Check your D2L e-mail!!!!!

Check harryzaims.com for new postings: tests-u-took.

There was a serious typo that we corrected on the fly. I'll edit that test and upload what it was supposed to be!

Dylan's Test 1 #1

$P_1:\ x+2y-z=7$ Find line of intersection

$P_2:\ 2x+3y+z=11$ 1st Find direction vector for

$\forall t\in\mathbb{R}$ $\boxed{\mathcal{L}=\bar{r}_0+t\bar{v}}$ in vector form.

Observe $\bar{v}\perp\bar{n}_1=\langle 1,2,-1\rangle$ and $n_2=\langle 2,3,1\rangle$

$\bar{n}_1=\langle 1,2,-1\rangle,1,2$

$\bar{n}_2=\langle 2,3,1\rangle,2,3$

$\boxed{\langle 5,-3,-1\rangle=\bar{v}}$

Now, just need a point common to both.

Let $z=0$ (Assume the line is NOT parallel to the xy-plane!)

If this doesn't work, try something else.

$x+2y=7$

$2x+3y=11$

$\left[\begin{array}{cc|c}1&2&7\\2&3&11\end{array}\right]\xrightarrow{-2R1+R2}\left[\begin{array}{cc|c}1&2&7\\0&-1&-3\end{array}\right]$

$x+2y=7$

$-y=-3$

$\boxed{y=3}$

$x+2y=x+2(3)=x+6=7$

$\boxed{x=1}$

So, $(1,3,0)\in P_1\cap P_2$

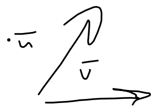
Let $\boxed{\bar{r}_0=\langle 1,3,0\rangle,}$

c. (5 pts) (Vector Equation of Plane) Form the vector $\overline{v} = \overrightarrow{AC}$, using A and C from part a. Then write a vector equation for the plane containing A, B and C. Sketch the plane using its intercepts.

$\bar{v} = \overrightarrow{AC} = \langle 1,1,1 \rangle$

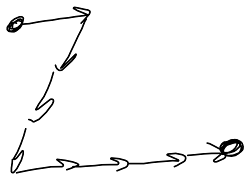
$\bar{u} = \overrightarrow{AB} = \langle -4,-4,0 \rangle$

$A(1,2,1) \longrightarrow \bar{r}_0 = \langle 1,2,1 \rangle$



$\mathcal{P}: \quad \bar{r}_0 + t\bar{u} + s\bar{v} \quad \forall\, (s,t) \in \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

10-15 pages of writing per day



5 moves ahead?

Nah! Think positionally,

I move ahead!

Questions on homework? Test 1?

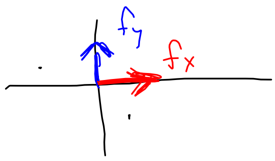
§ 14.4

f_x = "slope" in the x-direction

= change in f (i.e. z-coordinate) with respect to (wrt) a small change in x.

f_y = change in f wrt change in y.

Lookin' Down:



These two directions span a plane! Easy way to build eq'n of tan plane.

Section 14.4 Tangent Planes and Linear Approximations

Recall tangent line

$y - y_0 = m_{tan}(x - x_0)$

I say, DO THIS:

$y = m_{tan}(x - x_0) + y_0$

$y = f'(x_0)(x - x_0) + f(x_0)$

Tangent Plane @ $(x_0, y_0, f(x_0, y_0))$

$z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$= f_x(x - x_0) + f_y(y - y_0) + \underbrace{f(x_0, y_0)}_{z_0}$

Recall tangent lines in the plane:

See 14.4 Maple Handout (demonstrates
some techniques & commands.

Tangent Planes in 3-space also make for nice approximations:

This one has a cusp at the origin, its derivatives of all orders exist, but they aren't continuous at the origin.

Increment of *y*:

Increment of *z*:

If you want to play with these ideas (and formalisms), the #46 is the bomb.

If you *don't*, then the following is a very practical way to check for differentiability is given by:

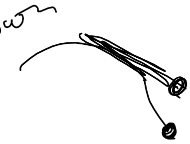
Differentials in the Plane:

The Differential of a surface in 3-space:

$\approx \Delta z$

Also called the "total differential."

Concave Down



Treat this similar to #20, below, but I include some other stuff (like the plane $x = 1$, the trace of this plane on the surface, and the tangent to the surface living in this plane.

$g(x,y) = 6 - x - x^2 - 2y^2 \Rightarrow g_x = -1 - 2x \Rightarrow g_x(1,2) = -1-2 = -3 = g_x(1,2)$

$L_{(1,2)}(x,y)$

$g_y = -4y \Rightarrow g_y(1,2) = -4(2) = -8$

g

$g(1,2) = 6 - 1 - 1^2 - 2(2)^2 = 6 - 2 - 8 = -4 = g(1,2) = z_0$

$z = g_x(1,2)(x-1) + g_y(1,2)(y-2) - 4 =$

$L_{(1,2)}(x,y) = 3(x-1) - 8(y-2) - 4$

Trace of the plane $x=1$

$g(1,y) = 6-1-1-2y^2 = 4-2y^2 = gtrace1$

Space curve: $gtrace1 = \langle 1, y, 4-2y^2 \rangle$

$= \langle 1, t, 4-2t^2 \rangle$ in Maple.

$g_y(1,2) = -8$

$x=1,\ y=y,\ z = -8(y-2)$

$x=1,\ y=t,\ z=-8(t-2) \rightsquigarrow \langle 1, t, -8(t-2) \rangle$

Tangent Plane Approx.

20. Find the linear approximation of the function $f(x, y) = \ln(x - 3y)$ at $(7, 2)$ and use it to approximate $f(6.9, 2.06)$. Illustrate by graphing f and the tangent plane.

$f(7,2)$

$f_x = \left(\frac{1}{x-3y}\right)(1) \qquad f_x(7,2) = \frac{1}{7-3(2)} = 1$

$f_y = \left(\frac{-3}{x-3y}\right) \qquad f_y(7,2) = \frac{-3}{7-6} = -3$

$(7, 2, f(7,2)) =$

$f(7,2) = \ln(7-6) = \ln(1) = 0 = z_0$

$P: \quad f_x(7,2)(x-7) + f_y(7,2)(y-2) + 0$

$z = 1(x-7) - 3(y-2)$

$f(6.9, 2.06) \approx (6.9-7) - 3(2.06-2)$

$-.1 - 3(.06) = -.1 - .18 = -.28$

Vector Approach is very similar

$\vec{r} = \vec{r}_0 + t\vec{u} + s\vec{v}$

$= \langle x_0, y_0, f(x_0,y_0) \rangle + t\langle 1, 0, f_x(x_0,y_0) \rangle + s\langle 0, 1, f_y(x_0,y_0) \rangle$

$z = f_x(x-x_0) + f_y(y-y_0) + z_0$

$yz + x\ln y = z^2$ Find f_x & f_y

(Assume $z = f(x,y)$, implicitly)

$f_x = z_x$

$yz' + \ln y + x \cdot \frac{1}{y} = 2zz'$ No! y is constant!

$yz_x - 2zz_x = -\ln y - \frac{x}{y}$

$z_x(y - 2z) = -\ln y - \frac{x}{y}$

$$z_x = \frac{-\ln y - \frac{x}{y}}{y - 2z} = \frac{\ln y + \frac{x}{y}}{2z - y}$$

f_y: $z + yz' + x \cdot \frac{1}{y} = 2zz'$

Keenan says:

$$\frac{\partial}{\partial y}\left[yz + x\ln y = z^2\right] \qquad \frac{d}{dx}\left[(f(x))^2\right] = 2f(x)^1 \cdot f'(x)$$

$$z + y\frac{\partial z}{\partial y} + x \cdot \frac{1}{y} = 2z\frac{\partial z}{\partial y}$$

$$y\frac{\partial z}{\partial y} - 2z\frac{\partial z}{\partial y} = -z - \frac{x}{y}$$

$$\frac{\partial z}{\partial y}(y - 2z) = -z - \frac{x}{y}$$

$$\frac{\partial z}{\partial y} = \frac{-z - \frac{x}{y}}{y - 2z} = \boxed{\frac{z + \frac{x}{y}}{2z - y} = \frac{\partial z}{\partial y}}$$

25–30 Find the differential of the function.

25. $z = x^3 \ln(y^2)$ #25 NA **26.** $v = y \cos xy$