S15.1, 15.2 questions?

TYPE I, TYPE II

General Idea of Volume using double integrals.

Test 2 questions?

$\sqrt{x+1} + \sqrt{25-x^2} + \sqrt{y^2-1}$

$x+1 \geq 0$

$x \geq -1$

$x \in [-1, \infty)$

$25 - x^2 \geq 0$

$25 \geq x^2$

$\sqrt{25} \geq \sqrt{x^2} = |x|$

$|x| \leq 5$

$x \leq 5$ and $x \geq -5$

$-5 \leq x \leq 5$

$x \in [-5, 5]$

AND

$-5 \quad -1 \quad 5$

N N N Y Y Y N

$x \in [-1, 5]$

$y^2 - 1 \geq 0$

$y^2 \geq 1$

$\vdots$

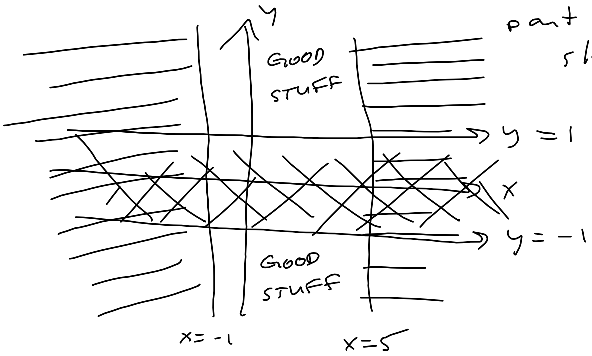
$|y| \geq 1$

$y \geq 1$ OR $y \leq -1$

$-1 \quad 1$

$y \in (-\infty, -1] \cup [1, \infty)$

Scratch out the bad stuff. Good STUFF is clean part of sketch



Include the Boundaries.

4. Suppose $f(x) = (x-3)^2(x+5)(x-7)(x+1) = x^5 - 7x^4 - 22x^3 + 178x^2 - 123x - 315$. I'm showing you both factored and expanded form to help you answer the following:

a. Solve the inequality $f(x) \leq 0$. Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.

$x = 3, -5, 7, -1$

x^5 ...

$-5, -1, 3, 7$

≤ 0

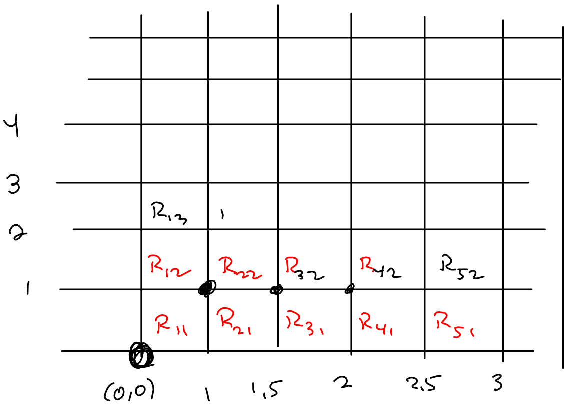
$- \quad + \quad - \quad - \quad +$

$-5 \quad -1 \quad 3 \quad 7$

$=0 \quad =0 \quad =0 \quad =0$

Y Y N Y Y Y Y Y N

$x \in (-\infty, -5] \cup [-1, 7]$



$$\Delta A \sum_{j=1}^{4} \sum_{i=1}^{5} f(x_{ij}, y_{ij}) = f(x$$

$$\lim_{m,n \to \infty} \sum_{j=1}^{n} \sum_{i=1}^{m} f(x_{ij}, y_{ij}) \Delta A = \iint_D f(x,y)\, dA$$

This is true only because we're choosing equal widths & heights on our Rectangles R_{ij}, so that $\Delta x \Delta y = \Delta A$ is constant!

That forces the mesh of the partition to approach zero, which is the point.

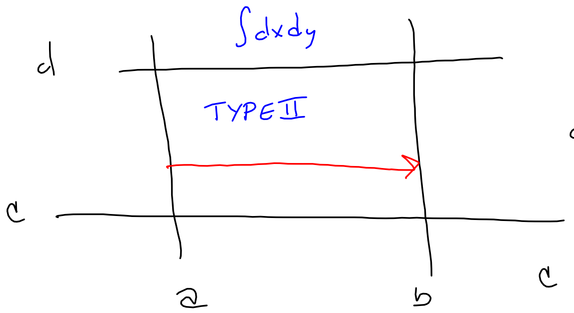
More generally it's

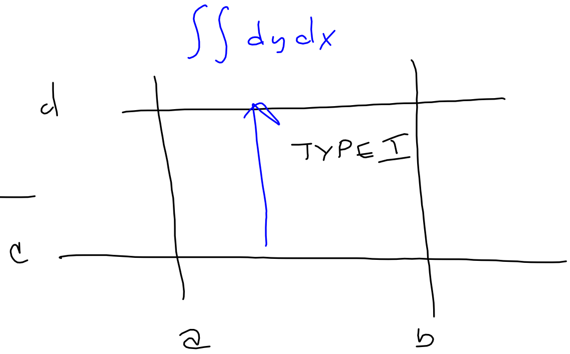
$$\lim_{\substack{|\Delta x_{ij}| \to 0 \\ |\Delta y_{ij}| \to 0 \\ \text{mesh} \to 0}} \sum \sum f(x_{ij}, y_{ij}) \underbrace{\Delta x_{ij} \Delta y_{ij}}_{\Delta A_{ij}}$$

FUBINI !

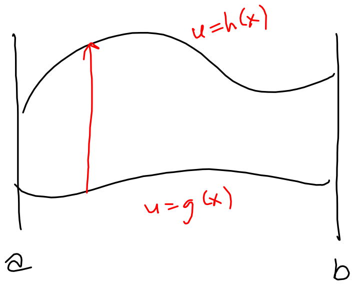
$$\int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx \quad \text{if}$$

you're integrating over a rectangle, with $x \in [a,b]$, $y \in [c,d]$



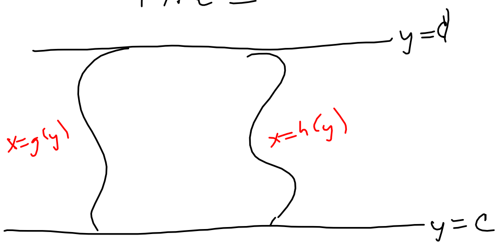


Type I: Region D



$$\int_a^b \int_{g(x)}^{g(x)} f(x,y)\,dy\,dx$$

TYPE II

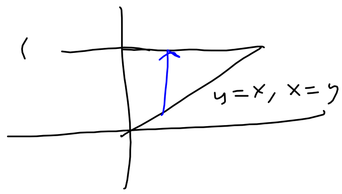


$$\int_c^d \int_{g(y)}^{h(y)} f(x,y)\,dx\,dy$$

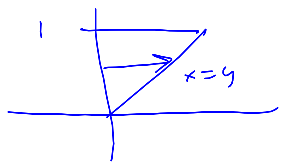
Book did

$$\iint_D \sin(y^2)\,dA = \int_0^1 \int_x^1 \sin(y^2)\,dy\,dx \quad \text{ouch!}$$

TI $\displaystyle\int_0^1 \int_x^1 \quad dy\,dx$ ugh!



TII $\displaystyle\int_0^1 \int_0^y \sin(y^2)\,dx\,dy$



$$= \int_0^1 \Big[x\sin(y^2)\Big]_0^y\,dy$$

$$= \int_0^1 \Big[y\sin(y^2)\Big]\,dy = \frac{1}{2}\int_0^1 (\sin(y^2))(2y\,dy)$$

$u = y^2$

$du = 2y\,dy$

$u = y^2 = 1$

$u = \pm\sqrt{y^2} = \pm 1$

$$= \frac{1}{2}\int_{y=0}^{y=1} \sin(u)\,du = -\frac{1}{2}\cos(u)\Big]_{0=y}^{1=y}$$

$$= -\frac{1}{2}\cos(y^2)\Big]_0^1 = -\frac{1}{2}(\cos(1) - \cos(0))$$

$$= -\frac{1}{2}\left[\cos(1) - 1\right]$$

Har-Dee-Har-Dee-Theta!

$\boxed{r\,dr\,d\theta}$ → Iterated Integrals in Polar coordinates.

§15.3

"Polar Rectangles"

TAKE-HOME DUE MONDAY.

§15.1, 15.2 " "

§15.3 " WEDNESDAY