Shorthand:

There is, there exists $\exists$

For all, for each, for every $\forall$

x is in the set S, *x* is an element of S $x \in S$

A implies B $A \Rightarrow B$

A implies B and B implies A, A holds if and only if B holds, A is necessary and sufficient to B $A \Leftrightarrow B$ A iff B

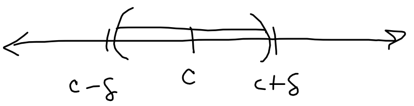
Such that, so that $\ni$

Recall the definition of limit from Calculus I:

Do ~~§4.4~~ §14.1 out of "assignments" directory.

$\lim_{x \to c} f(x) = L$ means

Given $\varepsilon > 0$, $\exists\ \delta > 0 \ni 0 < |x-c| < \delta$
$\Rightarrow |f(x) - L| < \varepsilon$



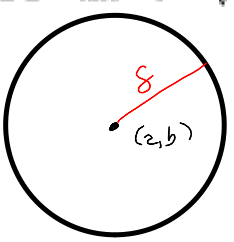
δ-ball in 1-D

1 Definition Let f be a function of two variables whose domain D includes points arbitrarily close to (a, b). Then we say that the **limit of $f(x, y)$ as (x, y) approaches (a, b)** is L and we write

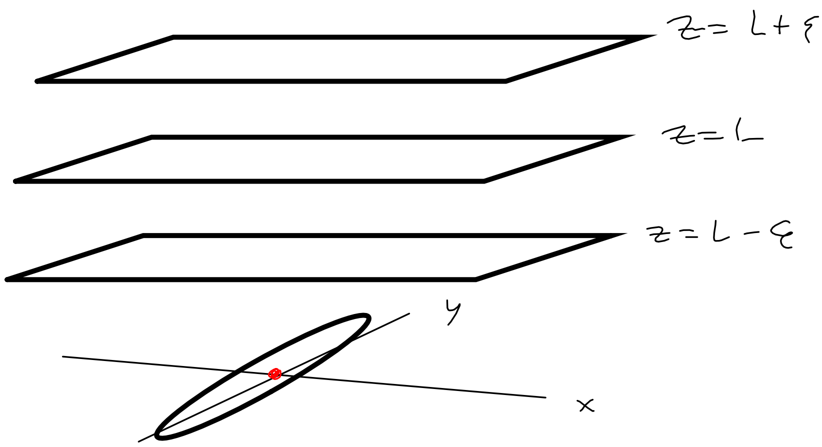
$$\lim_{(x, y) \to (a, b)} f(x, y) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that

$$\text{if} \quad (x, y) \in D \quad \text{and} \quad 0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \quad \text{then} \quad |f(x, y) - L| < \varepsilon$$



2-ball of radius δ, centered @ (a,b).



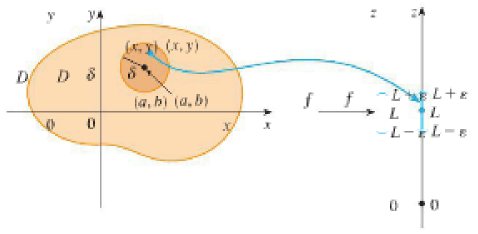


FIGURE 1

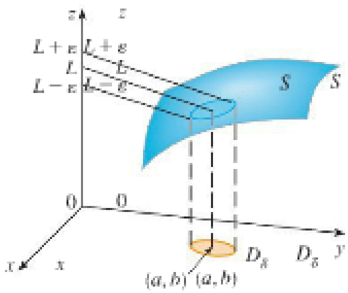


FIGURE 2

Better pic. than Mills's.

If $f(x, y) \to L_1$ as $(x, y) \to (a, b)$ along a path C_1 and $f(x, y) \to L_2$ as $(x, y) \to (a, b)$ along a path C_2, where $L_1 \neq L_2$, then $\lim_{(x,y)\to(a,b)} f(x, y)$ does not exist.

No matter how you approach (a,b), you need to get the same limit.

Standard "trick."

When the limit does NOT exist, sometimes the only practical way to show it is to be clever in how you make the approach to the limiting input value from a direction (or along a curve) where you get two different results, proving the limit doesn't exist.

EXAMPLE 1 Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.

$y = 0$: Approaching along x-axis

$$\frac{x^2}{x^2} = 1 \xrightarrow{(x,y) \to (0,0)} 1$$

$x = 0$ (y-axis approach)

$$\frac{-y^2}{y^2} = -1 \xrightarrow{(x,y) \to (0,0)} -1$$

(0,0)

$1 \neq -1$

Other approaches: $y = x$ OR $y = ax + b$ OR $x = ay + b$

a rotating line on
e 6 shows differ-
gin from different

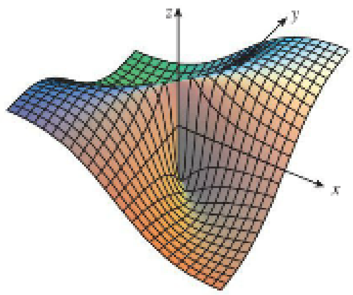


FIGURE 6
$f(x, y) = \dfrac{xy}{x^2 + y^2}$

Maybe y'all can help me grok the enrichment tool:

https://www.cengage.com/math/discipline_content/stewartcalc7/2008/14_cengage_tec/publish/deployments/transcendentals_7e/7e_m12_6a.html

will try to get this working.

EXAMPLE 4 Find $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$ if it exists. It DOES!!!

$y = x$ line : $\frac{3x^3}{2x^2} = \frac{3}{2}x \xrightarrow{(x,y)\to(0,0)} 0$

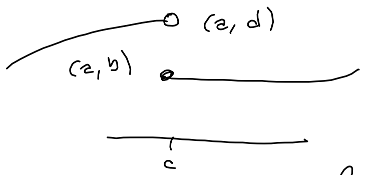
$y = 0$ line : $\frac{0}{x^2} = 0 \xrightarrow{(x,y)\to(0,0)} 0$

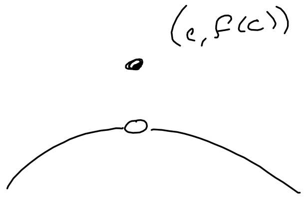
5–22 Find the limit, if it exists, or show that the limit does not exist.

5. $\lim_{(x,y)\to(1,2)} (5x^3 - x^2y^2)$ **6.** $\lim_{(x,y)\to(1,-1)} e^{-xy}\cos(x+y)$

f cnt^s $\Rightarrow \lim_{x\to c} f(x) = f(c)$

limit $\nexists$





$\lim_{x\to c^-} f(x) = d \neq c = \lim_{x\to c^+} f(x)$

$\lim_{x\to c} f(x)$ $\exists$, but $f(c) \neq \lim_{x\to c} f(x)$

10. $\lim_{(x,y)\to(0,0)} \dfrac{x^2 + \sin^2 y}{2x^2 + y^2}$

29–38 Determine the set of points at which the function is continuous.

29. $F(x, y) = \dfrac{\sin(xy)}{e^x - y^2}$ **30.** $F(x, y) = \dfrac{x - y}{1 + x^2 + y^2}$

31. $F(x, y) = \arctan(x + \sqrt{y})$ **32.** $F(x, y) = e^{x^2 y} + \sqrt{x + y^2}$

These are basically "Look for holes in the domain, and those will be where it's not continuous, generally speaking." 99.9% of the functions that have a formula you can write down are continuous on their entire domain.

39–41 Use polar coordinates to find the limit. [If (r, θ) are polar coordinates of the point (x, y) with $r \geq 0$, note that $r \to 0^+$ as $(x, y) \to (0, 0)$.]

39. $\lim_{(x,y)\to(0,0)} \dfrac{x^3 + y^3}{x^2 + y^2}$

40. $\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2)$

$(x,y) \longrightarrow (0,0)$

means $r \longrightarrow 0$

$x^3+y^3 = (x+y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$x^3 =$

$x^2+y^2 = r^2$

$r^2 - y^2 = x^2$

$x = \pm\sqrt{r^2 - y^2}$

$x^3 = \pm (r^2 - y^2)^{\frac{3}{2}}$