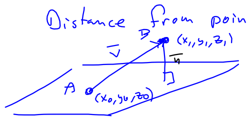
40. Find the point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$.

$x - y + z - 4 = 0$

Distance from point to plane



$\vec{n} = \langle a,b,c\rangle$

$ax + by + cz + d = 0$

$ax + by + cz = d$

$\vec{v} = \overrightarrow{AB} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0\rangle$

$$\left|\text{comp}_{\vec{n}}\vec{v}\right| = \frac{|\vec{v}\cdot\vec{n}|}{\|\vec{n}\|} = \frac{|a(x_1-x_0)+b(y_1-y_0)+c(z-z_0)|}{\|\vec{n}\|}$$

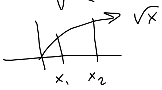
$$= \frac{|ax_1+by_1+cz_1-(ax_0+by_0+cz_0)|}{\|\vec{n}\|} = \frac{|ax_1+by_1+cz_1\ \ d|}{\|\vec{n}\|}$$

$$= \frac{|1(1)-1(2)+1(3)\ -4|}{\sqrt{1^2+1^2+1^2}} = \frac{|1-2+3\ \ -4|}{\sqrt{3}} = \boxed{\frac{|-2|}{\sqrt{3}}} = \frac{2}{\sqrt{3}}$$

That's chapter 12 way

§14.7 way   Minimize $d = \sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$

Find (x,y,z) Fact



is increasing

So we max/min $\sqrt{\text{~~}}$ by max/min ~~

Let $d^2 = f(x,y,z) = (x-1)^2+(y-2)^2+(z-3)^2$

$(x,y,z) \in \mathcal{P} \Rightarrow x - y + z = 4$

$\underline{z = 4 - x + y}$

Keenan

So minimize $f(x,y) = (x-1)^2 + (y-2)^2 + (4 - x + y - 3)^2$

Scratch: $\boxed{(y-x+1)^2} = ((y-x)+1)^2$ ⇒ why expand, fool?

$= (y-x)^2 + 2(y-x) + 1^2 = y^2 - 2xy + x^2 + 2y - 2x + 1$

$= (x-1)^2 + (y-2)^2 + (y-x+1)^2$

$f_x = 2(x-1) + 2(y-x+1)(-1)$

$= 2x - 2 - 2y + 2x - 2$

$= 4x - 2y - 4 \overset{\text{SET}}{=} 0$

$f_y = 2(y-2) + 2(y-x+1)$

$= 2y - 4 + 2y - 2x + 2$

$= 4y - 2x - 2 \overset{\text{SET}}{=} 0$

FACT: If $f(x,y)$ is max/min then $f_x(x_1,y_1) = f_y(x_1,y_1) = 0$

we get a system of equations

$4x - 2y = 4$
$-2x + 4y = 2$
$4x - 2y = 4$

$\longrightarrow$ Keenan

$x - 2y = -1$
$2x - y = 2$

$$\left[\begin{array}{cc|c} 1 & -2 & -1 \\ 2 & -1 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 3 & 4 \end{array}\right]$$

$3y = 4 \Rightarrow \boxed{y = \frac{4}{3}}$

$x - 2y = x - 2\left(\frac{4}{3}\right)$

$= x - \frac{8}{3} = -1 = -\frac{3}{3}$

$\boxed{x = \frac{5}{3}}$

$x - y + z = 4$

$\frac{5}{3} - \frac{4}{3} + z = 4$

$z = \frac{12-1}{3} = \boxed{\frac{11}{3} = z}$

~~$(a,b,c) = (1,1,4)$~~

~~$d = \sqrt{(1-1)^2+(1-2)^2+(4-3)^2}$~~

~~$= \sqrt{1^2+1^2} = \sqrt{2}$~~

We get

$(a,b,c) = \left(\frac{5}{3}, \frac{4}{3}, \frac{11}{3}\right)$

$$\sqrt{\left(\frac{5}{3}-1\right)^2 + \left(\frac{4}{3}-2\right)^2 + \left(\frac{11}{3}-3\right)^2}$$

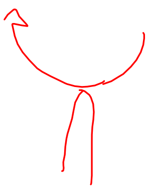
$$= \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{3\left(\frac{4}{9}\right)} = \frac{2}{3}\sqrt{3}$$

1^{st} Derivative Test:

$f(a,b)$ a max/min $\Longrightarrow f_x(a,b) = f_y(a,b) = 0$

"$\Longleftarrow$" is NOT generally true

We also have saddles



still, we do

$f_x = 0, f_y = 0$ to get

some candidate (a,b)'s

3 SECOND DERIVATIVES TEST Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ [that is, (a, b) is a critical point of f]. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

(a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

(b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

(c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

(d) If $D = 0$, we know nothing.

Let's apply this test to #40

We get

$(a, b, c) = \left(\frac{5}{3}, \frac{4}{3}, \frac{11}{3}\right)$

$f_x = 4x - 2y - 4$ $\quad f_y = 4y - 2x - 2$

$f_{xx} = 4$ $\quad$ Thx again, Keenan! $\quad f_{yy} = 4$

$f_{xy} = -2$ $\quad f_{yx} = -2$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - (f_{xy})^2 = (4)(4) - (-2)^2 = 16 - 4 = 12 > 0$$

$\Rightarrow$ Minimum.

Will, text me at
970-290-0550

5–18 Find the local maximum and minimum values and saddle point(s) of the function. If you have three-dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function.

5. $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$

6. $f(x, y) = x^3y + 12x^2 - 8y$

7. $f(x, y) = x^4 + y^4 - 4xy + 2$

8. $f(x, y) = e^{4y-x^2-y^2}$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - f_{xy}^2$$

5. $f(x,y) = 9-2x+4y-x^2-4y^2$

$f_x = -2-2x$ $\quad f_y = 4-8y$

$f_{xx} = -2$ $\quad f_{yy} = -8$

$f_{xy} = 0$ $\quad f_{yx} = 0$ ✓

$D = (-2)(-8) - (0)(0) = 16 > 0$

$f_{xx}(a,b) = -2 < 0 \ \forall \ (a,b)$.

So maximum when/if we find (a,b) candidates

$f_x = -2-2x \overset{SET}{=} 0 \Rightarrow$

$-2 = 2x$

$x = -1$

$f_y = 4-8y \overset{SET}{=} 0$

$\Rightarrow 8y = 4 \Rightarrow y = \frac{1}{2}$

So $x = -1, y = \frac{1}{2}$.

$\Rightarrow f(-1, \frac{1}{2}) = 9 - 2(-1) + 4(\frac{1}{2}) - (-1)^2 - 4(\frac{1}{2})^2 = 9+2+2-1-1$

$= \boxed{13-2 = 11 = Max}$

@ $(a,b) = (-1, \frac{1}{2})$