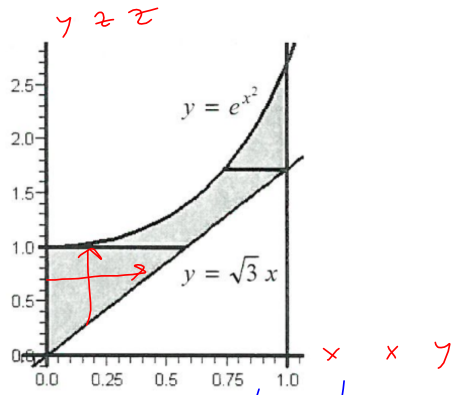
1. (15 pts) Evaluate the iterated integral $\int_0^1 \int_{\sqrt{3}x}^{e^{x^2}} 8xy\,dy\,dx$. A sketch of the Type I region $\boldsymbol{R}$ over which this integral is taken is given on the right, with some additional information you might find helpful for #2.



2. (15 pts) Re-write the integral in #1, to give you the volume under $f(x, y) = 4xy$, by viewing $\boldsymbol{R}$ as a Type II region. This will require 3 different iterated integrals. The extra horizontal lines are meant to be a hint. *Do not evaluate!!!*

(2) $\int_0^1 \int_0^{\frac{1}{\sqrt{3}}y} 8xy\,dx\,dy + \int_1^{\sqrt{3}} \int_{\sqrt{\ln(y)}}^{\frac{1}{\sqrt{3}}y} 8xy\,dx\,dy + \int_{\sqrt{3}}^{e^1} \int_{\sqrt{\ln(y)}}^{1} 8xy\,dx\,dy$

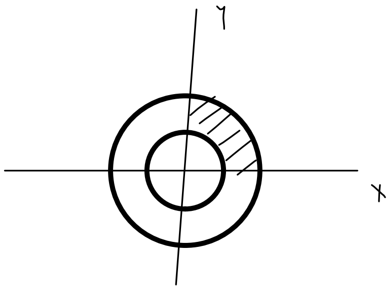
$y = \sqrt{3}\,x \Rightarrow x = \frac{1}{\sqrt{3}}y$

$y = e^{x^2} = y$

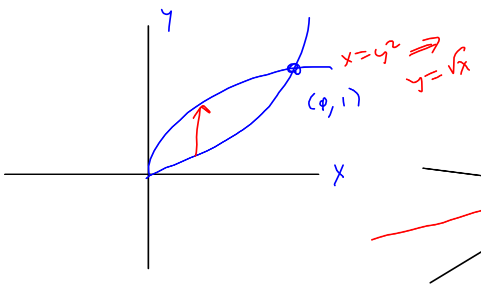
$x^2 = \ln y$

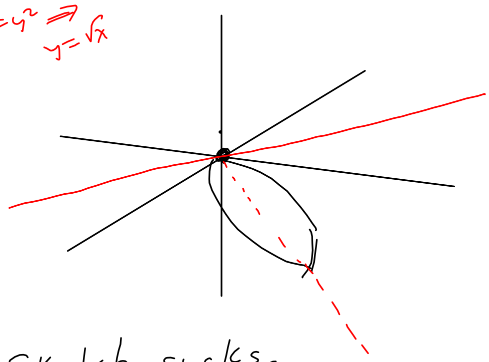
TAKE + :

$x = \pm\sqrt{\ln(y)} \therefore = (\ln(y))^{\frac{1}{2}}$



$$\int_0^{\frac{\pi}{2}}\int_1^2 (r\cos\theta + r\sin\theta)\, r\,dr\,d\theta$$





#4

$z = (x+y)$

$x + y - z = 0$

$(0,0,0)$

$(1,0,1)$

$(0,1,1)$

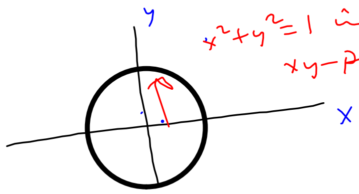
Sketch sucks.

$z = x+y$ plane isn't nice

$$\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x+y} xy \quad dz\,dy\,dx$$

#5

$z = 1-(x^2+y^2)$ $z=0$



$x^2+y^2=1$ in xy-plane

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{1-(x^2+y^2)} (x^3+xy^2)\,dz\,dy\,dx$$

$z=z$, $x=r\cos\theta$, $y=r\sin\theta$

$1-(x^2+y^2)=1-r^2$

$$\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{1-r^2}\left(r^3\cos^3\theta + r\cos\theta\, r^2\sin^2\theta\right) r\,dz\,dr\,d\theta$$

$$\bar{r} = \langle x(r,\theta), y(r,\theta)\rangle = \langle r\cos\theta, r\sin\theta\rangle$$

$$\bar{r}_r = \langle \cos\theta, \sin\theta, 0\rangle, \cos\theta, \sin\theta$$

$$\times \quad \bar{r}_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle, -r\sin\theta, r\cos\theta$$

$$\langle 0, 0, r\cos^2\theta - (-r\sin^2\theta)\rangle$$

$$= \langle 0, 0, r\rangle \quad \& \quad \|\bar{r}_r \times \bar{r}_\theta\| = r$$

$$\bar{r} = \langle r\cos\theta, r\sin\theta, z\rangle$$

$$\bar{r}_r = \langle \cos\theta, \sin\theta, 0\rangle$$

$$\bar{r}_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle$$

$$\bar{r}_z = \langle 0, 0, 1\rangle$$

$$\bar{r}_\theta \cdot (\bar{r}_r \times \bar{r}_z):$$

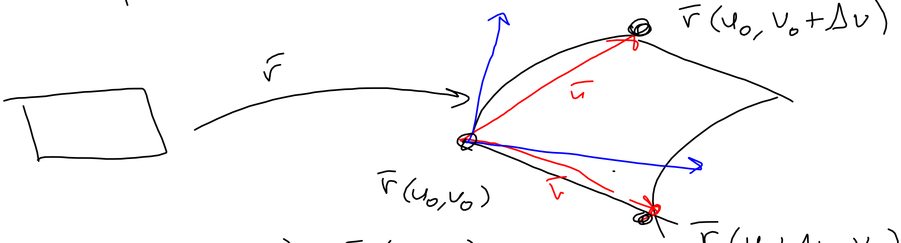
$$\bar{r}_r = \langle \cos\theta, \sin\theta, 0\rangle, \cos\theta, \sin\theta$$

$$\bar{r}_z = \langle 0, 0, 1\rangle, 0, 0$$

$$\langle \sin\theta, -\cos\theta, 0\rangle$$

$$\bar{r}_\theta \cdot (\bar{r}_r \times \bar{r}_z) = \langle -r\sin\theta, r\cos\theta, 0\rangle \cdot \langle \sin\theta, -\cos\theta, 0\rangle$$

$$= \left| -r\sin^2\theta - r\cos^2\theta \right| \quad \left| -r(\sin^2\theta + \cos^2\theta)\right| = r$$



$$\bar{u} = \bar{r}(u_0, v_0+\Delta v) - \bar{r}(u_0,v_0) \approx \bar{r}_v \Delta v$$

$$\bar{v} = \bar{r}(u_0+\Delta u, v_0) - \bar{r}(u_0,v_0) \approx \bar{r}_u \Delta u$$

$$\iint_R (x+y)e^{x^2-y^2}\,dA = \iint u e^{uv} \tfrac{1}{2}\,du\,dv$$

$R:$ $v=0$ to $v=2$ $\quad u=0$ to $u=3$

$x-y=0 \quad x+y=0$

$x-y=2 \quad x+y=3$

$$\frac{1}{2}\int_0^2\int_0^3 u e^{uv}\,du\,dv$$

$u = x+y$

$v = x-y$

$$\left[\begin{array}{c|c} x+y & u \\ x-y & v \end{array}\right] \qquad \left[\begin{array}{cc|c} 1 & 1 & u \\ 1 & -1 & v \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & u \\ 0 & -2 & v-u \end{array}\right]$$

$-2y = v-u$

$\boxed{y = \frac{u-v}{2}}$

$x+y=u$

$x + \frac{u-v}{2} = u$

$x = \frac{2u}{2} - \frac{u}{2} + \frac{v}{2} = \frac{u+v}{2} =$

$\bar{r} = \langle x(u,v), y(u,v)\rangle$

$= \langle \frac{1}{2}(u+v), \frac{1}{2}(u-v)\rangle$

$= \frac{1}{2}\langle u+v, u-v\rangle$

$\bar{r}_u = \frac{1}{2}\langle 1, 1, 0\rangle$, 1, 1

$\times \quad \bar{r}_v = \frac{1}{2}\langle 1, -1, 0\rangle$, 1, -1

$\frac{1}{4}\langle 0, 0, -2\rangle = \langle 0, 0, -\frac{1}{2}\rangle$

$\|\bar{r}_u \times \bar{r}_v\| = \frac{1}{2}$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$$