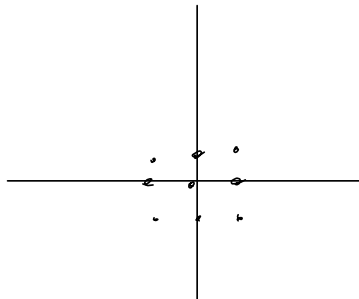


16.1 Plot a vector field.



16.3

16.2 Calculate a line integral

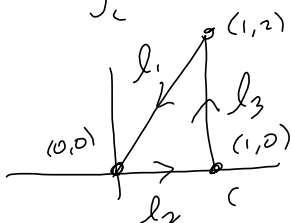
$$\langle xy, x^2y^3 \rangle \cdot \langle x'(t), y'(t) \rangle dt \quad \vec{F} = \langle P, Q \rangle \quad 16.5$$

$$16.4 \quad \int_C xy dx + x^2y^3 dy = \int \vec{F} \cdot d\vec{r} = \int \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt$$

$$= \int_C \langle P, Q \rangle \cdot \langle x'(t), y'(t) \rangle dt$$

$$= \int_C (Px'(t) + Qy'(t)) dt = \int Px'(t) dt + \int Qy'(t) dt$$

$$= \int_C P dx + Q dy$$



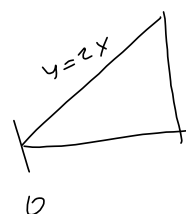
$$\langle 1-t \rangle \langle 1, 2 \rangle + t \langle 0, 0 \rangle$$

$$= \langle 1-t, 2-2t \rangle$$

$$= \langle 1-t, 2(1-t) \rangle$$

$$\iint_D (Q_x - P_y) dA$$

$$\int_0^1 \int_0^{2x} (2xy^3 - x) dy dx = \frac{2}{3}$$



S16.5 #13

$$16.5 \quad \vec{F} = \langle \overset{x}{y^2 z^3}, \overset{y}{2xy z^3}, \overset{z}{3xy^2 z^2} \rangle \quad \vec{G} = \langle \overset{x}{y^2 z^3}, \overset{y}{2xy z^3} \rangle$$

$$\langle 6xy z^2 - 6xy z^2, 3y^2 z^2 - 3y^2 z^2, 2yz^3 - 2yz^3 \rangle = \vec{0}$$

$$f_x = y^2 z^3 \Rightarrow f = xy^2 z^3 + g(y, z) \Rightarrow$$

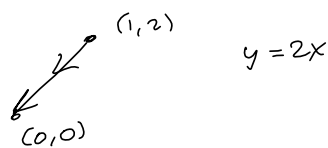
$$f_y = 2xy z^3 = 2xy z^3 + g_y(y, z) \Rightarrow g_y(y, z) = g'(z) \Rightarrow$$

$$f_z = 3xy^2 z^2 = 3xy^2 z^2 + g'(z) \Rightarrow g'(z) = 0 \Rightarrow g(z) = K = 0$$

$$\Rightarrow \boxed{f(x, y, z) = xy^2 z^3}$$

Joseph

$$\int_C xy \, dx$$



$$(1-t) \langle 1, 2 \rangle + t \langle 0, 0 \rangle$$

$$\int_0^1 x \cdot 2x \cdot (-dx) = \int_1^0 2x^2 \, dx \quad (\text{From Right to Left})$$

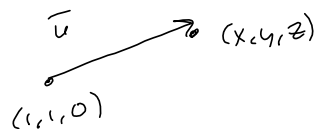
Test 4 Monday

11:00 Keenan, Dylan.

1:30 Joseph

$$\bar{n} = \bar{r}_u \times \bar{r}_v$$

$$\text{Let } (x, y, z) \in \mathcal{P} \Rightarrow \langle x-1, y-1, z \rangle = \bar{u} \in \mathcal{P}.$$



$$\bar{r} = \langle v, u^2 + v^2, u \rangle$$

$$\bar{r}_u = \langle 0, 2u, 1 \rangle, \quad 0, 2u$$

$$\bar{r}_v = \langle 1, 2v, 0 \rangle, \quad 1, 2v$$

---


$$\langle -2v, 1, -2u \rangle$$

$$\Rightarrow \textcircled{q} (1, 1, 0) ?$$

$$v = 1 \quad \rightarrow \quad u = 0, v = 1$$

$$u^2 + v^2 = 1$$

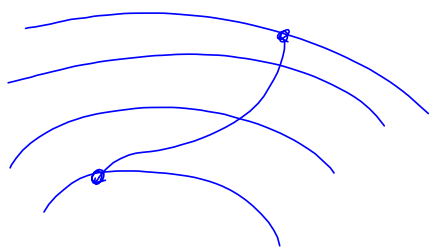
$$u = 0$$

$$\Rightarrow \bar{r}_u \times \bar{r}_v = \langle -2, 1, 0 \rangle = \bar{n}$$

$$\text{Then } \bar{n} \cdot \bar{x} = 0$$

$$\langle -2, 1, 0 \rangle \cdot \langle x-1, y-1, z \rangle = 0$$

is an eq'n of that tangent plane.



Fundamental Theorem of Line Integrals.

Show Conservative.

Find  $f$

$$\text{Eval. } \int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

Green's

$$\int_C \vec{F} \cdot d\vec{r} = \iint_D (\alpha_x - \beta_y) dA = \iint_D \text{curl } \vec{F} \cdot \vec{k} dA$$

Stokes'

$$\int_C \vec{F} \cdot d\vec{r} = \iint_D \text{curl } \vec{F} \cdot \vec{n} dS'$$

$$= \iint_D \text{curl } \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) dA$$

$$\begin{aligned} \nabla^2 f \quad -vs- \quad \nabla^2 F &= \nabla^2 \langle P, Q, R \rangle \\ \nabla \cdot \nabla f &= \langle \nabla^2 P, \nabla^2 Q, \nabla^2 R \rangle \\ \nabla \cdot \langle f_x, f_y, f_z \rangle & \\ = f_{xx} + f_{yy} + f_{zz} & \end{aligned}$$