7–18 Evaluate the double integral.

8. $\displaystyle\iint_D \frac{y}{x^5+1}\,dA$, $D = \{(x, y) \mid 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant x^2\}$

15. $\iint_D y^3 \, dA$,

D is the triangular region with vertices $(0, 2)$, $(1, 1)$, $(3, 2)$

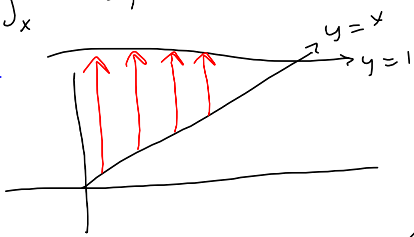
dx

Questions? §15.3?

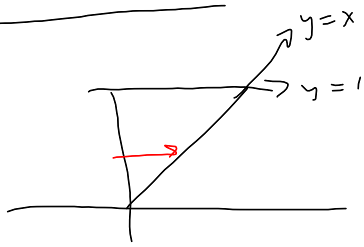
$$\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$$

TYPE 1

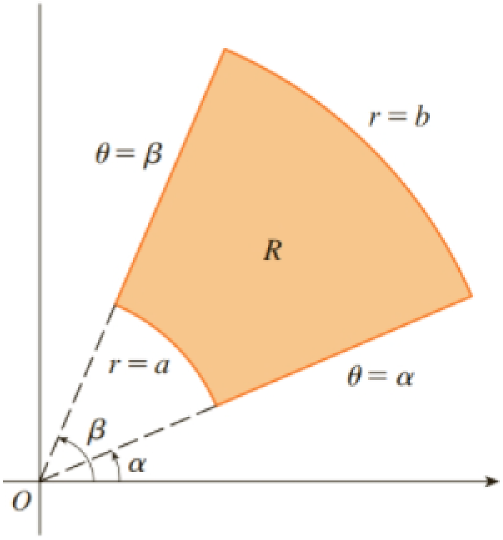
TYPE I



TYPE II



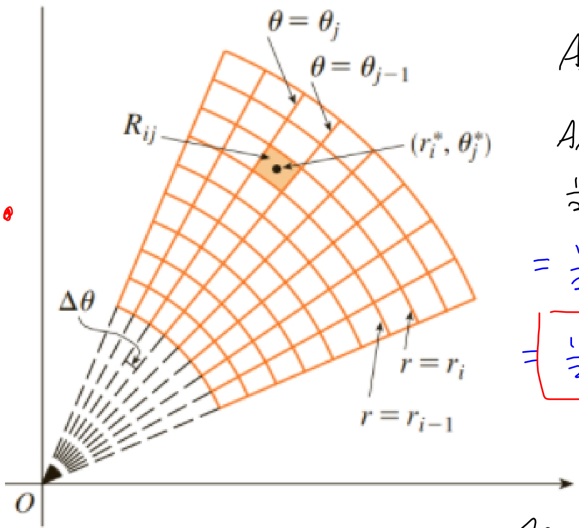
$$\int_0^1 \int_0^y e^{x/y}\, dx = \frac{e}{2} - \frac{1}{2}$$



$\beta-\alpha=\Delta\theta$

$b-a=\Delta r$

Area of sector $A=\frac{1}{2}r^2\theta$



Area of R_{ij} =

Area of bigger sector − Area of smaller.

$\frac{1}{2}r_i^2\Delta\theta - \frac{1}{2}r_{i-1}^2\Delta\theta$

$= \frac{1}{2}(r_i^2 - r_{i-1}^2)\Delta\theta$

$= \boxed{\frac{1}{2}(r_i + r_{i-1})}(r_i - r_{i-1})\Delta\theta$

$= r_i^* \Delta r \Delta\theta = \Delta A$

= increment of area in polar coordinates.

Area =

$$\text{Area} \approx \sum_{i=1}^{n}\sum_{k=1}^{m} r_k^* \Delta r \Delta\theta \xrightarrow{m,n\to\infty} \iint r\,dr\,d\theta$$

Integrating $f(x,y)$ over such a region, convert $x=r\cos\theta$, $y=r\sin\theta$, which gives

$$\iint f(x,y)\,dA = \iint f(r\cos, r\sin\theta)\underbrace{r\,dr\,d\theta}_{dA \text{ in polar coords.}}$$

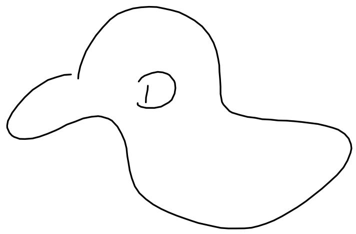
This is our 15.3; whereas it's 15.4 in previous editions, from which I lifted old work.

Applications (Our 15.4. Old 15.5)

Moment of Inertia

Center of Mass,

Let $\rho(x,y)$ be a density function for a lamina



Moment about y-axis

$$M_y = \iint_D x\,\rho(x,y)\,dA$$

Moment about x-axis

$$M_x = \iint_D y\,\rho(x,y)\,dA$$

$\bar{x}$ = x-coordinate of center of mass. $= \dfrac{M_y}{m}$

$\bar{y}$ = y- " " " " " , $= \dfrac{M_x}{m}$,

where $m = \displaystyle\iint_D \rho(x,y)\,dA$

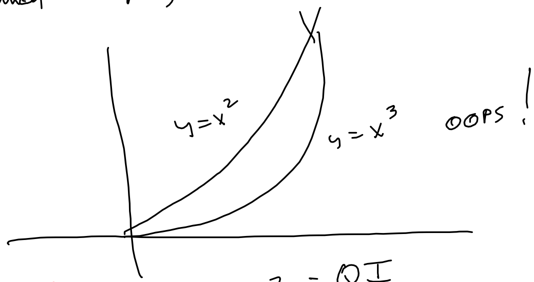
Total Charge $Q = \displaystyle\iint_D \rho(x,y)\,dA$, where ρ = charge density in coulombs/unit area

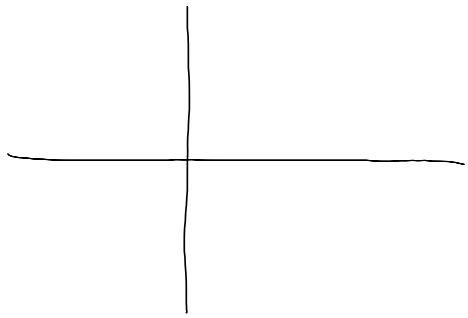
$\displaystyle\iint_D \rho(x,y)\,dA = 1$, when ρ is a probability density function.

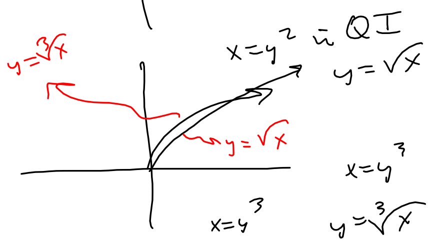
19–28 Find the volume of the given solid.

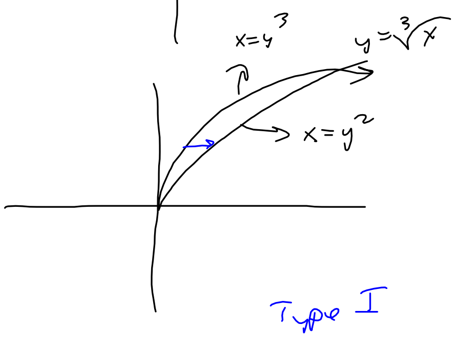
19. Under the plane $x + 2y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$

(20) Under $2x+y^2$, $x=y^2$, $x=y^3$







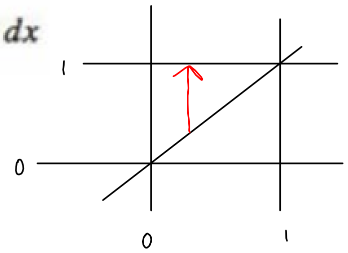


T II : $\displaystyle\int_0^1 \int_{x=y^3}^{x=y^2} (2x+y^2)\,dx\,dy = \frac{19}{210}$

Type I $\displaystyle\int_0^1 \int_{\sqrt{x}}^{\sqrt[3]{x}} (2x+y^2)\,dy\,dx = \frac{19}{210}$

45–50 Evaluate the integral by reversing the order of integration.

48. $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$



$= \int_0^1 \int_0^y e^{x/y}\, dx\, dy$

$u = \frac{x}{y} \quad du = \frac{1}{y}\, dx$

$\int e^u\, dx = \frac{1}{du/dx} \int e^u \frac{du}{dx}\, dx$

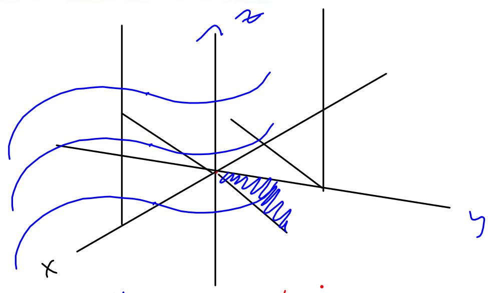
$= \int_0^1 y \int_0^y e^{\frac{x}{y}} \cdot \frac{1}{y}\, dx\, dy$

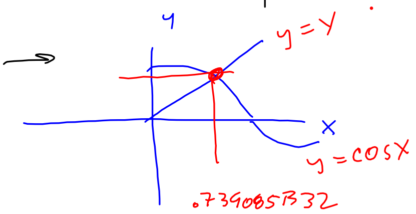
$= \int_0^1 y \left[e^{x/y} \right]_0^y dy$

$= \int_0^1 y \left[e^1 - e^0 \right] dy = (e^1 - 1) \int_0^1 y\, dy = (e-1) \frac{1}{2} y^2 \Big]_0^1 = \frac{1}{2}(e-1)$

= same as Maple.

30. Find the approximate volume of the solid in the first octant that is bounded by the planes $y = x$, $z = 0$, and $z = x$ and the cylinder $y = \cos x$. (Use a graphing device to estimate the points of intersection.)





.7390851332

TYPE I:

$$\int_0^{.7390851332}\int_x^{\cos(x)} x\,dy\,dx$$

TYPE II:

$$\cancel{\int_0^1\int_y^{.7390851332} x\,dx\,dy}$$

$$\cancel{\int_{.7390851332}^{\cos(x)}}$$

$$\int_0^{.739\ldots}\int_0^y x\,dx\,dy + \int_{.739\ldots}^1\int_0^{\arccos(y)} x\,dx\,dy$$

11 If $m \leq f(x, y) \leq M$ for all (x, y) in D, then

$$mA(D) \leq \iint_D f(x, y)\, dA \leq MA(D)$$

53–54 Use Property 11 to estimate the value of the integral.

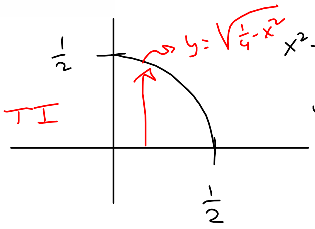
53. $\iint_Q e^{-(x^2+y^2)^2}\, dA$, Q is the quarter-circle with center the origin and radius $\frac{1}{2}$ in the first quadrant

Analysis :

"How big IS it?"

"How bad can it be?"

$$e^{-(x^2+y^2)^2} = \frac{1}{e^{(x^2+y^2)^2}}$$



$x^2 + y^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$y^2 = \frac{1}{4} - x^2$

$y = \pm\sqrt{\frac{1}{4} - x^2}$

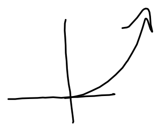
$Q\,\mathrm{I} \Rightarrow$

$y = \sqrt{\frac{1}{4} - x^2}$

TI :

$$\int_0^{\frac{1}{2}} \int_0^{\sqrt{\frac{1}{4}-x^2}} e^{-(x^2+y^2)^2}\, dy\, dx$$

$\frac{1}{e^{(x^2+y^2)^2}}$ is smallest when $e^{(x^2+y^2)^2}$ is biggest



e^{m} is an increasing function of m

make m smallest to make e^{m} smallest

.. m biggest to make e^{m} biggest.

$$m = \frac{1}{e^{\text{biggest}}} = \frac{1}{e^{\left(\left(\frac{1}{2}\right)^2\right)^2}} = \frac{1}{e^{1/16}}$$

$$M = \frac{1}{e^{\text{smallest}}} = \frac{1}{e^0} = 1$$

$r = \sqrt{x^2+y^2} = \frac{1}{2}$

$r^2 = x^2 + y^2 = \frac{1}{4}$

$(r^2)^2 = (x^2+y^2)^2 = \frac{1}{16} =$ max of $(x^2+y^2)^2$

$$\frac{1}{16} A \leq \int_0^{\frac{1}{2}} \int_0^{\sqrt{\frac{1}{4}-x^2}} e^{-(x^2+y^2)^2}\, dy\, dx \leq 1 A$$

$$A = \frac{1}{4}\left[\pi \left(\frac{1}{2}\right)^2\right] = \frac{1}{4} \cdot \frac{1}{4}\pi = \frac{1}{16}\pi$$

$$\left(\frac{1}{16}\right)\left(\frac{1}{16}\pi\right) \leq \iint_Q f\, dA \leq 1\left(\frac{1}{16}\pi\right)$$

$$\boxed{\frac{\pi}{256} \leq \iint_Q f\, dA \leq \frac{\pi}{16}}$$, where $f = f(x,y) = e^{-(x^2+y^2)^2}$