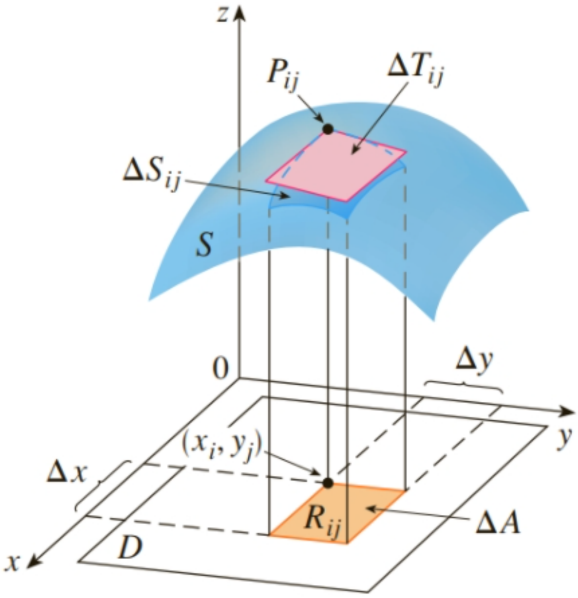
Section 15.5 Surface Area



$$A(S) = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \Delta T_{ij}$$

FIGURE 1

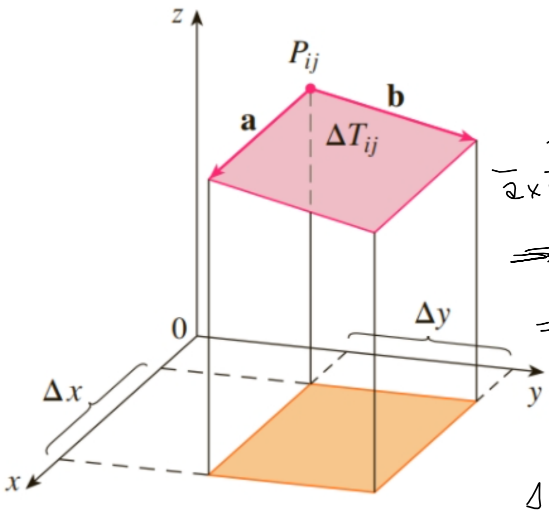


FIGURE 2

$$\vec{a} = \langle \Delta x, 0, f_x(x_i, y_j)\Delta x \rangle, \Delta x, 0$$

$$\times \quad \vec{b} = \langle 0, \Delta y, f_y(x_i, y_j)\Delta y \rangle, 0, \Delta y$$

$$\vec{a}\times\vec{b} = \langle -\Delta y \Delta x\, f_x(x_i, y_j), -\Delta x \Delta y\, f_y(x_i, y_j), \Delta x \Delta y \rangle$$

$$\Rightarrow \|\vec{a}\times\vec{b}\| = \Delta T_{ij}$$

$$= \sqrt{\Delta y^2 \Delta x^2 f_x(x_i, y_j)^2 + \Delta x^2 \Delta y^2 f_y(x_i, y_j) + \Delta x^2 \Delta y^2}$$

$$= \sqrt{f_x(x_i, y_j)^2 + f_y(x_i, y_j)^2 + 1}\ \Delta x \Delta y$$

$$\xrightarrow{\Delta x, \Delta y \to 0} \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1}\ dy\,dx$$

= increment of area on the surface.

2 The area of the surface with equation $z = f(x, y)$, $(x, y) \in D$, where f_x and f_y are continuous, is

$$A(S) = \iint_D \sqrt{[f_x(x, y)]^2 + [f_y(x, y)]^2 + 1}\, dA$$

3 $$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

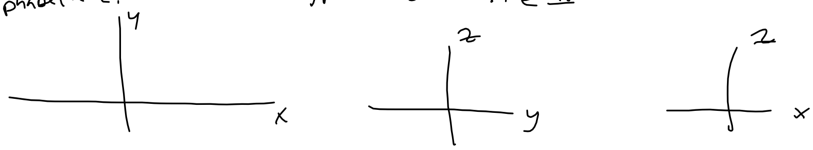
Liebniz

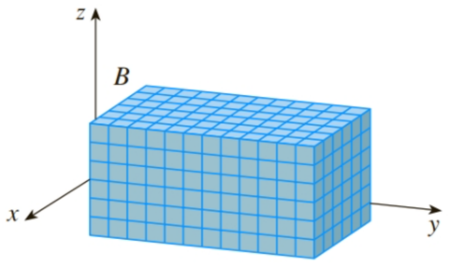
§15.6 Triple Integrals

RECALL: TYPE I, II

Now it'll be Type 1, Type 2, Type 3 over

Alphabetical! . Type I or TYPE II





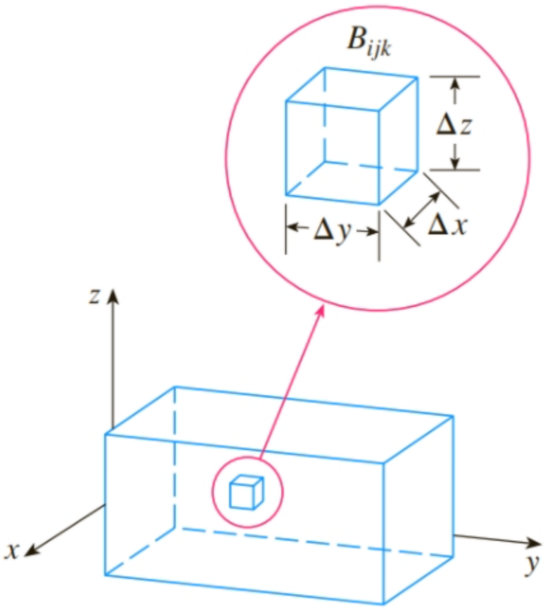


FIGURE 1

3 Definition The **triple integral** of f over the box B is

$$\iiint_B f(x, y, z)\, dV = \lim_{l, m, n \to \infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)\, \Delta V$$

if this limit exists.

$$\iiint_B f(x, y, z)\, dV = \lim_{l, m, n \to \infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_i, y_j, z_k)\, \Delta V$$

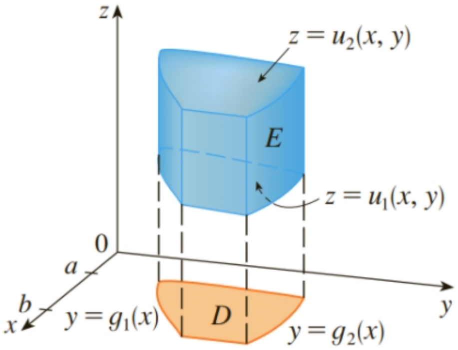


FIGURE 3
A type 1 solid region where the projection D is a type I plane region

$$\int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz\,dy\,dx$$

~~T I over T 1~~ T 1 over T I

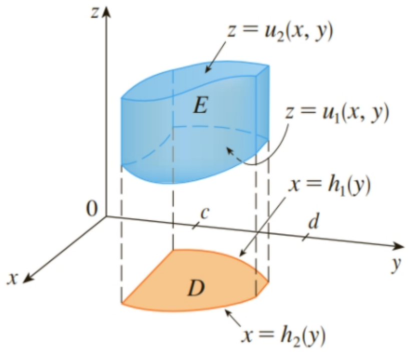


FIGURE 4
A type 1 solid region with a type II projection

$$\int_c^d \int_{h_1(y)}^{h_2(y)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz\,dx\,dy$$

Type 1 over Type II

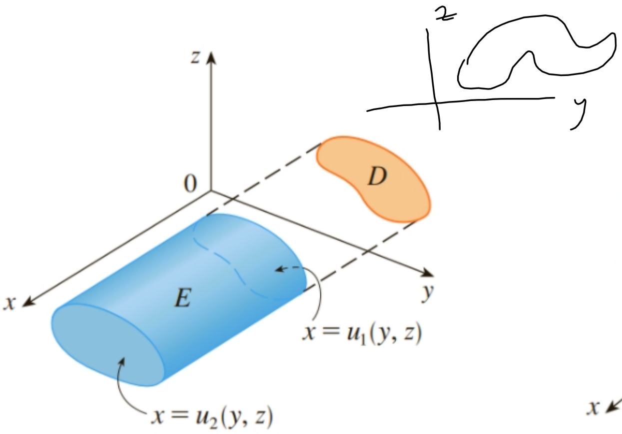


FIGURE 7
A type 2 region

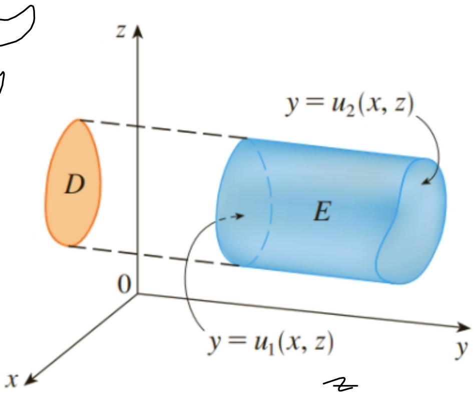
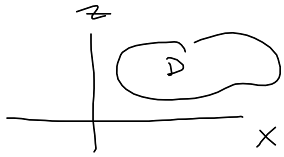


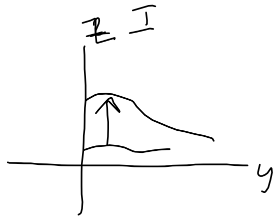
FIGURE 8
A type 3 region



$$\iint_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x,y,z)\,dx\right] dA$$

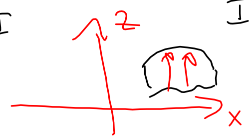
$$\iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x,y,z)\,dy\right] dA$$

I



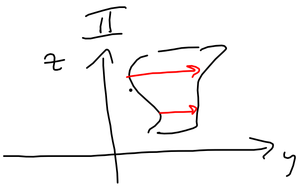
I : $dA = dz\,dy$

I



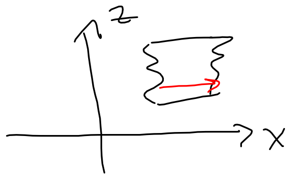
I ; $dA = dz\,dx$

II

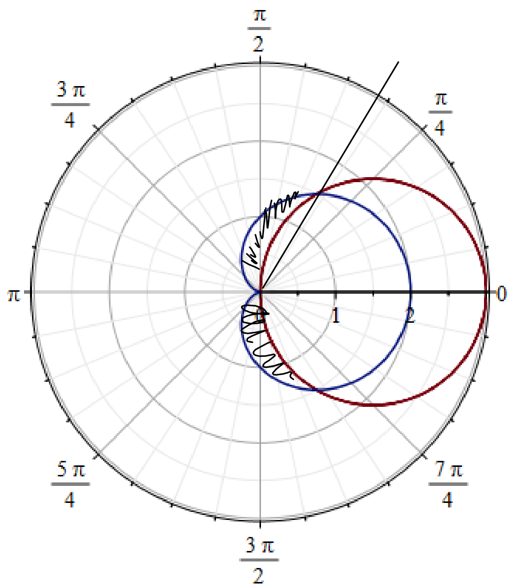


II : $dy\,dz$

II



II : $dx\,dz$



C 10:

$$\int_{\theta_1}^{\theta_2} \frac{1}{2} f(\theta)^2 \, d\theta$$

C 15

$$\int_{r_1}^{r_2} \int_{\theta_1}^{\theta_2} f(r,\theta) \, r \, dr \, d\theta$$

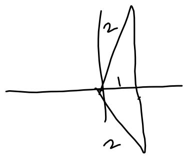
$$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\text{OUTER}^2 - \text{INNER}^2\right) d\theta$$

$$+ \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} (1+\cos\theta)^2 \, d\theta$$

$3\cos\theta = 1+\cos\theta$

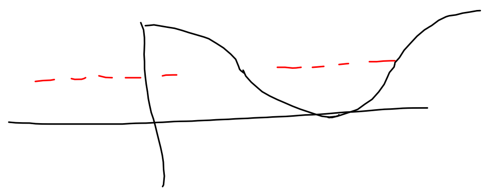
$2\cos\theta = 1$

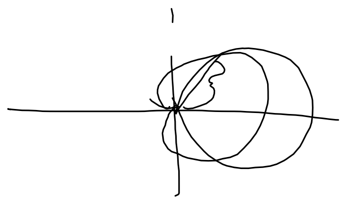
$\cos\theta = \frac{1}{2}$



$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$1+\cos\theta$





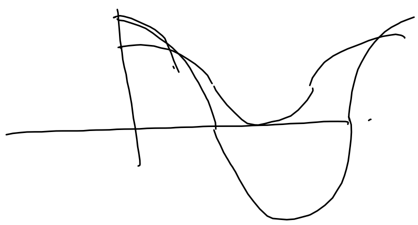
$\theta \in \left[\frac{\pi}{3}, \pi\right]$

$1+\cos\frac{\pi}{3} = \frac{3}{2}$

$$\int_{\frac{\pi}{3}}^{\pi} \int_0^{\frac{3}{2}} r \, dr \, d\theta$$

$$- \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^{3\cos\theta} r \, dr \, d\theta$$

Polar rectangles



Calc 15 way

$f(x,y) = 1 = f(r,\theta)$

$\int 1 \cdot r\,dr\,d\theta$

Due Dates: Thru 15.5 by Friday

15.1, 15.2, $\underbrace{15.3, 15.4}_{\text{TODAY}}$, $\underbrace{15.5, 15.6}_{\text{Friday}}$

monday