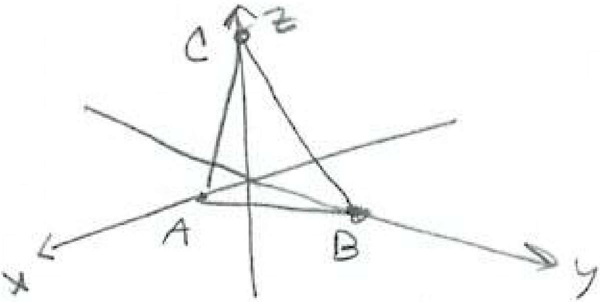
203 §12.5 II #s 41, 44, 48-51, 53, 58, 59, 61, 63-65, 67-9, 71, 73, 75, 76

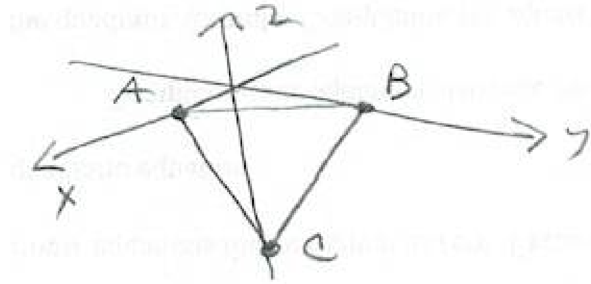
(41) Use intercepts to sketch $2x+5y+z=10$

$A(5,0,0), B(0,2,0), C(0,0,10)$



(44) $6x+5y-3z=15$

$A(\frac{5}{2},0,0), B(0,3,0), C(0,0,-5)$



(48) Where does line thru $(-3,1,0)$ & $(-1,5,6)$ intersect $2x+y-z=-2$?

$L: (1-t)\langle -3,1,0\rangle + t\langle -1,5,6\rangle$

$= \langle -3,1,0\rangle + \langle 3t,-t,0\rangle + \langle -t,5t,6t\rangle$

$= \bar{r}(t) = \langle 2t,4t,6t\rangle + \langle -3,1,0\rangle$

$= \langle 2t-3, 4t+1, 6t\rangle$

$2(2t-3)+4t+1-6t = -2$

$4t-6-2t+1 = 2t-5 = -2 \Rightarrow$

$2t = 3 \Rightarrow t = \frac{3}{2}$

$2(0)+7-9 = -2$ ✓

$\langle 2(\frac{3}{2})-3, 4(\frac{3}{2})+1, 6(\frac{3}{2})\rangle$

$= \langle 3-3, 6+1, 9\rangle = \langle 0,7,9\rangle \rightsquigarrow \boxed{(0,7,9)}$

203 §12.5 II #s 49-51, 53, 58, 59, 61, 63-65, 67-9, 71, 73, 75, 76

(49) Find direction #s for line of intersection of $x+y+z=1$ & $x+z=0$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \end{array}\right]$$

$y=1,\ x=-z=-t,$ $\Rightarrow$

$\langle x, y, z\rangle = \langle -t, 1, t\rangle$

Direction #s are $-1, 0, 1$

$1, 0, -1$

for line of intersection $\Rightarrow$

Direction #s are $\boxed{-1, 0, 1}$ OR

(50) Find cosine of the angle between the planes $x+y+z=0$ & $x+2y+3z=1$

$\bar{n}_1 = \langle 1, 1, 1\rangle, \bar{n}_2 = \langle 1, 0, 1\rangle$

$$\cos\theta = \frac{1+2+3}{\sqrt{3}\sqrt{14}} = \frac{6}{\sqrt{42}} \text{ OR } \boxed{\frac{\sqrt{42}}{7} = \cos\theta}$$

Angle between their normals = angle between the planes.

(51) $\parallel$, $\perp$, neither? If neither, find angle between them, to one decimal.

$x+4y-3z=1,\ -3x+6y+7z=0$

$\bar{n}_1 = \langle 1, 4, -3\rangle, \bar{n}_2 = \langle -3, 6, 7\rangle$

Not multiples $\Rightarrow$ not $\parallel$

$\bar{n}_1 \cdot \bar{n}_2 = -3+24-21=0 \Rightarrow \boxed{\perp}$

Same instructions #s 51-56

203 §12.5 II #s 53, 58, 59, 61, 63–65, 67–9, 71, 73, 75, 76

(53) $x+2y-z=2, \quad 2x-2y+z=1$

$\bar{n}_1 = \langle 1,2,-1\rangle, \quad \bar{n}_2 = \langle 2,-2,1\rangle$

Not $\parallel$, by inspection.

$\bar{n}_1 \cdot \bar{n}_2 = 2-4-1 = -3 \neq 0 \Rightarrow \not\perp$.

$$\cos\theta = \frac{-3}{\sqrt{1+4+1}\sqrt{4+4+1}} = \frac{-3}{\sqrt{6}\sqrt{9}} = \frac{-3}{3\sqrt{6}} \Rightarrow$$

$$\theta = \arccos\left(-\frac{1}{3\sqrt{6}}\right) \approx 65.90515745^\circ \approx \boxed{65.9^\circ \approx \theta}$$

114.0940426°

$\approx 114.1^\circ$

≈ 1.150261992

≈ 1.2 radians

(58) (a) Find parametric eq'ns for line of intersection of the planes & (b) Find $\angle$ between planes

$3x-2y+z=1, \quad 2x+y-3z=3$

$$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 3 \\ 3 & -2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -3 & 3 \\ 0 & -7 & 11 & -7 \end{array}\right]$$

$-7y+11z=-7 \Rightarrow y - \frac{11}{7}z = 1 \Rightarrow$

$y = \frac{11}{7}z + 1$

(b) $\cos\theta = \dfrac{\bar{n}_0 \cdot \bar{n}_1}{\|\bar{n}_0\|\|\bar{n}_1\|}$

$= \dfrac{6-2-3}{\sqrt{14}\sqrt{14}} = \dfrac{1}{14}$

$\Rightarrow \theta \approx 85.90395624$

$\approx \boxed{85.9^\circ}$

$2x + \frac{11}{7}z + 1 - 3z = 3$

$2x - \frac{10}{7}z = 2$

$2x = \frac{10}{7}z + 2$

$x = \frac{5}{7}z + 1$

$x = \frac{5}{7}t + 1$

$y = \frac{11}{7}t + 1$

$z = t$

Check: $(6, 12, 7)$

$3(6) - 2(12) + 7 = 1$ ✓

$2(6) + 12 - 3(7) = 3$ ✓

203 §12.5 II #s 59, 61, 63-5, 67-9, 71, 73, 75, 76

59 $5x-2y-2z=1$, $4x+y+z=6$

Find symmetric eq'ns of intersection

$$\left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 4 & 1 & 1 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 0 & 13 & 13 & 26 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 0 & 1 & 1 & 2 \end{array}\right] \qquad \begin{aligned} 5x-2y-2z&=1 \\ y+z&=2 \\ y&=2-z \end{aligned}$$

$5x-2(2-z)-2z=1$

$5x-4+2z-2z=1$ $\quad (1, 2-t, t)$

$5x=5$ $\quad x=1,\ y=2-t,\ z=t$

$x=1$ $\quad \boxed{x=1,\ 2-y=z}$

61 Find eq'n for plane consisting of all points equidistant from $(1,0,-2)$ & $(3,4,0)$

Direction vector for line segment between the two points is $\bar{n}$. midpoint is pt on plane.

$\bar{n}=\langle 3-1, 4-0, 0-(-2)\rangle = \langle 2,4,2\rangle = \bar{n}$

midpt: $\left(\frac{1+3}{2}, \frac{0+4}{2}, \frac{-2+0}{2}\right) = (2,2,-1)$

So $2(x-2)+4(y-2)+2(z+1)=0$

$2x-4+4y-8+2z+2=0$

$\boxed{2x+4y+2z=10}$

OR $\boxed{x+2y+z=5}$

203 #s 63-5, 67-9, 71, 73, 75, 76

63 Find eq'n of plane with x-int a, y-int b, z-int c

$A(a,0,0)$, $B(0,b,0)$, $C(0,0,c)$

$\bar{u} = \overrightarrow{AB} = \langle -a, b, 0\rangle$, $\bar{v} = \langle -a, 0, c\rangle$

$\bar{u} = \langle -a, b, 0\rangle, -a, b$

$\times\ \bar{v} = \langle -a, 0, c\rangle, -a, 0$

$\bar{n} = \langle bc, ac, ab\rangle \longrightarrow$

$bc(x-a) + acy + abz = 0$

$bcx - abc + acy + abz = 0$

$$\boxed{bcx + acy + abz = abc}$$

64 a Find pt @ which the lines intersect:

$\bar{r}_1 = \langle t+1, -t+1, 2t\rangle$

$\bar{r}_2 = \langle -s+2, s, 2\rangle$

$t+1 = -s+2$

$-t+1 = s$

$2t = 2$

$\boxed{t=1}$

$2 = -s+2$

$\boxed{s=0}$

$\bar{r}_1(1) = \langle 2, 0, 2\rangle$

$\bar{r}_2(0) = \langle 2, 0, 2\rangle$

$\boxed{A(2,0,2) \text{ is intersection}}$

b Eqn of plane containing them.

$\bar{u} = \langle 1, -1, 2\rangle, 1, -1$

$\bar{v} = \langle -1, 1, 0\rangle, -1, 1$

$\bar{n} = \langle -2, -2, 0\rangle$

oops! b cont'd

$-2(x-2) - 2(y) = 0$

$-2x + 4 - 2y = 0$

$\boxed{2x+2y=4}$

203 5/12.5 II #s 65, 67-9, 71, 73, 75, 76

(65) Find parametric eq'ns for the line thru A(0,1,2) that is ∥ to plane P and ⊥ to the line $\mathcal{L}_1$, where

$\mathcal{L}_1$: $\bar{r}(t) = \langle t+1, -t+1, 2t \rangle = \bar{r}_0 + t\bar{u}$

P: $x+y+z=2$, $\bar{n} = \langle 1,1,1 \rangle$ ⊥ to P.

$\bar{u} = \langle 1,-1,2 \rangle$ is direction vector for $\mathcal{L}_1$.

$\bar{r}_0 = \langle 1,1,0 \rangle \in \mathcal{L}_1 \cap P$ is convenient, since

$1+1+0 = 2 \Rightarrow B(1,1,0) \in P$.

$\bar{n}$ is perpendicular to $\mathcal{L}_2$ = line we want. Let $\bar{v}$ be direction vector for $\mathcal{L}_2$

Then $\bar{n} \cdot \bar{v} = \langle 1,1,1 \rangle \cdot \langle v_1, v_2, v_3 \rangle = 0$

$v_1 + v_2 + v_3 = 0$

Also $\bar{u} \cdot \bar{v} = \langle 1,-1,2 \rangle \cdot \langle v_1, v_2, v_3 \rangle$

$= v_1 - v_2 + 2v_3 = 0 \Rightarrow$

$v_1 + v_2 + v_3 = v_1 - v_2 + 2v_3 \Rightarrow$

$v_2 + v_3 = -v_2 + 2v_3 \Rightarrow$

$2v_2 = v_3 \Rightarrow$

$v_2 = \frac{1}{2}v_3 \Rightarrow \langle v_1, \frac{1}{2}v_3, v_3 \rangle = \bar{v}$

$v_1 + v_2 + v_3 = 0 \Rightarrow$ So $\bar{v} = \langle -\frac{1}{2}v_3, \frac{1}{2}v_3, v_3 \rangle$

So, $v_1 + \frac{1}{2}v_3 = 0$ $v_3 = 2 \Rightarrow$

$v_1 + \frac{1}{2}v_3 = 0$

$\Rightarrow v_1 = -\frac{1}{2}v_3$ $\boxed{\bar{v} = \langle 1, -1, 2 \rangle}$

$\langle 1, -1, 2 \rangle, 1, -1$

$\times \langle 1, 1, 1 \rangle, 1, 1$

$\langle -4, 0, 2 \rangle$

$x = -4t, y = 1, z = 2t+2$

203 §12.5 IV #s 65, 67-9, 71, 73, 75, 76

#65 cont'd :

Now Build eq'n for $\mathcal{L}_2$

$\vec{w}_0 = \langle 0, 1, 2 \rangle$ from $A(0,1,2) \in \mathcal{L}_2$.

So $\mathcal{L}_2$ is $\vec{w} = \vec{w}_0 + t\vec{v}$

$= \langle 0, 1, 2 \rangle + t \langle 1, -1, 2 \rangle$

$= \langle t, -t+1, 2t+2 \rangle$

$\boxed{x = t,\ y = -t+1,\ z = 2t+2}$

NOT QUITE
see #75 fixed
in Solutions

(67) Which of the following planes are ∥?
Are any identical?

$P_1 : 3x + 6y - 3z = 6$

$x + 2y - z = 2$

$P_2 : 4x - 12y + 8z = 5$

$x - 3y + 2z = \frac{5}{4}$

$P_3 : 9y = 1 + 3x + 6z$

$-6z - 3x + 9y = 1$

$-3x + 9y - 6z = 1$

$x - 3y + 2z = -\frac{1}{3}$

$P_4 : z = x + 2y - 2$

$-x - 2y + z = -2$

$x + 2y - z = 2$

$\boxed{P_1 = P_4,\ P_2 \parallel P_3}$

203 §12.5 II #s 68-9, 71, 73, 75, 76

(68) Which of the following 4 lines are $\parallel$? Are any identical?

$\mathcal{L}_1$: $x = 1+6t, y = 1-3t, z = 12t+5$

$\bar{u}_1 = \langle 6, -3, 12 \rangle$, $\bar{r}_{01} = \langle 1, 1, 5 \rangle$

$\frac{1}{3}\bar{u}_1 = \langle 2, -1, 4 \rangle$

$\mathcal{L}_2$: $x = 1+2t, y = t, z = 1+4t$

$\bar{u}_2 = \langle 2, 1, 4 \rangle$, $\bar{r}_{02} = \langle 1, 0, 1 \rangle$

$\mathcal{L}_3$: $t = 2x-2 = 4-4y = z+1$

$\bar{u}_3 = \langle \frac{1}{2}, -\frac{1}{4}, -1 \rangle \Rightarrow 4\bar{u} = \langle 2, -1, -4 \rangle$

~~$\bar{r}_{03}$~~

$x = \frac{t+2}{2} = \frac{1}{2}t + 1,$

$y = \frac{t-4}{-4} = -\frac{1}{4}t + 1$ $\qquad \bar{r}_{03} = \langle 1, 1, -1 \rangle$

$z = t-1$

$\mathcal{L}_4$: $\bar{r} = \langle 3, 1, 5 \rangle + t\langle 4, -2, 8 \rangle$

$x = 4t+3$

$y = -2t+1$

$z = 8t+5$

$\bar{r}_{04} = \langle 3, 1, 5 \rangle,$

$\bar{u}_4 = \langle 2, -1, 4 \rangle$

$\mathcal{L}_1 \parallel \mathcal{L}_3$

$\mathcal{L}_4$ $\langle 3, 1, 5 \rangle$

$\mathcal{L}_2$ $\langle 1, 0, 1 \rangle$

$\langle 2, 1, 4 \rangle$

Any identical? $\boxed{\mathcal{L}_2 = \mathcal{L}_4}$

203 § 12.5 II #s 68-9, 71, 73, 75, 76

#68 cont'd

$\mathcal{L}_1, \mathcal{L}_3$ $\quad 6t+1 = \frac{s+2}{2} \rightarrow 12t+2 = s+2$

$12t = s$

$t = -\frac{1}{4}s + 1 \rightarrow t = -\frac{1}{4}\cdot 12t + 1$

$t = -3t+1$

$4t = 1$

$t = \frac{1}{4} \Rightarrow s = 3$

$12t+5 = s-1$

$12(\frac{1}{4})+5 = 8$

$= 3-1 = 2?$ $\times$

$\mathcal{L}_1 \neq \mathcal{L}_3$

$\mathcal{L}_1, \mathcal{L}_4$: $\quad 6t+1 = 4s+3 \rightarrow 6t = 4s+2$

$t = \frac{4s+2}{6} = \frac{2s+1}{3}$

$1-3t = -2s+1$

$12t+5 = 8s+5$

$1-3\left(\frac{2s+1}{3}\right) = -2s+1$

$1-2s-1 = -2s+1$

$-2s = -2s+1$

$0 = 1$ $\times$

$\mathcal{L}_1 \neq \mathcal{L}_4$

$\mathcal{L}_2 = \mathcal{L}_4$.

$\mathcal{L}_3, \mathcal{L}_4$: $\quad \frac{t+2}{2} = 4s+3$

$\frac{t-4}{-1} = -2s+1$

$t-1 = 8s+5 \rightarrow t = 8s+6$

Try:

$\frac{8s+6+2}{2} = 4s+3$

$8s+8 = 8s+6$

$0 = -2$ $\times$

So $\mathcal{L}_3 \neq \mathcal{L}_4$

~~None identical~~

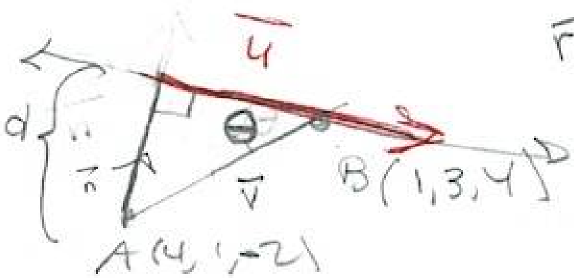
2

203 §12.5 II #s 69, 71, 73, 75, 76

69) Use formula in 12.4.45 to find distance from point to line

Let $\boxed{d = \text{distance desired.}}$

69) $A(4,1,-2)$ $x = 1+t,\ y = 3-2t,\ z = 4-3t$



$\bar{r} = \bar{r}_0 + t\bar{u}$, where
$\bar{r}_0 = \langle 1, 3, 4\rangle$, &
$\bar{u} = \langle 1,-2,-3\rangle$

$\bar{v} = \overrightarrow{AB} = \langle -3, 2, 6\rangle$ $\|\bar{u}\|\|\bar{v}\|\sin\theta = \|\bar{u}\times\bar{v}\|$

$\frac{d}{\|\bar{v}\|} = \sin\theta$ $\Rightarrow \sin\theta = \frac{\|\bar{u}\times\bar{v}\|}{\|\bar{u}\|\|\bar{v}\|}$

$\Rightarrow d = \|\bar{v}\|\sin\theta = \|\bar{v}\|\frac{\|\bar{u}\times\bar{v}\|}{\|\bar{u}\|\|\bar{v}\|}$

$= \frac{\|\bar{u}\times\bar{v}\|}{\|\bar{u}\|} = \frac{\|\langle -6,3,-4\rangle\|}{\|\langle 1,-2,-3\rangle\|}$

$= \frac{\sqrt{36+9+16}}{\sqrt{1+4+9}}$

$= \frac{\sqrt{61}}{\sqrt{14}}$ or $\frac{\sqrt{61}\sqrt{14}}{14}$

$\boxed{= \frac{\sqrt{854}}{14} = d}$

Scratch

$\bar{u} = \langle 1,-2,-3\rangle, 1, -2$

$\bar{v} = \langle -3,2,6\rangle, -3, 2$

$\|\bar{u}\times\bar{v}\| = \langle -6, 3, -4\rangle$

$2\,\underline{|854}$
$7\,\underline{|427}$
61

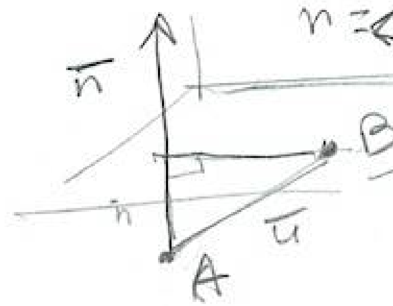
61
$\underline{14}$
244
$\underline{610}$
854

203 §12.5 IV #s 71, 73, 75, 76

(71) Find distance from pt to plane

$A(1,-2,4)$, $\mathcal{P}: 3x+2y+6z=5$

$\boxed{\bar{n}=\langle 3,2,6\rangle}$ $\quad (\frac{5}{3},0,0)=B\in\mathcal{P}$



want $|\text{comp}_{\bar{n}}\bar{u}|$

$\bar{u}=\overrightarrow{AB}=\langle \frac{2}{3},2,-4\rangle$

~~Direction's all we need, so~~ NO

~~$\bar{v}=3\bar{u}=\langle 2,6,-12\rangle$~~ will suffice.

So $|\text{comp}_{\bar{n}}\bar{v}| = \dfrac{|\bar{n}\cdot\bar{v}|}{\|\bar{n}\|} = \dfrac{|\langle 3,2,6\rangle\cdot\langle \frac{2}{3},2,-4\rangle|}{\sqrt{3^2+2^2+6^2}}$

$2\lfloor 184$, $2\lfloor 92$, $2\lfloor 46$, 23 $\quad 72 - 18 = 54$

$= \dfrac{|2+4-24|}{\sqrt{9+4+36}} = \dfrac{|-18|}{\sqrt{49}} = \boxed{\dfrac{18}{7}=d}$

$= \boxed{\dfrac{18}{7}=d}$

$\bar{n}=\langle 2,-3,1\rangle$

(73) Find distance between $\mathcal{P}_1$ & $\mathcal{P}_2$ $\quad \mathcal{P}_1: 2x-3y+z=4$

$\mathcal{P}_2: 4x-6y+2z=3$

$\mathcal{P}_1: (2,0,0)=A$ $\quad \overrightarrow{BA}=\langle 2,\frac{1}{2},0\rangle = \bar{u}$

$\mathcal{P}_2: (0,-\frac{1}{2},0)=B$

want $|\text{comp}_{\bar{n}}\bar{u}| = \dfrac{|\bar{n}\cdot\bar{u}|}{\|\bar{n}\|} = \dfrac{|\langle 2,-3,1\rangle\cdot\langle 2,\frac{1}{2},0\rangle|}{\sqrt{4+9+1}}$

$= \dfrac{4-\frac{3}{2}}{\sqrt{14}} = \dfrac{5}{2\sqrt{14}} = \boxed{\dfrac{5\sqrt{14}}{28}=d}$