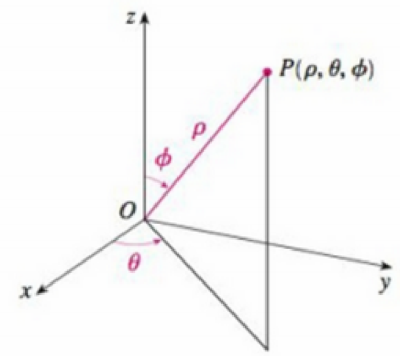
The **spherical coordinates** (ρ, θ, ϕ) of a point P in space are shown in Figure 1, where $\rho = |OP|$ is the distance from the origin to P, θ is the same angle as in cylindrical coordinates, and ϕ is the angle between the positive z-axis and the line segment OP. Note that

$$\rho \geqslant 0 \qquad 0 \leqslant \phi \leqslant \pi$$

$\sqrt{1-x^2}$ ugh!



$\theta = \frac{3\pi}{2}$

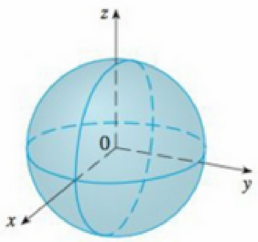


FIGURE 2 $\rho = c$, a sphere

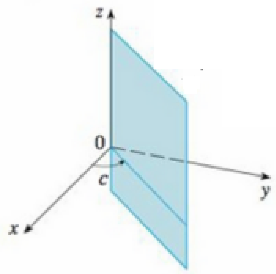
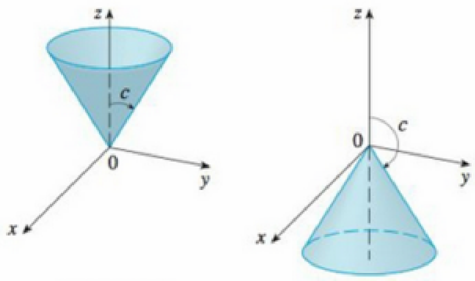


FIGURE 3 $\theta = c$, a half-plane



$0 < c < \pi/2$ $\qquad$ $\pi/2 < c < \pi$

FIGURE 4 $\phi = c$, a half-cone

Questions? Do 15.9

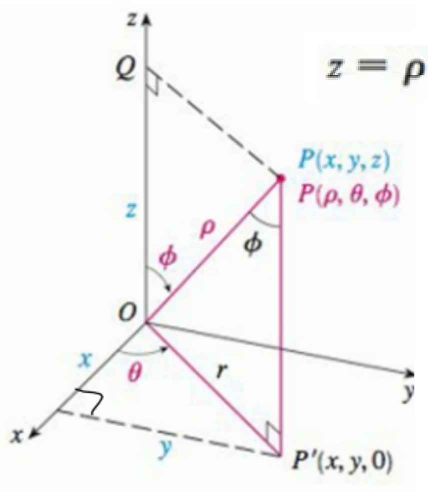


FIGURE 5

$z = \rho \cos \phi \qquad r = \rho \sin \phi$

$x = \underbrace{\rho \sin \phi}_{r} \cos \theta \qquad y = \rho \sin \phi \sin \theta \qquad z = \rho \cos \phi$

$$\rho^2 = x^2 + y^2 + z^2$$

$x = r\cos\theta$

$\frac{r}{\rho} = \sin\phi$

$r = \rho\sin\phi$

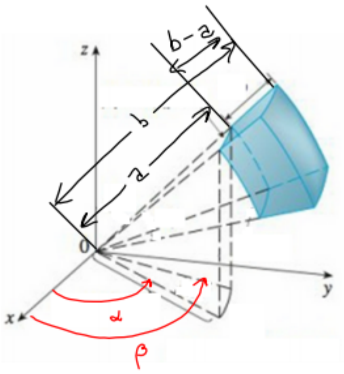
1–2 Plot the point whose spherical coordinates are given. Then find the rectangular coordinates of the point.

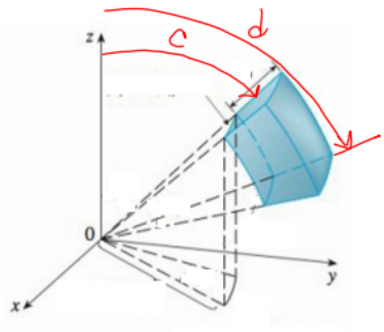
2. (a) $(5, \pi, \pi/2)$ (b) $(4, 3\pi/4, \pi/3)$

In the spherical coordinate system the counterpart of a rectangular box is a **spherical wedge** = spherical "rectangular box."

$$E = \{(\rho, \theta, \phi) \mid a \leqslant \rho \leqslant b,\ \alpha \leqslant \theta \leqslant \beta,\ c \leqslant \phi \leqslant d\}$$

where $a \geqslant 0$, $\beta - \alpha \leqslant 2\pi$, and $d - c \leqslant \pi$.





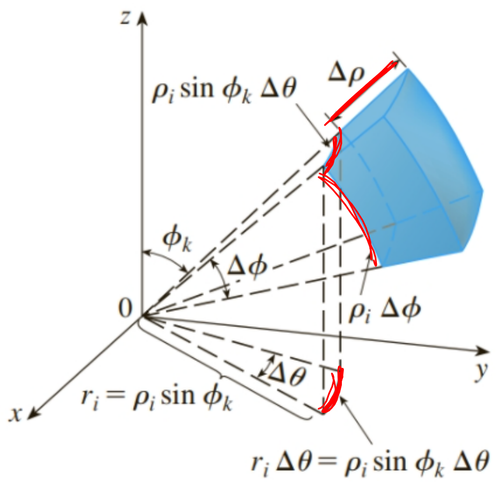


FIGURE 7

Keenan sez

Volume $\approx \Delta\rho\,(\rho \sin\phi\,\Delta\theta)\,(\rho\,\Delta\phi)$

$= \rho^2 \sin\phi\,\Delta\rho\,\Delta\theta\,\Delta\phi$

$\xrightarrow{\Delta\rho,\,\Delta\phi,\,\Delta\theta \,\to\, 0} \rho^2 \sin\phi\,d\rho\,d\theta\,d\phi$

is the increment of volume in spherical coords

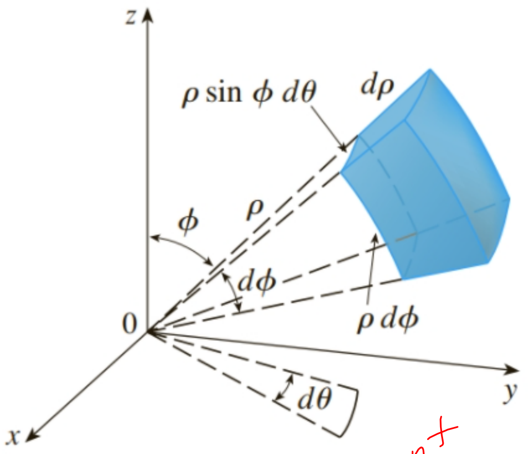


FIGURE 8

increment

Volume $\boxed{\text{element}}$ in spherical coordinates: $dV = \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi$

$$x = \overbrace{\rho \sin \phi}^{r\cos\theta} \cos \theta$$
$$y = \rho \sin \phi \sin \theta$$
$$z = \rho \cos \phi$$

3 $$\iiint_E f(x, y, z) \, dV$$

$$= \int_c^d \int_\alpha^\beta \int_a^b f(\rho \sin \phi \, \cos \theta, \rho \sin \phi \, \sin \theta, \rho \cos \phi) \, \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi$$

where E is a spherical wedge given by

$$E = \{(\rho, \theta, \phi) \mid a \leqslant \rho \leqslant b, \ \alpha \leqslant \theta \leqslant \beta, \ c \leqslant \phi \leqslant d\}$$