Type 3 over TYPE I

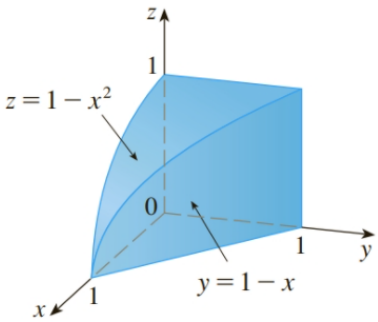
$P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3! = 6$

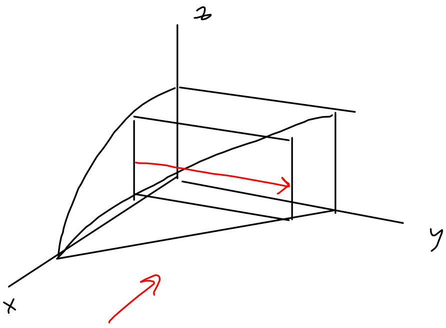
The figure shows the region of integration for the integral

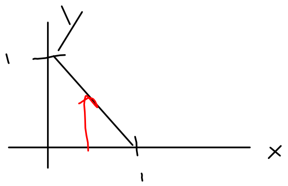
$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z)\, dy\, dz\, dx$$

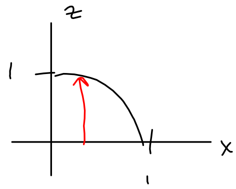
$\{(x,y,z) \mid 0 \le y \le 1-x,\ 0 \le z \le 1-x^2,\ 0 \le x \le 1\}$

Rewrite this integral as an equivalent iterated integral in the five other orders.

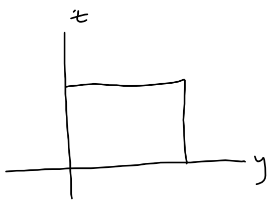


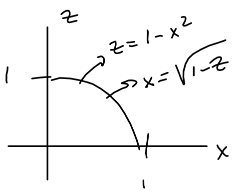


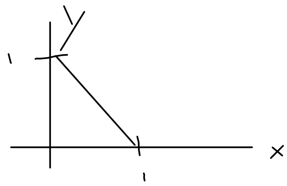


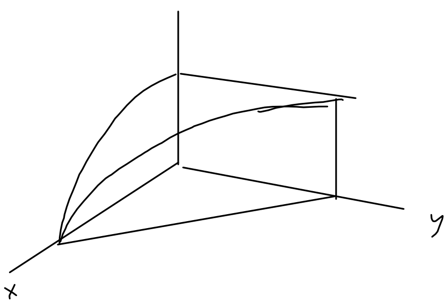


$dx\, dy\, dz$

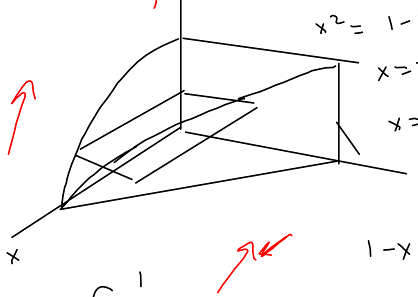








$dx\, dy\, dz$



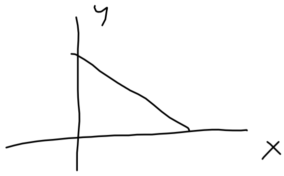
$z = 1-x^2$

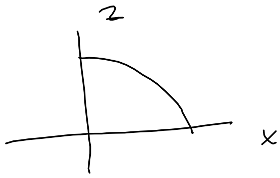
$x^2 = 1-z$

$x = \pm\sqrt{1-z}$

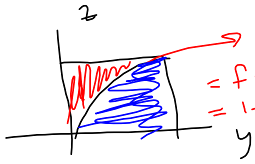
$x = \sqrt{1-z}$

$1-x = 1-\sqrt{1-z}$





$$\int_0^1 \qquad dz$$



$z = f(y)$

$= f(y(x)) = f(y(x(z)))$

$= 1-\sqrt{1-z}$

$z = 1-x^2$

$x^2 = 1-z$

$x = \sqrt{1-z}$

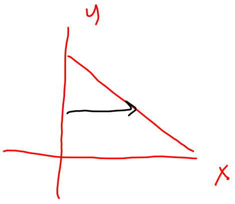
$y = 1-x$

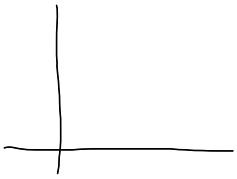
$= 1-\sqrt{1-z}$

dxdydz - toughie! Type 2 over Type II

$$\int_0^1\int_{-\mathrm{sqrt}(1-z)+1}^1\int_0^{1-y} 1\,dx\,dy\,dz+\int_0^1\int_0^{1-\mathrm{sqrt}(1-z)}\int_0^{\mathrm{sqrt}(1-z)} 1\,dx\,dy\,dz$$

$$\frac{5}{12}$$





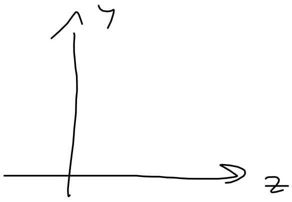
To see this:

$1-\sqrt{1-z}$

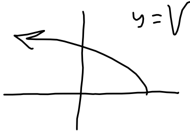
$=1-\sqrt{-(z-1)}$

$y=\sqrt{z}$

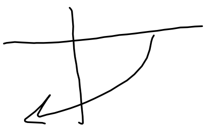
$y=\sqrt{-z}$



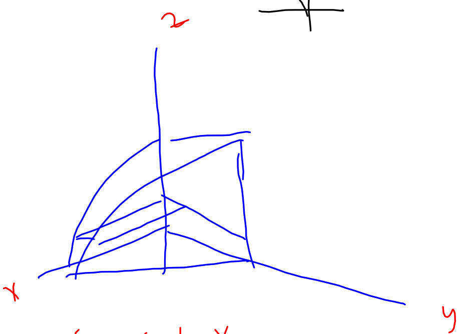
$y=\sqrt{-(z-1)}$



$y=-\sqrt{-(z-1)}$



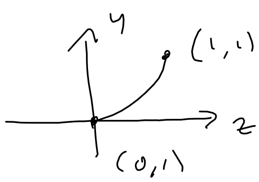
$y=1-\sqrt{1-z}$



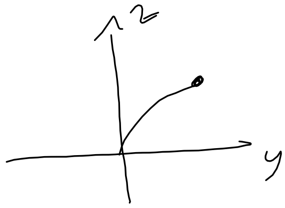
$0\le y\le 1-x$

$0\le z\le 1-x^2$

$0\le x\le 1$



$y=1-\sqrt{1-z}$



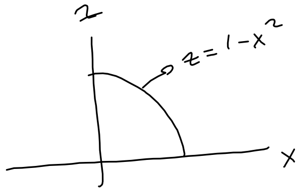
dx dy dz

$\int_0^1 \qquad dz$

Joseph

$dz\,dx\,dy$

T I over T II



$$\int_0^1 \int_0^{1-y} \int_0^{1-x^2} dz\,dx\,dy$$

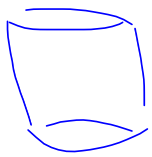
Solving $z = f(y)$ to get $y = g(z)$

Next up:

Cylindrical coordinates

$(x,y,z) \longmapsto (r\cos\theta, r\sin\theta, z)$

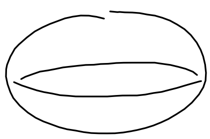
Good for solids that are right circular cylinders



Spherical Coords

(ρ, θ, ϕ)

"r" θ angle measured from positive z-axis.



$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$