$$\int_0^1 \int_x^{2x} \int_0^x dy\, dz\, dx$$

$$\int_0^1 \int_0^x \int_x^{2x} yz\cos(x^5)\, dz\, dy\, dx = \frac{3\sin(1)}{20}$$

Sorry for tedium of S'15.5

Setups were the main thing, there Maple would've served for the evaluation.

15.9 Area of a parallelogram w/ the edges $\bar{r}_u \Delta u$, $\bar{r}_v \Delta v$ is $\underbrace{(\bar{r}_u \times \bar{r}_v)\Delta u \Delta v}_{\text{Increment of area}}$

Volume of parallelepiped

$\bar{r}_u \Delta u, \bar{r}_v \Delta v, \bar{r}_w \Delta w$

$$\left.\begin{aligned} &|\bar{r}_u \cdot (\bar{r}_v \times \bar{r}_w)|\Delta u \Delta v \Delta w \\ &= |\bar{r}_v \cdot (\bar{r}_u \times \bar{r}_w)|\Delta u \Delta v \Delta w \\ &= |\bar{r}_w \cdot (\bar{r}_u \times \bar{r}_w)|\Delta u \Delta v \Delta w \end{aligned}\right\} \text{increment of volume.}$$

Think of it this way & be much better off in C16, when we'll have vector-valued funcs. over vector fields.

$$\vec{r} = \langle \overset{x=?}{e^{-r}\sin\theta}, \overset{y=?}{e^{r}\cos\theta}, 0 \rangle \qquad e^{r-r} = e^0 = 1$$

$$\vec{r}_r = \langle -e^{-r}\sin\theta, e^r\cos\theta, 0\rangle, -e^{-r}\sin\theta, e^r\cos\theta$$

$$\times \quad \vec{r}_\theta = \langle e^{-r}\cos\theta, -e^r\sin\theta, 0\rangle, e^{-r}\cos\theta, -e^r\sin\theta$$

$$\langle 0, 0, \sin^2\theta - \cos^2\theta \rangle$$

$$\|\vec{r}_r \times \vec{r}_\theta\| = \sqrt{(\sin^2\theta - \cos^2\theta)^2} = \boxed{|\sin^2\theta - \cos^2\theta|}$$

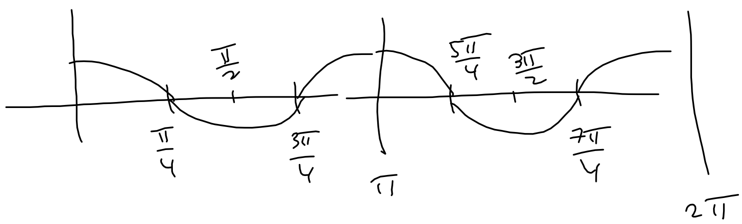
$$\cos^2\theta - \sin^2\theta = \cos(2\theta) \qquad |(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)|$$

is the Jacobian

Getting clever, to no purpose,

$$|\sin^2\theta - \cos^2\theta| = |-\cos(2\theta)| = |\cos(2\theta)|$$

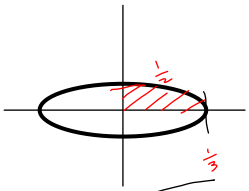
$$= \begin{cases} \cos^2\theta - \sin^2\theta, & \theta \in [0, \frac{\pi}{4}] \cup [\frac{3\pi}{4}, \frac{5\pi}{4}] \cup [\frac{7\pi}{4}, 2\pi] \\ \sin^2\theta - \cos^2\theta, & \theta \in (\frac{\pi}{4}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{7\pi}{4}) \end{cases}$$



$$\iint_R \sin(9x^2+4y^2)\, dA \qquad R = \{(x,y) \mid 9x^2+4y^2 \le 1\}$$

IDIOT CHECK

$$9x^2+4y^2 = 1 \Rightarrow \frac{x^2}{\frac{1}{9}} + \frac{y^2}{\frac{1}{4}} = 1$$

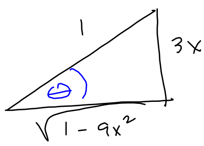


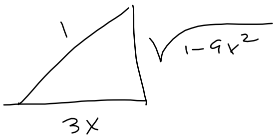
$$4y^2 = 1-9x^2$$

$$y^2 = \frac{1-9x^2}{4}$$

$$y = \pm \frac{\sqrt{1-9x^2}}{2}$$

$$\int_0^{\frac{1}{3}} \int_0^{\frac{1}{2}\sqrt{1-9x^2}} \sin(9x^2+4y^2)\, dy\, dx$$





$$\frac{3x}{1} = \sin\theta$$

$$x = \tfrac{1}{3}\sin\theta$$

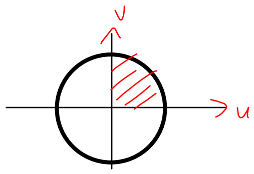
Keenan suggests we make this sucker a circle.

$$9x^2+4y^2 = 1$$

$$u = 3x, \quad v = 2y \Rightarrow$$

$$u^2+v^2 = 1$$

$$u^2+v^2 \le 1 = D$$



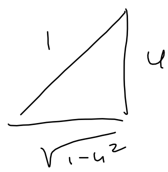
$$x = \tfrac{1}{3}u$$

$$y = \tfrac{1}{2}v$$

$$\bar{r} = \langle \tfrac{1}{3}u, \tfrac{1}{2}v \rangle$$

$$\left| \begin{matrix} \bar{r}_u \\ \bar{r}_v \end{matrix} \right| = \left| \begin{matrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{matrix} \right| = \frac{1}{6}$$

$$\int_0^1 \int_0^{\sqrt{1-u^2}} \sin(u^2+v^2)\left(\tfrac{1}{6}\right) dv\, du$$



$$u^2+v^2 = 1$$

$$v = \sqrt{1-u^2}$$

$$r^2 = 1$$

$$u = r\cos\theta$$

$$v = r\sin\theta$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} \sin(r)$$

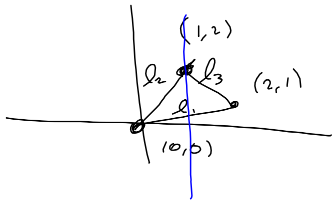
$$0 \le r \le 1$$

$$0 \le \theta \le \frac{\pi}{2}$$

$$\int_0^{\frac{\pi}{2}} \int_0^1 \sin(r^2)\, r\, dr\, d\theta$$

$(0,0), (2,1), (1,2)$

$\iint_R (x-3y)\,dA$



l_1: $y = \frac{1-0}{2-0}(x-0)+0$

$y = \frac{1}{2}x$

l_2: $y = 2x$

l_3: $y = \frac{1-2}{2-1}(x-2)+1 = -\frac{1}{1}(x-2)+1 = -x+2+1 = -x+3$

Enclosed by $y=-x+3 \Longrightarrow x+y=3$

$y = \frac{1}{2}x$ $\quad x-2y=0$

$y = 2x$ $\quad 2x-y=0$

$\boxed{\begin{array}{l} u = x+y \\ v = x-2y \end{array}}$ $\Longrightarrow x = u-y$

$\Longrightarrow v = u-y-2y = u-3y$

$\left[\begin{array}{cc|c} 1 & 1 & u \\ 1 & -2 & v \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & u \\ 0 & -3 & v-u \end{array}\right]$

$y = \frac{v-u}{-3} = \boxed{\frac{u-v}{3} = y}$

$x+y = x+\frac{u-v}{3} = u$

$x = \frac{2u}{3}+\frac{v}{3} = \boxed{\frac{2u+v}{3} = x}$

$\int_0^1 \int_{\frac{x}{2}}^{2x} (x-3y)\,dy\,dx + \int_1^2 \int_{\frac{x}{2}}^{3-x} (x-3y)\,dy\,dx$

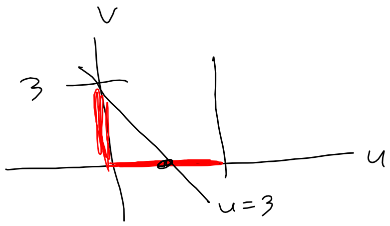
l_1: $y = \frac{1}{2}x \quad 0 \le x \le 1$

$\frac{1}{2}x - y = 0$

$x-2y = 0 = v$

l: $y = 2x$

$u = 2x-y = 0$



l_3: $3-x = 3-\frac{2u+v}{3} = y = \frac{u-v}{3}$

$9-2u-v = u-v$

$-3u = -9$

$u = 3$

$x=1$ goes to

$\frac{2u+v}{3} = 1$

$2u+v = 3$

u	v
0	3
$\frac{3}{2}$	0

$\begin{array}{l} u = ax+by \\ v = cx+dy \end{array}$ to $\begin{array}{l} x = g(u,v) \\ y = h(u,v) \end{array}$

& should be able to calculate Jocobean.