I think we've covered all the Chapter 15 material. We can do examples and answer questions, but that's about all I plan to do.

15. A solid lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. Write a description of the solid in terms of inequalities involving spherical coordinates.

$x^2+y^2+z^2-z = 0$

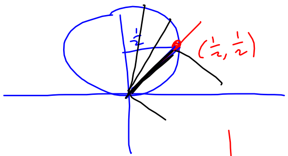
$x^2+y^2+z^2-z+(\frac{1}{2})^2 = \frac{1}{4}$

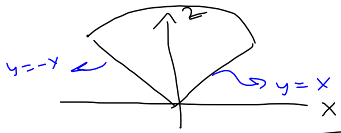
$x^2+y^2+(z-\frac{1}{2})^2 = (\frac{1}{2})^2$

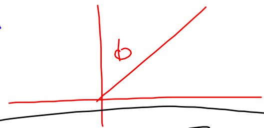
How to describe the cone.

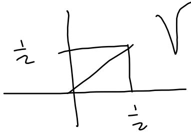
Look @ xz-plane, where y=0:

$z = \sqrt{x^2} = |x|$









$\sqrt{(\frac{1}{2})^2+(\frac{1}{2})^2} = \sqrt{2(\frac{1}{4})}$

$= \frac{1}{2}\sqrt{2}$

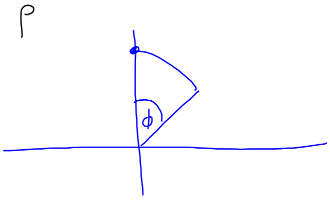
ϕ from 0 to $\frac{\pi}{4}$

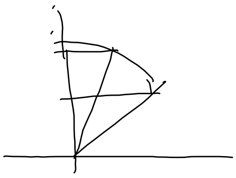
$0 \le \theta \le 2\pi$

$\frac{\sqrt{2}}{2} \le \rho \le 1$

→ This is different interpretation than I used before.

ρ





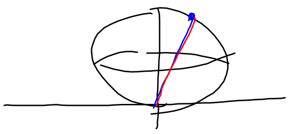
$(z-\frac{1}{2})^2 + y^2 + x^2 = \frac{1}{4}$

$(z-\frac{1}{2})^2 = (\frac{1}{4} - x^2 - y^2)$

$z - \frac{1}{2} = \sqrt{\frac{1}{4} - x^2 - y^2}$ Top $\frac{1}{2}$ of sphere.

$z = \sqrt{1/4 - x^2 - y^2} + \frac{1}{2}$

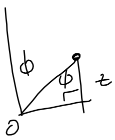
what's distance from top of sphere to origin?



(x,y,z) on sphere.

$(x, y, \sqrt{\frac{1}{4} - x^2 - y^2} + \frac{1}{2})$

Need a better description



$\frac{z}{\rho} = \cos\phi$

$z = \rho\cos\phi$

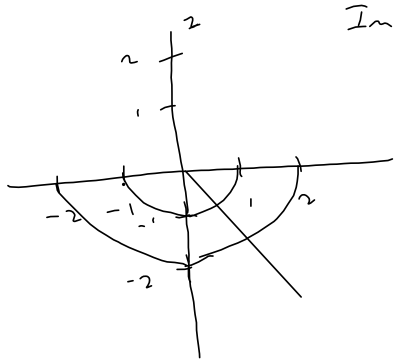
17–18 Sketch the solid whose volume is given by the integral and evaluate the integral.

18. $\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_1^2 \boxed{\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}$

Increment of volume in sphericals?

26. Evaluate $\iiint_E xyz \, dV$, where E lies between the spheres $\rho = 2$ and $\rho = 4$ and above the cone $\phi = \pi/3$.

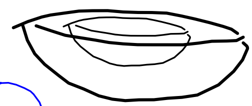
In the xz-plane

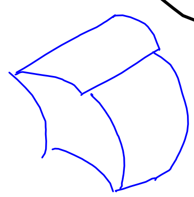


$$\frac{1}{2}\left[\frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3\right]$$

$$= \frac{1}{2}\left(\frac{4}{3}\pi\left(2^3 - 1^3\right)\right)$$

$$= \frac{2\pi}{3}(7) = \frac{14\pi}{3}$$





Jacobian for $u = x+y$, $v = 2x+3y$

we need $x(u,v)$ & $y(u,v)$ to write $\bar{r}_u$ & $\bar{r}_v$

$x + y = u \longrightarrow y = u - x$

$2x+3y = v \longrightarrow 2x + 3(u-x) = v$

$2x+3u-3x = v$

$-x = v-3u$

$\boxed{x = 3u-v}$

$y = u-(3u-v) = -2u+v = y$

$\bar{r} = \langle 3u-v, -2u+v \rangle$

Jacobian $= \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \| \bar{r}_u \times \bar{r}_v \|$

$$\begin{array}{rl} \bar{r}_u & \langle 3, -2, 0 \rangle, 3, -2 \\ \times \ \bar{r}_v & \langle -1, 1, 0 \rangle, -1, 1 \\ \hline & \langle 0, 0, 1 \rangle \ !? \end{array}$$

So Jacobian is 1!

$x + y = u$
$2x+3y = v$

$$\left[\begin{array}{cc|c} 1 & 1 & u \\ 2 & 3 & v \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & u \\ 0 & 1 & -2u+v \end{array}\right]$$

$$\boxed{\sim \left[\begin{array}{cc|c} 1 & 0 & 3u-v \\ 0 & 1 & -2u+v \end{array}\right]}$$

Gauss-JORDAN

$\boxed{y = -2u+v}$

$\Rightarrow x+y = x + (-2u+v) = u$

$\Rightarrow \boxed{x = 3u - v}$

Gauss

I want to do something tougher, where YOU find the change-of-variable.

$$\iint \frac{x+3y}{x-3y} \sim$$

Let $u = x+3y$
$v = x-3y$

$$\iint \frac{u}{v}$$

I want SOME meat on the bones of the Jacobean.