Cauchy-Schwarz $\qquad \sum x_i y_i \le \sqrt{\sum x_i^2}\,\sqrt{\sum y_i^2}$

$$\left|\int f(x)g(x)\,dx\right| \le \int |f(x)g(x)|\,dx$$

$$\left(\ |x+y| \le |x| + |y|\ \right)$$

$$\int f(x)g(x)\,dx \le \sqrt{\int f(x)^2\,dx}\,\sqrt{\int g(x)^2\,dx}$$

More general inequality Hölder's inequality:

$$\int f(x)g(x)\,dx \le \sqrt[p]{\int f(x)^p\,dx}\,\sqrt[q]{\int g(x)^q\,dx}$$

where $\quad \frac{1}{p} + \frac{1}{q} = 1$

$\frac{3}{4} + \frac{1}{4}$

$p = \frac{4}{3},\ q = 4 \qquad \frac{1}{\frac{4}{3}} + \frac{1}{4} = 1$ ✓

3. Find $\frac{\partial z}{\partial x}$ for the equation $y\sin\left(xy^3\right)+x^2yz^2=2xyz$ in 2 ways:

   a. (5 pts) Use implicit differentiation, holding y constant and treating z as an implicit function of x.

   b. (5 pts) Form a function $F(x,y,z)$ and find $\frac{\partial z}{\partial x}$ for the level surface $F(x,y,z)=0$.

$$z_x = \frac{\partial z}{\partial x}$$

$$\left(y\cos(xy^3)\right)(3xy^2) + 2xyz^2 + 2x^2yz\,z_x = 2yz + 2xy\,z_x$$

$$2xy\,z_x - 2x^2yz\,z_x = 3xy^3\cos(xy^3) + 2xyz^2 - 2yz$$

$$z_x\left[\sim\right] = \sim$$

$$z_x = \frac{\sim}{\left[\sim\right]} = \frac{3xy^3\cos(xy^3) + 2xyz^2 - 2yz}{2xy - 2x^2yz}$$

(b) $y\sin(xy^3) + x^2yz^2 = 2xyz \Rightarrow$

$$\boxed{F(x,y,z) \equiv y\sin(xy^3) + x^2yz^2 - 2xyz} = 0$$

$$\boxed{F_x = y^4\cos(xy^3) + 2xyz^2 + 2xyz}$$

Formula $\boxed{7}$ -vs- Formula $\boxed{6}$

$$\boxed{F_z = 2x^2yz - 2xy}$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}} = -\frac{\cancel{y}\left(y^3\cos(xy^3) + 2xz^2 + 2xz\right)}{\left(2x^2z - 2x\right)\cancel{y}}$$