203 §12.4 #s 1, 4, 7, 13, 15, 20, 22, 27, 28, 33,
37, 41, 46

#s 1-7 Find $\bar{a} \times \bar{b}$ & verify it's $\perp$ to $\bar{a}$ & $\bar{b}$

(1) $\bar{a} = \langle 2, 3, 0 \rangle$, $\bar{b} = \langle 1, 0, 5 \rangle$

$\bar{a} = \langle 2, 3, 0 \rangle, 2, 3$

$\times\ \bar{b} = \langle 1, 0, 5 \rangle, 1, 0$

$\boxed{\bar{c} = \langle 15, -10, -3 \rangle = \bar{a} \times \bar{b}}$

$\bar{a} \cdot \bar{c} = 30 - 30 + 0 = 0$ ✓

$\bar{b} \cdot \bar{c} = 15 + 0 - 15 = 0$ ✓

(4) $\bar{a} = \langle 3, 3, -3 \rangle$, $\bar{b} : \langle 3, -3, 3 \rangle$

$\bar{a} = \langle 3, 3, -3 \rangle, 3, 3$

$\bar{b} = \langle 3, -3, 3 \rangle, 3, -3$

$\boxed{\bar{c} = \langle 0, -18, -18 \rangle = \bar{a} \times \bar{b}}$

$\bar{a} \cdot \bar{c} = 0 - 54 + 54 = 0$ ✓

$\bar{b} \cdot \bar{c} = 0 + 54 - 54 = 0$

(7) $\bar{a} = \langle t, 1, \frac{1}{t} \rangle, t, 1$

$\times\ \bar{b} = \langle t^2, t^2, 1 \rangle, t^2, t^2$

$c = \boxed{\langle 1-t, 0, t^3 - t^2 \rangle = \bar{a} \times \bar{b}}$

$\bar{a} \cdot \bar{c} = t - t^2 + 0 + t^2 - t = 0$ ✓

$\bar{b} \cdot \bar{c} = t^2 - t^3 + 0 + t^3 - t^2 = 0$ ✓

203 §12.4 #s 13, 15, 20, 22, 27, 28, 33, 37, 41, 46

(13) state if each is meaningful. If not, state why. If so, explain if it's vector or scalar

(a) $\bar{a}\cdot(\bar{b}\times\bar{c})$ scalar

(b) $\bar{a}\times(\bar{b}\cdot\bar{c})$ vector × scalar ?!

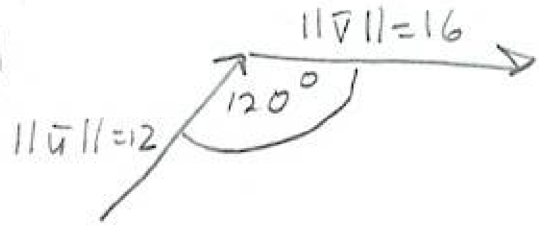
(c) $\bar{a}\times(\bar{b}\times\bar{c})$ vector

(d) $\bar{a}\cdot(\bar{b}\cdot\bar{c})$ vector · scalar ?!

(e) $(\bar{a}\cdot\bar{b})\times(\bar{c}\cdot\bar{d})$ scalar × scalar ?!

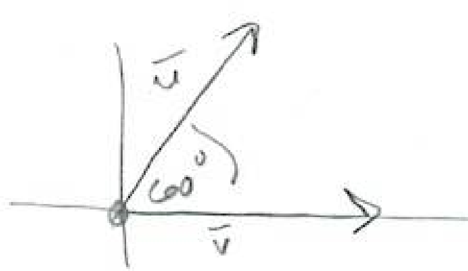
(f) $(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d})$ scalar.

(15)



Find $\|\bar{u}\times\bar{v}\|$ & determine if it's into or out of the page.

INTO PAGE



$\|\bar{u}\times\bar{v}\| = \|\bar{u}\|\,\|\bar{v}\|\sin 60^\circ$

$= (16\cdot 12)\frac{\sqrt{3}}{2} = \boxed{96\sqrt{3}}$

203 § 12.4 #s 20, 22, 27, 28, 33, 37, 41, 46

(20) Find 2 unit vectors $\perp$ to both $\bar{j} - \bar{k}$ & $\bar{i} + \bar{j}$

$\bar{u} = \langle 0, 1, -1 \rangle$, 0, 1

$\bar{v} = \langle 1, 1, 0 \rangle$, 1, 1

$\bar{u} \times \bar{v} = \boxed{\langle 1, -1 -1 \rangle = \bar{w}}$

$\boxed{\pm \frac{\bar{w}}{\sqrt{3}} \text{ are } \perp \text{ to both}}$

(22) Show $(\bar{a} \times \bar{b}) \cdot \bar{b} = 0 \ \forall \ \bar{a}, \bar{b} \in V^3$

Case 1: $\bar{a} \parallel \bar{b} \Rightarrow \bar{a} \times \bar{b} = \bar{0}$

& $\bar{0} \cdot \bar{b} = 0 \ \forall \ \bar{b} \in V^3$

Case 2: $\bar{a} \times \bar{b}$ is $\perp$ to $\bar{a}$ & $\bar{b}$.

(27) Find area of parallelogram wit vertices $A(-3, 0)$, $B(-1, 3)$, $C(5, 2)$, $D(3, -1)$

$\bar{u} = \overrightarrow{AB}$, $= \langle 2, 3 \rangle$

$\bar{v} = \overrightarrow{AD} = \langle 6, -1 \rangle$

$\in V^3$!

$\bar{u} = \langle 2, 3, 0 \rangle$, 2, 3

$\times \bar{v} = \langle 6, -1, 0 \rangle$, 6, −1

$\bar{u} \times \bar{v} = \langle 0, 0, -20 \rangle \Rightarrow \boxed{\text{Area} = 20}$

203 §12.4 HS 28, 33, 37, 41, 46

28) Find area of parallelogram

$P(1,0,2)$, $Q(3,3,3)$, $R(7,5,8)$, $S(5,2,7)$

$\overrightarrow{PQ} = \bar{u} = \langle 2,3,1 \rangle, 2, 3$

$\overrightarrow{PS} = \bar{v} = \langle 4,2,5 \rangle, 4, 2$

$\bar{u} \times \bar{v} = \langle 13,-6,-8 \rangle \Rightarrow$

$\text{Area} = \sqrt{13^2+6^2+8^2} = \sqrt{169+36+64} = \boxed{\sqrt{269}}$

Check: $\overrightarrow{PR} = \langle 6,5,6 \rangle$ is diagonal (biggest)

37) Use scalar triple product to show vectors are coplanar.

$\bar{u} = \langle 1,5,-2 \rangle$, $\bar{v} = \langle 3,-1,0 \rangle$, $\bar{w} = \langle 5,9,-4 \rangle$

~~$\bar{u} \cdot (\bar{v} \times \bar{w}) = \langle 1,5,-2 \rangle$~~

$\bar{v} = \langle 3,-1,0 \rangle, 3, -1$

$\times\ \bar{u} = \langle 1,5,-2 \rangle, 1, 5$

$\bar{v} \times \bar{u} = \langle 2,6,16 \rangle \Rightarrow \bar{u} \times \bar{v} = \langle -2,-6,-16 \rangle$

$\bar{w} \cdot (\bar{v} \times \bar{u}) = \langle 5,9,-4 \rangle \cdot \langle 2,6,16 \rangle$

$= 10 + 54 - 64 = 0 \Rightarrow$

$\boxed{\text{volume} = 0 \Rightarrow \text{same plane}}$

203 §12.4 #s 41, 46

41) A wrench 30 cm long lies on positive y-axis & grips a bolt at the origin. A force is applied in the direction $\langle 0,3,-4\rangle$. Find magnitude of force needed to supply 100 N·m of torque to the bolt.

$\bar{r} = \langle 0, .3, 0\rangle$

$\bar{F}$ in direction of $\langle 0,3,-4\rangle \Longrightarrow$

$$\cos\theta = \frac{\langle 0,.3,0\rangle \cdot \langle 0,3,-4\rangle}{\sqrt{.3^2}\sqrt{3^2+4^2}} = \frac{.9}{.3\sqrt{25}} = \frac{.9}{(.3)(5)} = \frac{.9}{1.5} = \frac{3}{5}$$

$$= \frac{9}{45} = \frac{1}{5} \Longrightarrow \theta \approx 126.869.8976$$

$$\text{Torque} = \|\bar{\tau}\| = 100 = \|\bar{r}\|\,\|\bar{F}\|\sin\theta$$

$$\approx .3\|\bar{F}\|\sin 126.8\ldots^\circ \approx 100.$$

$$\approx \|\bar{F}\| \approx \frac{100}{.3\sin 126.8\ldots^\circ}$$

$$\approx 416.\bar{6} \approx \boxed{417\text{ N}}$$

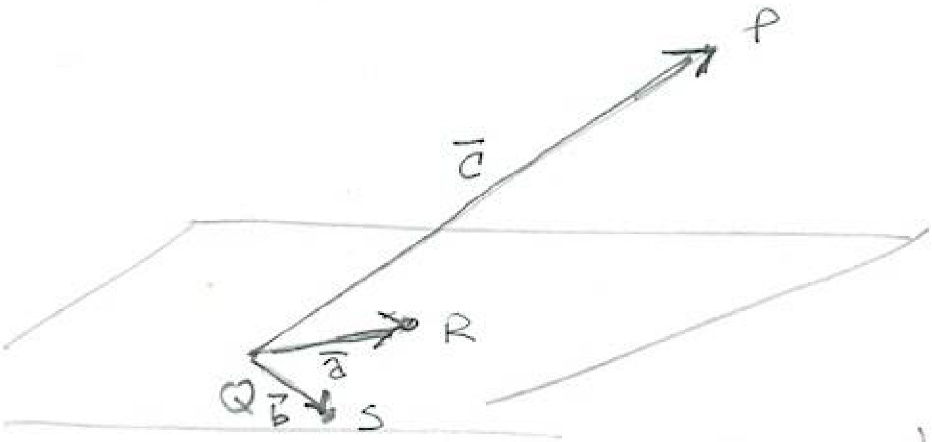
203 §12.4 # 46

(46) $P \notin \mathcal{P}$ = Plane containing Q, R, S

Show that distance from P to $\mathcal{P}$ is

$$d = \frac{|\bar{a} \cdot (\bar{b} \times \bar{c})|}{\|\bar{a} \times \bar{b}\|}, \text{ where}$$

$\bar{a} = \overrightarrow{QR}$, $\bar{b} = \overrightarrow{QS}$, $\bar{c} = \overrightarrow{QP}$



$\bar{n} = \bar{a} \times \bar{b}$, we Project $\bar{c}$ onto $\bar{n}$

for its distance/length, i.e. $|\text{comp}_{\bar{n}} \bar{c}|$

$$= \left| \frac{\bar{c} \cdot \bar{n}}{\|\bar{n}\|} \right| = \frac{|\bar{c} \cdot (\bar{a} \times \bar{b})|}{\|\bar{a} \times \bar{b}\|}$$

$$= \frac{|\bar{a} \cdot (\bar{b} \times \bar{c})|}{\|\bar{a} \times \bar{b}\|} \text{ by Theorem.}$$

(b) $P(2,1,4)$, $Q(1,0,0)$, $R(0,2,0)$, $S(0,0,3)$

$\overrightarrow{PQ} = \bar{a} = \langle -1,-1,-4 \rangle$, $\bar{b} = \overrightarrow{QR} = \langle -1, 2, 0 \rangle = \bar{b}$,

$\bar{c} = \overrightarrow{QS} = \langle -1, 0, 3 \rangle$ $\times$ $\begin{matrix} \bar{b} & \langle -1, 2, 0 \rangle, -1, 2 \\ \bar{c} & \langle -1, 0, 3 \rangle, -1, 0 \end{matrix}$

~~$\bar{a} \cdot (\bar{b} \times \bar{c})$~~ $= \langle -1,-1,4 \rangle \cdot \langle 6, 3, 2 \rangle$

203 § 13.4 #46b

46b cntd

$$\frac{|\bar{a}\cdot(\bar{b}\times\bar{c})|}{\|\bar{b}\times\bar{c}\|} = \frac{|\langle -1,-1,4\rangle \cdot \langle 6,3,2\rangle|}{\sqrt{6^2+3^2+2^2}}$$

$$= \frac{|-6-3+8|}{\sqrt{36+9+4}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$$

~~Book version~~

~~$\bar{c} = \overrightarrow{QR} = \langle$~~