203 §12.3 #s 13, 14, 16, 19, 23, 33, 34, 40, 42

(13) (a) Show that $\bar{i}\cdot\bar{j} = \bar{j}\cdot\bar{k} = \bar{k}\cdot\bar{i} = 0$

(b) Show that $\bar{i}\cdot\bar{i} = \bar{j}\cdot\bar{j} = \bar{k}\cdot\bar{k} = 1$

(a) $\bar{i}\cdot\bar{j} = \langle 1,0,0\rangle \cdot \langle 0,1,0\rangle = 0+0+0 = 0$

$\bar{j}\cdot\bar{k} = \langle 0,1,0\rangle \cdot \langle 0,0,1\rangle = 0+0+0 = 0$

$\bar{k}\cdot\bar{i} = \langle 0,0,1\rangle \cdot \langle 1,0,0\rangle = 0+0+0 = 0$

(b) $\bar{i}\cdot\bar{i} = \langle 1,0,0\rangle \cdot \langle 1,0,0\rangle = 1+0+0 = 1$

$\bar{j}\cdot\bar{j} = \langle 0,1,0\rangle \cdot \langle 0,1,0\rangle = 0+1+0 = 1$

$\bar{k}\cdot\bar{k} = \langle 0,0,1\rangle \cdot \langle 0,0,1\rangle = 0+0+1 = 1$

(14) a hamburgers, b hot dogs, c drinks are sold at \$4, \$2.50, & \$1, respectively for each. If $\bar{A} = \langle a,b,c\rangle$ &

$\bar{P} = \langle 4, 2.5, 1\rangle$, then

$\bar{A}\cdot\bar{P} = 4a + 2.5b + c$ = total revenue from sales.

2-03 §12.3 #s 16, 19, 23, 33, 34, 40, 42

#s 15-20

Find the angle between the vectors

Exact & then to nearest degree.

(16) $\bar{a} = \langle -2, 5 \rangle$, $\bar{b} = \langle 5, 12 \rangle \Longrightarrow$

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{\|\bar{a}\| \, \|\bar{b}\|} = \frac{-10+60}{\sqrt{4+25}\,\sqrt{25+144}}$$

$$= \frac{50}{\sqrt{29}\,\sqrt{169}} = \frac{50}{13\sqrt{29}} \Longrightarrow$$

$$\theta = \arccos\left(\frac{50}{13\sqrt{29}}\right) \approx 44.42127443^\circ$$

$$\approx \boxed{44^\circ}$$

(19) $\bar{a} = \langle 4, -3, 1 \rangle$, $\bar{b} = \langle 2, 0, -1 \rangle$

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{\|\bar{a}\| \, \|\bar{b}\|} = \frac{8+0-1}{\sqrt{16+9+1}\,\sqrt{4+1}} = \frac{7}{\sqrt{26}\sqrt{5}}$$

$$\theta = \arccos\left(\frac{7}{\sqrt{5}\sqrt{26}}\right) \approx 52.12501635^\circ \approx \boxed{52^\circ}$$

(23) $\perp$, $\parallel$, or neither?

(a) $\bar{a} = \langle 9, 3 \rangle$, $\bar{b} = \langle -2, 6 \rangle$

$\bar{a} \cdot \bar{b} = -18 + 18 = 0 \Longrightarrow \boxed{\perp}$

(b) $\bar{a} = \langle 4, 5, -2 \rangle$, $\bar{b} = \langle 3, -1, 5 \rangle$

$\bar{a} \cdot \bar{b} = 12 - 5 - 10 = -3$ $\boxed{\text{Neither}}$

$\|\bar{a}\| = \sqrt{16+25+4} = \sqrt{45}$

$\|\bar{b}\| = \sqrt{9+1+25} = \sqrt{35}$

203 §12.3 #s 23, 33, 34, 40, 42

(23) (c) $\bar{a} = \langle -8, 12, 4\rangle$, $\bar{b} = \langle 6, -9, -3\rangle$

$= -4\langle 2, -3, -1\rangle$ $= 3\langle 2, -3, -1\rangle$

are $\boxed{\parallel}$

(d) $\bar{a} = \langle 3, -1, 3\rangle$, $\bar{b} = \langle 5, 9, -2\rangle$

$\bar{a} \cdot \bar{b} = 15 - 9 - 6 = 0$ $\boxed{\perp}$

#s 33-37 Direction cosines & direction angles, to nearest degree.

(33) $\bar{u} = \langle 2, 1, 2\rangle$

$\cos\alpha = \frac{2}{\sqrt{4+1+4}} = \boxed{\frac{2}{3}} \Rightarrow \alpha \approx 48.1896851^\circ$

$\approx \boxed{48^\circ \approx \alpha}$

$\boxed{\cos\beta = \frac{1}{3}} \Rightarrow \beta \approx 70.52877937 \Rightarrow$

$\boxed{\beta \approx 71^\circ}$

$\boxed{\cos\gamma = \frac{2}{3} \Rightarrow \gamma \approx 48^\circ}$

(34) $\bar{u} = \langle 6, 3, -2\rangle$, $\|\bar{u}\| = \sqrt{36+9+4} = \sqrt{49} = 7$

$\boxed{\cos\alpha = \frac{6}{7}} \Rightarrow \alpha \approx 31.0027191 3^\circ$

$\approx \boxed{31^\circ \approx \alpha}$

$\boxed{\cos\beta = \frac{3}{7}} \Rightarrow \beta \approx 64.62306647^\circ \approx \boxed{65^\circ \approx \beta}$

$\cos\gamma = -\frac{2}{7} \Rightarrow \gamma \approx 106.6015496^\circ \approx \boxed{107^\circ \approx \gamma}$

§12.3 #s 40, 42

#s 39–44 Find $\text{comp}_{\bar{a}} \bar{b}$ & $\text{proj}_{\bar{a}} \bar{b}$ ($\bar{b}$ onto $\bar{a}$)

(40) $\bar{a} = \langle 1, 4 \rangle$, $\bar{b} = \langle 2, 3 \rangle$

$$\text{comp}_{\bar{a}} \bar{b} = \frac{\bar{a} \cdot \bar{b}}{\|\bar{a}\|} = \frac{2+12}{\sqrt{17}} = \frac{14}{\sqrt{17}} = \text{comp}_{\bar{a}} \bar{b}$$

$$\text{proj}_{\bar{a}} \bar{b} = \boxed{\frac{14}{17} \langle 1, 4 \rangle}$$

(42) $\bar{a} = \langle -1, 4, 8 \rangle$, $\bar{b} = \langle 12, 1, 2 \rangle$

$$\text{comp}_{\bar{a}} \bar{b} = \frac{-12+4+16}{\sqrt{1+16+64}} = \frac{8}{\sqrt{81}} = \boxed{\frac{8}{9}}$$

$$\text{proj}_{\bar{a}} \bar{b} = \boxed{\frac{8}{81} \langle -1, 4, 8 \rangle}$$

(45) Show that $\text{orth}_{\bar{a}} \bar{b} = \bar{b} - \text{proj}_{\bar{a}} \bar{b}$ is $\perp$ to $\bar{a}$.

$$\bar{b} - \text{proj}_{\bar{a}} \bar{b} = \bar{b} - \frac{\bar{b} \cdot \bar{a}}{\|\bar{a}\|^2} \bar{a} \longrightarrow$$

$$\bar{a} \cdot \text{orth}_{\bar{a}} \bar{b} = \bar{a} \cdot \bar{b} - \bar{a} \cdot \bar{a} \left(\frac{\bar{b} \cdot \bar{a}}{\|\bar{a}\|^2} \right) =$$

$$= \bar{a} \cdot \bar{b} - \|\bar{a}\|^2 \left(\frac{\bar{b} \cdot \bar{a}}{\|\bar{a}\|^2} \right) =$$

$$= \bar{a} \cdot \bar{b} - \bar{b} \cdot \bar{a} = 0 \quad \blacksquare$$

$$\bar{a} \cdot \bar{b} - \bar{b} \cdot \bar{a}$$