Section 14.8 Lagrange Multipliers

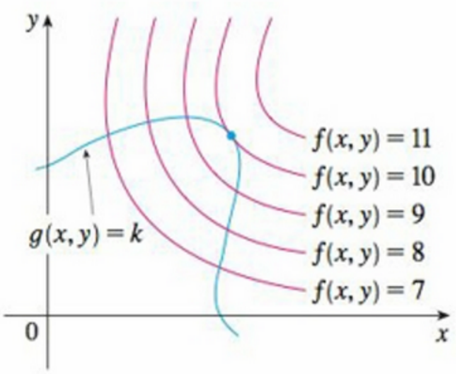


FIGURE 1

Trying to maximize or minimize f, subject to some sort of constraint g.

The big observation to make is that the tangent lines are the same for the level curves and the curve corresponding to

$g = k$.

13. $f(x, y, z, t) = x + y + z + t$; $x^2 + y^2 + z^2 + t^2 = 1$

$f_x = 1 \overset{\text{SET}}{=} \lambda g_x = 2\lambda x$

$f_y = 1 = \lambda g_y = 2\lambda y$

$f_z = 1 = \lambda g_z = 2\lambda z$

$f_t = 1 = \lambda g_t = 2\lambda t$

$\underbrace{x^2+y^2+z^2+t^2=1}$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2} = y = z = t$

$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, $(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$

$2\lambda x = 1 \Rightarrow x = \frac{1}{2\lambda}$

$y = \frac{1}{2\lambda}$

$z = \frac{1}{2\lambda}$

$t = \frac{1}{2\lambda}$

$x = y = z = t$

Want one of these in a take-home setting.

14.3 #53

$$w = \sqrt{u^2+v^2} = (u^2+v^2)^{\frac{1}{2}}$$

$$\Longrightarrow w_u = \tfrac{1}{2}(u^2+v^2)^{-\frac{1}{2}}(2u) = \frac{u}{(u^2+v^2)^{\frac{1}{2}}} = u(u^2+v^2)^{-\frac{1}{2}}$$

$$\Longrightarrow w_{uv} = -\tfrac{1}{4}(u^2+v^2)^{-\frac{3}{2}}(2v)(2u) = \frac{-uv}{(u^2+v^2)^{3/2}} = \frac{-uv}{\sqrt{(u^2+v^2)^3}}$$

$$= \frac{-uv}{(u^2+v^2)\sqrt{u^2+v^2}}$$

$$w_v = \tfrac{1}{2}(u^2+v^2)^{-\frac{1}{2}}(2v)$$

$$w_{vu} = -\tfrac{1}{4}(u^2+v^2)^{-\frac{3}{2}}(2u)(2v) = -\frac{uv}{(u^2+v^2)^{3/2}}$$

$$w_u = \frac{u}{(u^2+v^2)^{\frac{1}{2}}} = u(u^2+v^2)^{-\frac{1}{2}}$$

NOT ASKED

$$w_{uu} = \underline{(u^2+v^2)^{-\frac{1}{2}}} + u\left(-\tfrac{1}{2}(u^2+v^2)^{-\frac{3}{2}}\right)(2u)$$

$$= \frac{1}{(u^2+v^2)^{\frac{1}{2}}} \cdot \frac{(u^2+v^2)}{(u^2+v^2)} - \frac{u^2}{(u^2+v^2)^{3/2}} = \frac{u^2+v^2-u^2}{(u^2+v^2)^{3/2}} = \frac{v^2}{(u^2+v^2)^{3/2}}$$

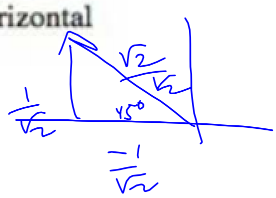
$14.6

34. Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where x, y, and z are measured in meters, and you are standing at a point with coordinates (60, 40, 966). The positive x-axis points east and the positive y-axis points north.

(a) If you walk due south, will you start to ascend or descend? At what rate? → negative y-direction $\qquad \langle 0,-1\rangle = \bar{u}$

(b) If you walk northwest, will you start to ascend or descend? At what rate? → $\langle -1,1\rangle \rightarrow \frac{1}{\sqrt{2}}\langle -1,1\rangle = \frac{\sqrt{2}}{2}\langle -1,1\rangle = \bar{v}$

(c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?



~~(a)~~ $\nabla f = \langle -.01x, -.02y\rangle$

(c) $\nabla f(60,40) = \langle -.01(60), -.02(40)\rangle$

$= \langle -.6, -.8\rangle$ Descending (a)

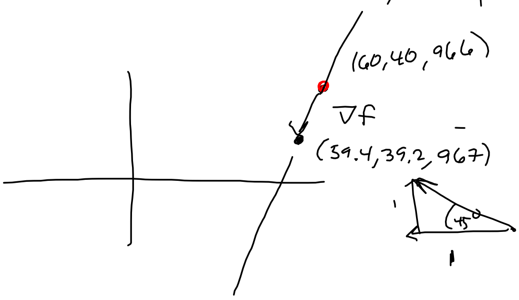
$\|\nabla f(60,40)\| = \sqrt{.6^2 + .8^2} = \sqrt{.36 + .64} = \sqrt{1} = 1$ $\frac{\text{ft up}}{\text{ft in direc. of } \nabla f}$

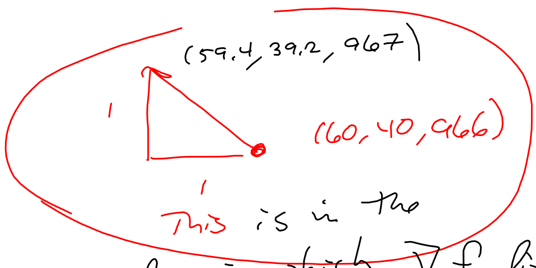
This is part c!

I'm going in the direction of the gradient!

So the angle of inclination is

$\arctan(1) = \frac{\pi}{4}$





This is in the plane in which ∇f lies.

(a) Due South: $\bar{u} = \langle 0,1\rangle$

$\nabla f(60,40) \cdot \bar{u} = \langle -.6, -.8\rangle \cdot \langle 0,1\rangle = -.8$

Descending at a rate of $\frac{.8\text{ ft}}{\text{ft}}$

(b) Northwest $\quad \bar{v} = \frac{1}{\sqrt{2}}\langle -1, 1\rangle$

$\nabla f(60,40) \cdot \bar{v} = \langle -.6, -.8\rangle \cdot \frac{1}{\sqrt{2}}\langle -1, 1\rangle$

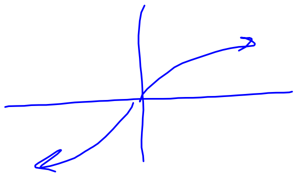
$= \frac{1}{\sqrt{2}}(.6 - .8) = \frac{1}{\sqrt{2}}(-.2) = \frac{-.2}{\sqrt{2}}$

Descending at $\frac{.2}{\sqrt{2}}$ $\frac{\text{ft vert}}{\text{ft horiz}}$

Section 14.8 #6

$f(x,y) = e^{xy}$, s.t. $x^3+y^3=16$

plot3d ([exp (x*y), $x^3+y^3=16$])



No.

$y^3 = 16-x^3$

$y = \sqrt[3]{16-x^3}$

fplot := plot3d ([exp(x*y), $\sqrt[3]{16-x^3}$], x = ..., y = ...)

Alternative:

plot3d (f(x,y), ...)

cplot := implicitplot3d ($x^3+y^3=16$, x=..., y=...)

display([fplot, cplot], ...)

5–18 Find the local maximum and minimum values and saddle point(s) of the function. If you have three-dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function.

5. $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$

6. $f(x, y) = x^3y + 12x^2 - 8y$

$3x^2y + 24x = 0$

$x^2y + 8x = 0$

$x(xy + 8) = 0$

$x = 0, \quad xy = -8$

$y = -\frac{8}{x}$

$solve(fx(x, y) = 0)$

$\{x = 0, y = y\}, \left\{x = -\frac{8}{y}, y = y\right\}$

$solve(fy(x, y) = 0)$

$2, -1 - I\sqrt{3}, -1 + I\sqrt{3}$

$\boxed{f_y = x^3 - 8 \stackrel{SET}{=} 0}$

$x = 2$ OR $x \notin \mathbb{R}$

$x = 2$ & $y = -\frac{8}{x} = -4$ when $x = 2$

$(2, -4)$

$$x^3 \cdot y + 12 \cdot x^2 - 8y$$

$(x^3 - 8)\, y + 12x^2 \stackrel{SET}{=} K$

$f_{xx} f_{yy} - (f_{xy})^2$ says saddle point there

$f_x = 3x^2y + 24x^2$

$f_{xx} = 6xy + 48x$

$f_{xy} = 3x^2 = f_{yx}$

$f_y = x^3 - 8$

$f_{yy} = 0$

$f_{yx} = 3x^2$ ✓

$D(2, -4) = (6(2)(-4) + 48(2))(0) - (3x^2)^2 < 0$

Saddle!