sqrt{9+4+36}} = \frac{18}{\sqrt{49}} = \boxed{\frac{18}{7}}$$

203 §13.4 1

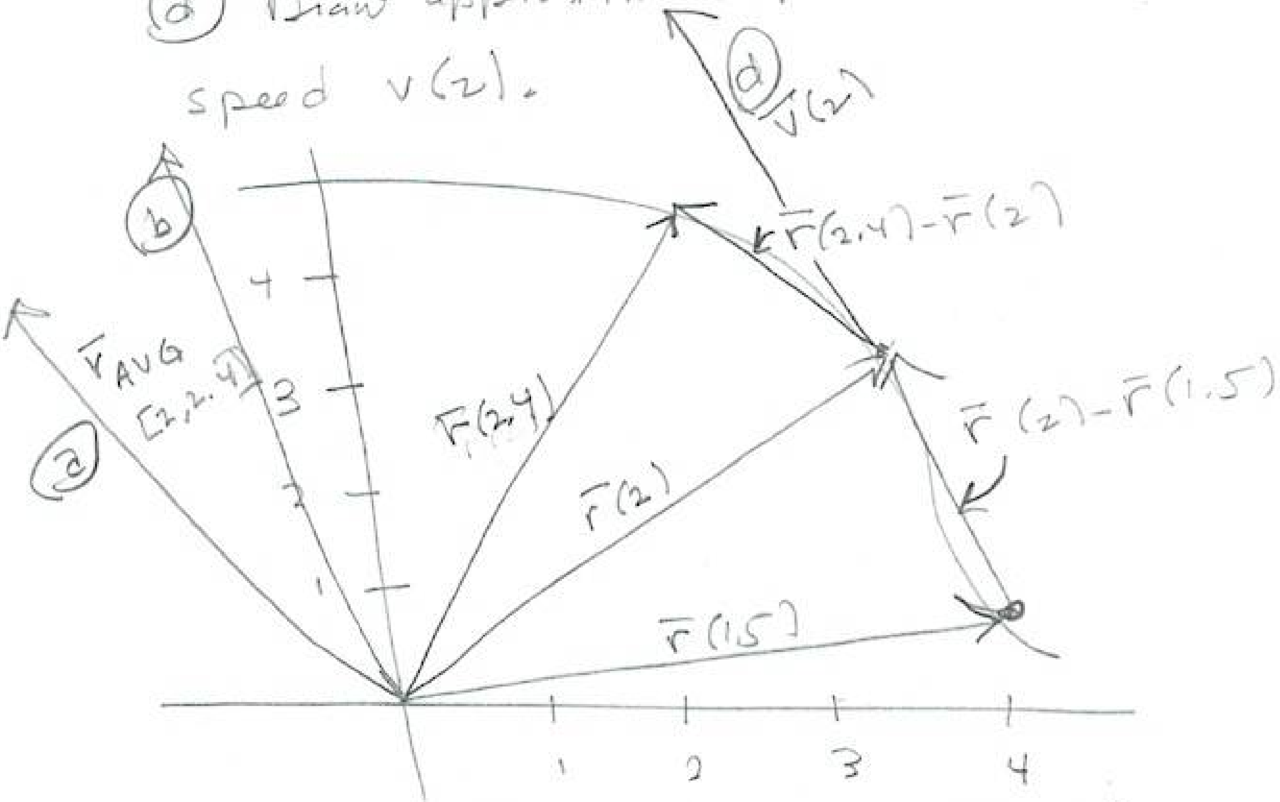
② The figure shows the path of a particle that moves w/ position vector $\bar{r}(t)$ at time t.

ⓐ Draw a vector that represents average velocity for $t \in [2, 2.4]$

ⓑ Draw $\bar{v}_{AVG}$ over $[1.5, 2]$

ⓒ Find expression for $\bar{v}(t) = \bar{r}'(t)$

ⓓ Draw approximate $\bar{v}(2)$ & estimate the speed $v(2)$.



ⓐ $$\frac{\bar{r}(2.4)-\bar{r}(2)}{.4} = \frac{5}{2}\left(\bar{r}(2.4)-\bar{r}(2)\right)$$

$$.4 \rightarrow \frac{4}{10} = \frac{2}{5}$$

ⓑ $$\frac{\bar{r}(2)-\bar{r}(1.5)}{.5} = 2\left(\bar{r}(2)-\bar{r}(1.5)\right)$$

203 S13.4

#2 cont'd

ⓒ $\boxed{\bar{v}(2) = \lim_{h\to 0} \frac{\bar{r}(2+h)-\bar{r}(2)}{h}}$

ⓓ See fig.

#s 3-8 Find $\bar{v}$, $\bar{a}$ & v. Sketch the path & draw $\bar{v}$ & $\bar{a}$ for specified values of t.

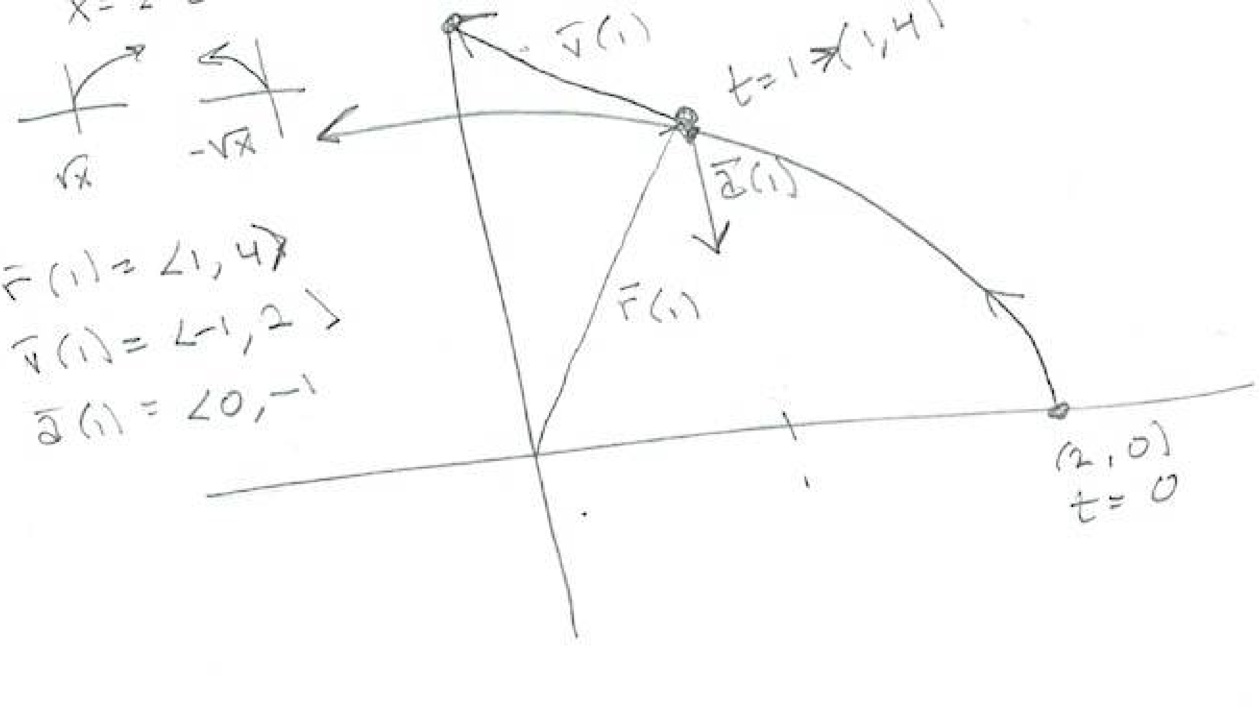
④ $\bar{r} = \langle 2-t, 4\sqrt{t}\rangle$, $t=1$

$\bar{v}(t) = \bar{r}'(t) = \langle -1, 4(\frac{1}{2\sqrt{t}})\rangle = \boxed{\langle -1, 2t^{-\frac{1}{2}}\rangle = \bar{v}}$

$\bar{a}(t) = \bar{r}''(t) = \boxed{\langle 0, -t^{-\frac{3}{2}}\rangle = \bar{a}}$

$\boxed{v = \sqrt{1 + 4t}}$

$x = 2-t \Longrightarrow t = 2-x \Longrightarrow y = 4\sqrt{2-x} = 4\sqrt{-(x-2)}$



$\bar{r}(1) = \langle 1, 4\rangle$

$\bar{v}(1) = \langle -1, 2\rangle$

$\bar{a}(1) = \langle 0, -1$

203 §13.4

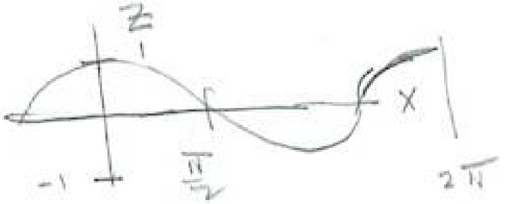
⑧ $\bar{r}(t) = \langle t, 2\cos t, \sin t \rangle$, $t = 0$

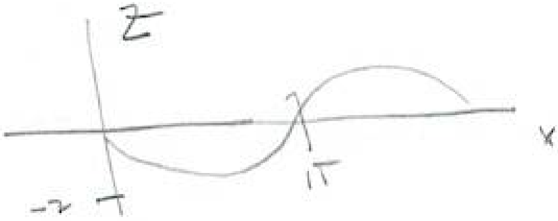
ellipsoidal

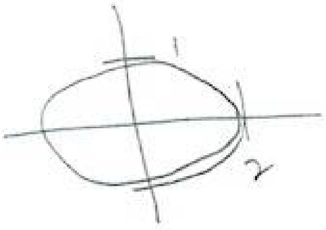
$\bar{v}(t) = \bar{r}'(t) = \langle 1, -2\sin t, \cos t$

$\bar{a}(t) = r''(t) = \langle 0, -2\cos t, -\sin t \rangle$

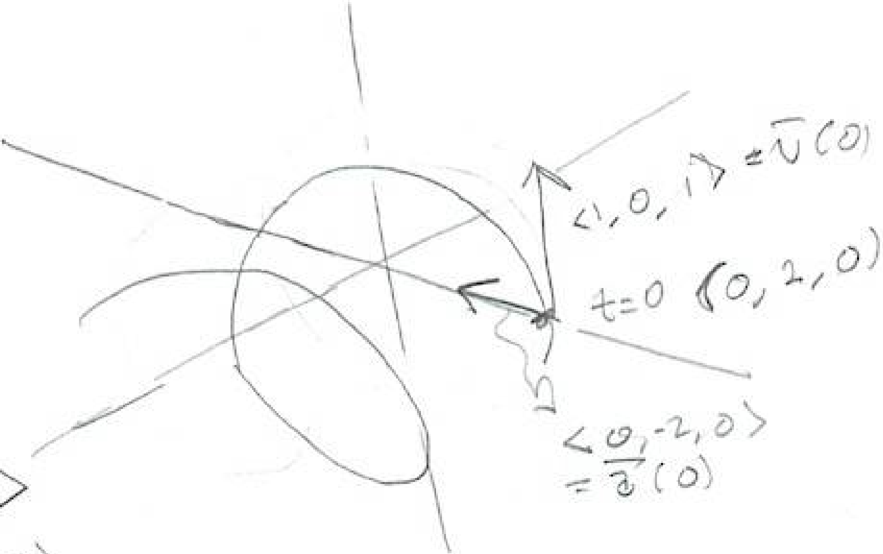
$v(t) = \sqrt{1 + 4\sin^2 t + \cos^2 t}$







$\bar{r}(0) = \langle 0, 2, 0 \rangle$



$\bar{v}(0) = \langle 1, 0, 1 \rangle$

$\bar{a}(0) = \langle 0, -2, 0 \rangle$

203 §13.4

#s 9-14 Find $\bar{v}$, $\bar{a}$, v of particle w/ given $\bar{r}$

(11) $\bar{r} = \langle \sqrt{2}\, t, e^{t}, e^{-t} \rangle$

$$\boxed{\bar{v} = \langle \sqrt{2}, e^{t}, -e^{-t} \rangle}$$

$$\boxed{\bar{a} = \langle 0, e^{t}, e^{-t} \rangle}$$

$$v = \sqrt{\sqrt{2}^{2} + e^{2t} + e^{-2t}} = \sqrt{e^{2t} + e^{-2t} + 2}$$

$$= \sqrt{e^{2t} + 2e^{2t}e^{-2t} + e^{-2t}} = \sqrt{(e^{t} + e^{-t})^{2}}$$

$$= \boxed{e^{t} + e^{-t} = v}$$

#s 15,16 Fin $\bar{v}$, $\bar{r}$, given $\bar{a}$, $\bar{v}(t_0)$, $\bar{r}(t_0)$

(15) $\bar{a} = \langle 1, 2, 0 \rangle$, $\bar{v}(0) = \langle 0, 0, 1 \rangle$, $\bar{r}(0) = \langle 1, 0, 0 \rangle$

$$\Longrightarrow \bar{v}(t) = \int \langle 1, 2, 0 \rangle dt + \bar{C} = \langle t, 2t, 0 \rangle + \bar{C}$$

$$\bar{v}(0) = \langle 0, 0, 0 \rangle + \bar{C} = \langle 0, 0, 1 \rangle \Longrightarrow \bar{C} = \langle 0, 0, 1 \rangle \Longrightarrow$$

$$\Longrightarrow \boxed{\bar{v}(t) = \langle t, 2t, 1 \rangle}$$

$$\bar{r}(t) = \langle \tfrac{1}{2}t^{2}, t^{2}, t \rangle + \bar{D}$$

$$\bar{r}(0) = \langle 0, 0, 0 \rangle + \bar{D} = \langle 1, 0, 0 \rangle = \bar{D} \Longrightarrow$$

$$\boxed{\bar{r}(t) = \langle \tfrac{1}{2}t^{2} + 1, t^{2}, t \rangle}$$

203 §13.4 # <u>19 cntd</u>

$$x = \frac{1}{250\sqrt{3}} t$$

$$t = \frac{1}{250\sqrt{3}} x$$

$$y = \frac{250}{250\sqrt{3}} x - \frac{49}{10}\left(\frac{1}{250\sqrt{3}}\right)^2 x^2$$

$$= \frac{x}{\sqrt{3}} - \frac{49}{10}\left(\frac{1}{250\sqrt{3}}\right)^2 x^2$$

$x \approx 11046.243$, $y \approx 3188.7755$

$250/9.8$ ⓐ $x \approx 22,092$, $y \approx 0$

22,092 meters is its range

ⓑ Max height is 3188.7755 m

Which doesn't make sense.

In the air 51.0204081 6

seconds

ⓒ Speed at impact.

$$\left\| \vec{v}\left(\frac{250}{4.9}\right) \right\| = \left\| \langle 250\sqrt{3}, 250 - 9.8 \cdot \left(\frac{250}{4.9}\right) \rangle \right\|$$

$$= \| \langle 250\sqrt{3}, 250 - 500 \rangle \|$$

$$= \| \langle 250\sqrt{3}, -250 \rangle \|$$

$$= \sqrt{(250\sqrt{3})^2 + 250^2} = 250\sqrt{}$$

$$= \sqrt{250^2 \cdot 3 + 250^2} = 2 \cdot 250 = \boxed{500 \tfrac{m}{s}}$$

As expected.

Note $\vec{v}\left(\frac{250}{4.9}\right) = \langle 250\sqrt{3}, -250$

comes in at 30° angle

203 S'13.4

(19) The position function is $\bar{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$

When is speed a minimum

$\bar{v}(t) = \langle 2t, 5, 2t-16 \rangle$ $\longrightarrow$

$v = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256}$

$= \sqrt{8t^2 - 64t + 281}$ To minimize v, minimize

$\begin{array}{r} 416 \\ 8 \\ \hline 128 \end{array}$

$v^2 = 8t^2 - 64t + 41$

$\frac{d}{dt}\left[v^2\right] = 16t - 64 \overset{SET}{=} 0 \longrightarrow t = 4$

$\Longrightarrow$ min v is $v(4) = \sqrt{8(4)^2 - 64(4) + 281}$

$= \sqrt{128 - 256 + 281} = \sqrt{281 - 128}$ $\boxed{= \sqrt{153} = v_{min}}$

(23) Projectile w/ $v_0 = 500$ m/s & angle of elevation 30°. Find (a) range and (b) max height an (c) speed at impact.

$\bar{r}(t) = \langle (500\cos(30^\circ))t, (500\sin(30^\circ))t - \frac{1}{2}gt^2 \rangle$

$= \langle 500 \cdot \frac{\sqrt{3}}{2} t, 250t - 4.9t^2 \rangle$

(b) $\frac{dy}{dt} = 250 - 9.8t \overset{SET}{=} 0 \Longrightarrow t = \frac{250}{9.8} = \frac{2500}{98}$

$= \frac{1250}{49} = t$

$(250\sqrt{3}$

≈ 25.5102040852? Seems too big! No. It's not. Just seems weird.

$-t(4.9t - 250) = 0$

$4.9t = 250$

$t = \frac{250}{4.9}$ OK m

203 § 13.4

#s 33-8 Find a_T and a_N for $\bar{r}$.

(33) $\bar{r}(t) = \langle 3t-t^3, 3t^2, 0 \rangle$

$$a_N = \frac{\|\bar{r}' \times r''\|}{\|\bar{r}'\|}, \quad a_T = \frac{\bar{r}' \cdot r''}{\|\bar{r}'\|}$$

$\bar{r}' = \langle 3-3t^2, 6t, 0 \rangle$, $3-3t^2, 6t$

$\times \ \bar{r}'' = \langle -6t, 6, 0 \rangle$, $-6t, 6$

$\bar{r}' \times \bar{r}'' = \langle 0, 0, 18-18t^2+36t^2 \rangle$

$= \langle 0, 0, 18t^2+18 \rangle$

$\|\bar{r}'\| = \sqrt{(3t^2-3)^2+(6t)^2}$

$= \sqrt{9(t^2-2t+1)+9(4t^2)}$

$= 3\sqrt{t^2-2t+1+4t^2} = \sqrt{5t^2-2t+1}$

So, $\boxed{a_T = \dfrac{-6t(3-3t^2)+36t^2}{\sqrt{5t^2-2t+1}}}$ &

$$\boxed{a_N = \frac{\langle 0, 0, 18t^2+18 \rangle}{\sqrt{5t^2-2t+1}}}$$

203 S' 13.4

(36) $\bar{r} = \langle t, t^2, 3t \rangle$

$\bar{r}' = \langle 1, 2t, 3 \rangle$, 1, 2t $\qquad \bar{r}' \cdot \bar{r}'' = 4t$

$\times\ \bar{r}'' = \langle 0, 2, 0 \rangle$, 0, 2

$\bar{r}' \times \bar{r}'' = \langle -6, 0, 2 \rangle$

$\|\bar{r}'\| = \sqrt{1 + 4t^2 + 9} = \sqrt{4t^2+10}$

$\longrightarrow \boxed{a_T = \frac{4t}{\sqrt{4t^2+10}}}$ & $a_N = \frac{\|\langle -6, 0, 2 \rangle\|}{\sqrt{4t^2+10}}$

$= \frac{\sqrt{36+4}}{\sqrt{4t^2+10}} = \boxed{\frac{2\sqrt{10}}{\sqrt{4t^2+10}} = a_N}$

(42) Mass as a function of time?

$F(t) = m(t)\,a(t)$

$= m(t)\frac{d\bar{v}}{dt} = \frac{dm(t)}{dt} v_e$, where $\bar{V}_e$ = velocity of exhaust gas. (constant vector value)

So $\int_0^t d\bar{v} = \int_0^t \frac{dm}{m} \bar{v}_e \ln m$

$\longrightarrow \bar{v}(t) - \bar{v}(0) = \ln(m(t)) - \ln(m(0))$

$\bar{v}(t) = \ln\left(\frac{m(t)}{m(0)}\right)\bar{v}_e + \bar{v}(0)$

$\longrightarrow \bar{v}(t) = \ln\left(\frac{m(0)}{m(t)}\right)\vec{V}_0$

$= \bar{v}(0) - \ln($

203 S'13.4

#42 cntd

(b) ~~When~~ What fraction of its initial mass would the rocket burn getting up to $2\bar{v}_e$, if $\bar{v}(0) = \bar{0}$?

$$\bar{V}(t) = \underbrace{\bar{V}(0)}_{=0} - \ln\left(\frac{m(0)}{m(t)}\right)\bar{v}_e = 2\bar{v}_e$$

~~$\Rightarrow \left(-\ln\left(\frac{m(0)}{m(t)}\right) = 2\right)\bar{v}_e$~~ → ~~Eventually~~

$\Rightarrow -\ln\left(\frac{m(0)}{m(t)}\right) = 2$ → $\frac{m_0}{m(t)} > 1$

since $m_0 > m \underset{\neq}{>} 0$

$\Rightarrow \ln\left(\frac{m_0}{m(t)}\right) = -2$

$\Rightarrow \frac{m_0}{m(t)} = \frac{1}{e^2}$

So Need

$\Rightarrow m(t) = e^2 m_0$

Doesn't make sense.

This appears to be saying you can't do it. Seems weird.

$m(0) > m(t)$

$-\ln\left(\frac{m_0}{m(t)}\right)\bar{v}_e = -2\bar{v}_e$ Makes more sense.

$\ln\left(\frac{m_0}{m(t)}\right) = 2$

$\frac{m_0}{m(t)} = e^2 \Rightarrow m(t) = \frac{1}{e^2} m_0$

$\frac{1}{e^2}$ of mass ejected.

203 §13.4

#42 cntd.

Not quite. $\frac{m_0}{e^2} = m(t) \Longrightarrow$

$m_0 - \frac{m_0}{e^2} = \frac{e^2 m_0 - m_0}{e^2} = \frac{e^2-1}{e^2} m_0$ was burned.

So $\frac{e^2-1}{e^2}$ of the mass of the vehicle is needed to get up to $2\bar{V}_e$'s magnitude

i.e. $2\|\bar{V}_e\|$.