§ 12.5 I #s 2, 6, 7, 10–14, 16, 17, 21, 23, 24, 26, 27,
30, 31, 35, 36

(2) Find a vector eq'n & parametric eq'ns for the line

thru $(6,-5,2)$ & $\parallel$ to $\langle 1, 3, -\frac{2}{3} \rangle$

$\bar{r}_0 = \langle 6,-5,2 \rangle$, $\bar{v} = \langle 1, 3, -\frac{2}{3} \rangle$

Vector eq'n: $\bar{r}(t) = \bar{r}_0 + t\bar{v}$, $t \in \mathbb{R}$

Parametric: $x = t+6$, $y = 3t-5$, $z = -\frac{2}{3}t + 2$

#s 6–12 Find parametric & symmetric eq'ns for the line

(6) thru $(0,0,0)$ & $(4,3,-1)$

vec: $\bar{r}_0 = (0,0,0)$, $\bar{u} = \langle 4, 3, -1 \rangle$

par. $x = 4t$, $y = 3t$, $z = -t$

symm: $\frac{x}{4} = \frac{y}{3} = -z$

(7) thru $(0, \frac{1}{2}, 1)$ & $(2, 1, -3)$ $\bar{r}_0 + t\bar{u}$

$\bar{r}_0 = \langle 2, 1, -3 \rangle$, $\bar{u} = \langle 2, \frac{1}{2}, -4 \rangle$

par. $x = 2t + 2$, $y = \frac{1}{2}t + 1$, $z = -4t - 3$

sym: $\frac{x-2}{2} = \frac{y-1}{\frac{1}{2}} = \frac{z+3}{-4}$

(10) thru $(2,1,0)$ & $\perp$ to $\langle 1,1,0 \rangle$ & $\langle 0,1,1 \rangle$

$\langle 1,1,0 \rangle, 1, 1$
$\times \langle 0,1,1 \rangle, 0, 1$

$\bar{u} = \langle 1, -1, 1 \rangle$

$\bar{r}_0 = \langle 2, 1, 0 \rangle$

$x = t + 2$, $y = -t + 1$, $z = t$

$x - 2 = \frac{y-1}{-1}$, $z = t$

§12.5 I #s 11-14, 16, 17, 21, 23, 24, 26, 27,
30, 31, 35, 36

(11) thru $(-6, 2, 3)$ & $\parallel$ to $\frac{1}{2}x = \frac{1}{3}y = z+1$

$\bar{u} = \langle 2, 3, 1 \rangle$ $\quad \bar{r}_0 = \langle -6, 2, 3 \rangle$

par: $x = 2t-6,\ y = 3t+2,\ z = t+3$

sym: $\frac{x+6}{2} = \frac{y-2}{3} = z-3$

(12) Line of intersection of $x+2y+3z = 1$ & $x-y+z=1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 2 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & 2 & 0 \end{array}\right]$$

$3y + 2z = 0$

$3y = -2z$

$y = -\frac{2}{3}z$

$x - y + z = 1$

$x - (-\frac{2}{3}z) + z = 1$

$x + \frac{2}{3}z + z = 1$

$x + \frac{5}{3}z = 1$

$x = -\frac{5}{3}z + 1$

par: $x = -\frac{5}{3}t + 1,\ y = -\frac{2}{3}t,\ z = t$

sym: $-\frac{3}{5}(x-1) = -\frac{3}{2}y = z$

(14) Is the line thru $(-2, 4, 0)$ & $(1, 1, 1)$ $\perp$ to the line thru $(3, -1, -8)$ & $(2, 3, 4)$

$\bar{u} = \langle 3, -3, 1 \rangle,\ \bar{v} = \langle 1, -4, -12 \rangle$

$\bar{u} \cdot \bar{v} = 3 + 12 - 12 = 3 \neq 0 \Rightarrow$ not $\perp$

§ 12.5 I #s 16, 17, 21, 23, 24, 26, 27, 30, 31, 35, 36

(16) Find line thru (2, 4, 6) that's
$\perp$ to $x - y + 3z = 7$

(a) $\bar{n} = \langle 1, -1, 3\rangle = \bar{u}$, $\bar{r}_0 = \langle 2, 4, 6\rangle$

parametric eq'n: $x = t+2,\ y = -t+4,\ z = 3t+6$

(b) where does this line intersect the coordinate planes?

xy- : $z = 3t + 6 = 0$
$z=0$ $\quad 3t = -6$
$t = -2$

$\boxed{(x, y, z) = (0, 6, 0)}$

xz- : $-t + 4 = y = 0$
$y = 0$ $\quad t = 4$

$\boxed{(x, y, z) = (6, 0, 18)}$

yz- : $t = -2$
$x = 0$

$\boxed{(x, y, z) = (0, 6, 0)}$ ✓

(17) Find vector eq'n for line segment thru FROM $(6, -1, 9)$ & TO $(7, 6, 0)$

$\boxed{\bar{r} = (1-t)\langle 6, -1, 9\rangle + t\langle 7, 6, 0\rangle \quad 0 \le t \le 1}$

#s 19-22 Determine if L_1 & L_2 are parallel, skew or intersecting. If intersecting, find point.

§12.5 I #s 21, 23, 24, 26, 27, 30, 31, 35, 36

(21) L_1: $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-1}{-3}$

L_2: $\frac{x-3}{1} = \frac{y+4}{3} = \frac{z-2}{-7}$

L_1 par.: $x = t+2,\ y = -2t+3,\ z = -3t+1$

L_2 par.: $x = s+3,\ y = 3s-4,\ z = -7s+2$

x: $t+2 = s+3$

$t = s+1$

y: $-2(s+1)+3 = 3s-4$

$-2s-2+3 = 3s-4$

$-5s+1 = -4$

$-5s = -5$

$s = 1 \Rightarrow t = s+1 = \boxed{2 = t}$

z: $-3t+1 = -7s+1$ L_1: $t=2$: $\langle 4,-1,-5\rangle$

$-3(2)+1 = -7(1)+1$ L_2: $s=1$; $\langle 4,-1,-5\rangle$ ✓

$5 = 5$ ✓

$\boxed{\therefore (x,y,z) = (4,-1,-5)}$ is intersection

#s 23-40 Find eq'n of the plane.

(23) thru $(0,0,0)$ & $\perp$ to $\vec{n} = \langle 1,-2,5\rangle$

$\boxed{x - 2y + 5z = 0}$

§12.5 I #'s 24, 26, 27, 30, 31, 35, 36

(24) Thru $(5,3,5)$ with $\bar{n} = \langle 2,1,-1\rangle$

$\bar{n}\cdot(\bar{r}-\bar{r}_0) = 0$

$2(x-5) + 1(y-3) - 1(z-5) = 0$

$2x - 10 + y - 3 - z + 5 = 0$

$\boxed{2x + y - z = 8}$

(26) thru $(2,0,1)$ & $\perp$ to $x = 3t,\ y = 2-t,$

$z = 4t+3 \qquad \bar{n} = \langle 3,-1,4\rangle$

$3(x-2) - 1(y) + 4(z-1) = 0$

$3x - 6 - y + 4z - 4 = 0$

$\boxed{3x - y + 4z = 10}$

(27) Thru $(1,-1,-1)$ & $\parallel$ to $5x - y - z = 6$

$\bar{n} = \langle 5,-1,-1\rangle$

$5(x-1) - 1(y+1) - 1(z+1) = 0$

$5x - 5 - y - 1 - z - 1 = 0$

$\boxed{5x - y - z = 7}$

§12.5 I 30, 31, 35, 86

30 Contains $x = 1+t$, $y = 2-t$, $z = 4-3t$

& ∥ to $5x+2y+z=1$ $\quad \bar{n} = \langle 5, 2, 1 \rangle$

$t=0$ gives $(x_0, y_0, z_0) = \bar{r}_0 = \langle 1, 2, 4 \rangle$

$$5(x-1)+2(y-2)+1(z-4)=0$$

$$5x-5+2y-4+z-4=0$$

$$\boxed{5x+2y+z=13}$$

Not sure about this. Does this contain the line described

$5(t+1)+2(2-t)+1(4-3t)$ It works!

$= 5t+5+4-2t+4-3t = 13$ I'm just not

$5+4+4=13$ ✓ sure it wasn't

CONTRIVED to work!

31 Plane thru $(0,1,1)$, $(1,0,1)$, $(1,1,0)$

$$\bar{u} = \langle -1, 1, 0 \rangle, -1, 1$$

$$\times \ \bar{v} = \langle 0, 1, -1 \rangle, 0, 1$$

$$\bar{n} = \langle -1, -1, -1 \rangle$$

$$\bar{r}_0 = \langle 0, 1, 1 \rangle \Rightarrow$$

$$-1(x)-1(y-1)-1(z-1)=0$$

$$-x-y+1-z+1=0$$

$$-x-y-z=-2$$

$$\boxed{x+y+z=2}$$

203 §12.5 I #s 35, 36

(35) The plane thru $(3,5,-1)$ & contains $x=4-t,\ y=2t-1,\ z=-3t$

$\bar{u} = \langle 4-3, -1-5, 1\rangle = \langle 1,-6,1\rangle$ is "in" ($\parallel$) the plane, and the direction vector $\bar{v} = \langle -1,2,-3\rangle$ is "in" ($\parallel$) to it.

So
$$\bar{u} = \langle 1,-6,1\rangle,\ 1,-6$$
$$\bar{v} = \langle -1,2,-3\rangle,\ -1,2$$
$$\bar{n} = \langle 16, 2, -4\rangle$$

Then $16(x-3)+2(y-5)-4(z+1)=0$

$\rightarrow 16x-48+2y-10-4z-4=0$

$$\boxed{16x+2y-4z=62}$$

(36) Plane thru $(6,-1,3)$ & contains

$\frac{x}{3} = y+4 = \frac{z}{2} \rightarrow$

$x=3t,\ y=t-4,\ z=2t \rightarrow$ direction vector $\bar{v}=\langle 3,1,2\rangle$

$\bar{v}$ is direction vector

$\bar{u}$ is vector from $(0,-4,0)$ to $(6,-1,3)$

$$\bar{u} = \langle 6,3,3\rangle,\ 6,\ 3$$
$$\times\ \bar{v} = \langle 3,1,2\rangle,\ 3,\ 1$$
$$\bar{n} = \langle 3,-3,-3\rangle$$

Then $3(x-6)-3(y+1)-3(z-3)=0 \rightarrow \boxed{3x-3y-3z=12}$

$3x-18-3y-3-3z+9=0$