

~~14.1 #s 13, 15, 17-19, 23, 24, 36, 41, 43, 45, 53, 54, 57, 58~~ use assignment sheet.

Definition A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

Definition If f is a function of two variables with domain D , then the **graph** of f is the set of all points (x, y, z) in \mathbb{R}^3 such that $z = f(x, y)$ and (x, y) is in D .

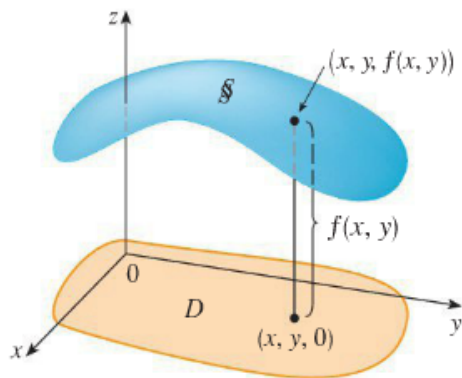
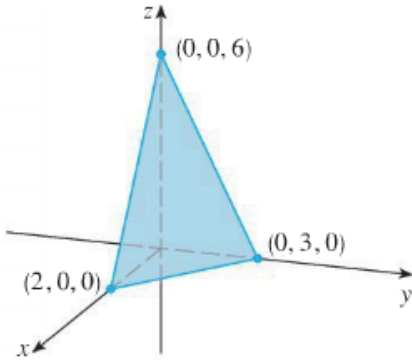


FIGURE 5

EXAMPLE 5 Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$.



$$z = 6 - 3x - 2y$$

$$3x + 2y + z = 6$$

$(0, 0, 6)$
 $(0, 3, 0)$
 $(2, 0, 0)$

FIGURE 6

EXAMPLE 6 Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

$x^2 + y^2 = 1$
 $y = \pm \sqrt{1 - x^2}$
 $y = \sqrt{1 - x^2}$ Top half
 $y = -\sqrt{1 - x^2}$ bottom half

$z = \sqrt{9 - x^2 - y^2} \rightarrow \text{So } z \geq 0$
 $z^2 = 9 - x^2 - y^2$
 $x^2 + y^2 + z^2 = 9$

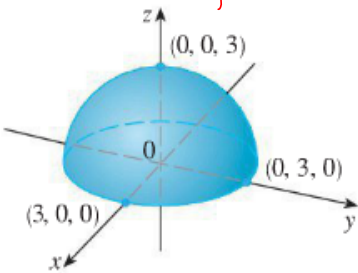
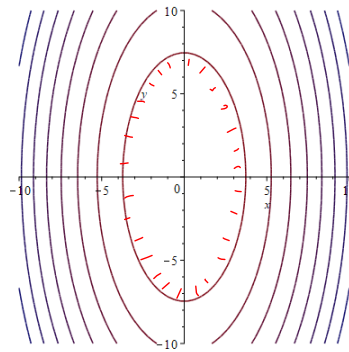
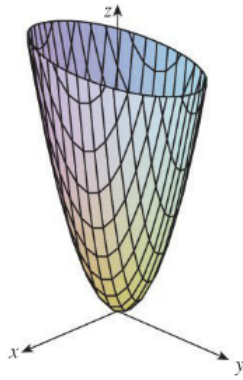


FIGURE 7

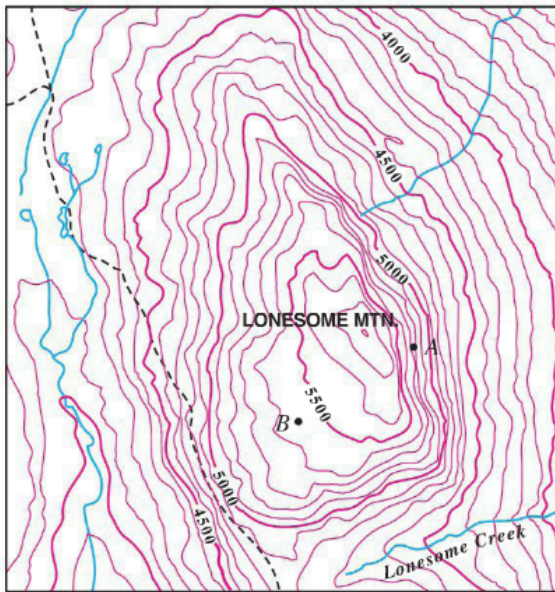
Graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$

EXAMPLE 8 Find the domain and range and sketch the graph of $h(x, y) = 4x^2 + y^2$.



$z = 4x^2 + y^2$

Definition The level curves of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f).



Topographic Maps!

FIGURE 12

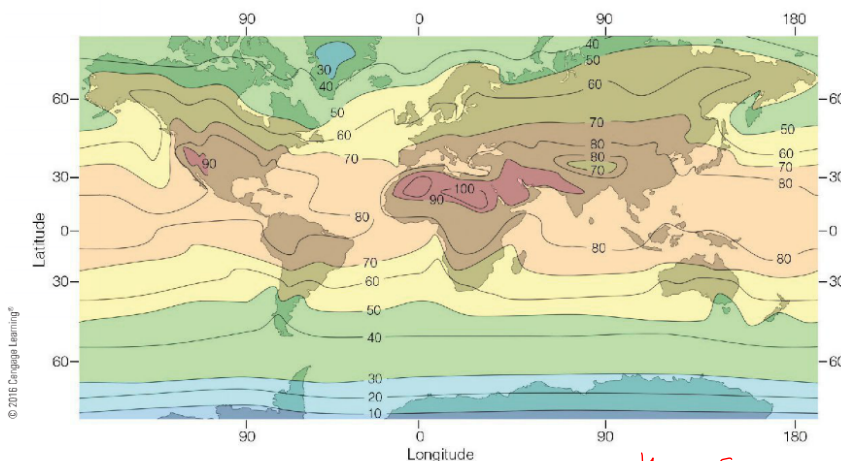
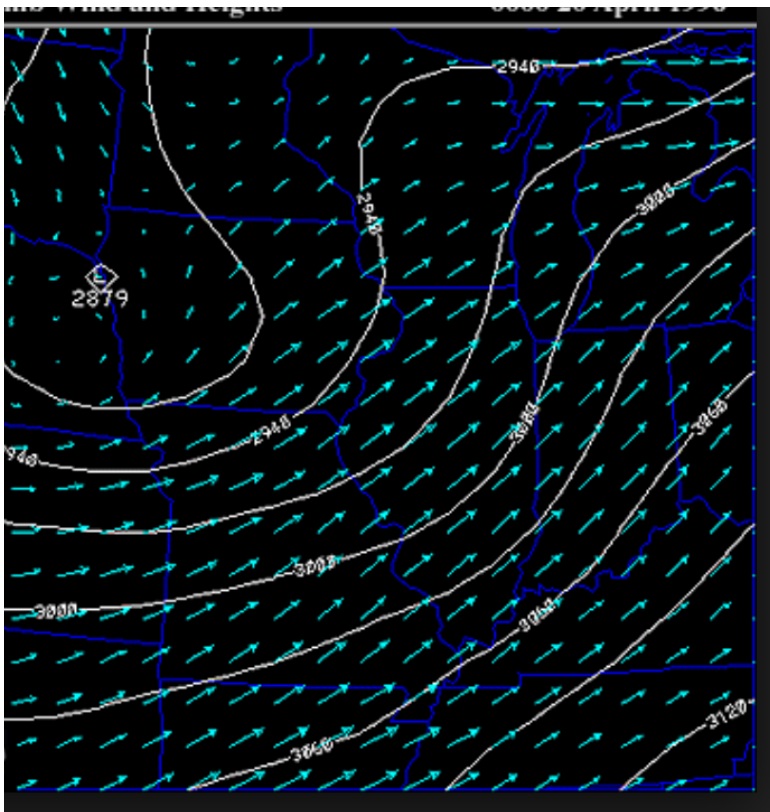


FIGURE 13 Average air temperature near sea level in July ($^{\circ}$ F)

isotherms



Δ Peak?
Your first Vector
Field!

EXAMPLE 12 Sketch some level curves of the function $h(x, y) = 4x^2 + y^2 + 1$

This is our friend, the elliptical paraboloid, opening up, with vertex at $(0, 0, 1)$.

Just like Example 8, shifted up 1 unit!

11-20 Find and sketch the domain of the function.

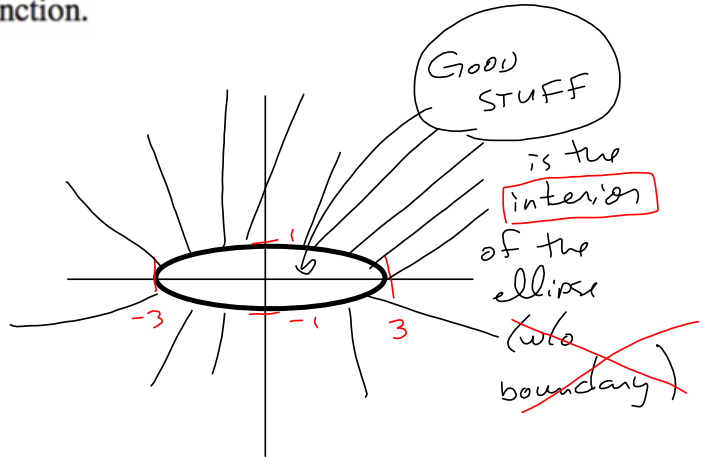
13. $f(x, y) = \ln(9 - x^2 - 9y^2)$

We need: $9 - x^2 - 9y^2 > 0$

$x^2 + 9y^2 < 9$

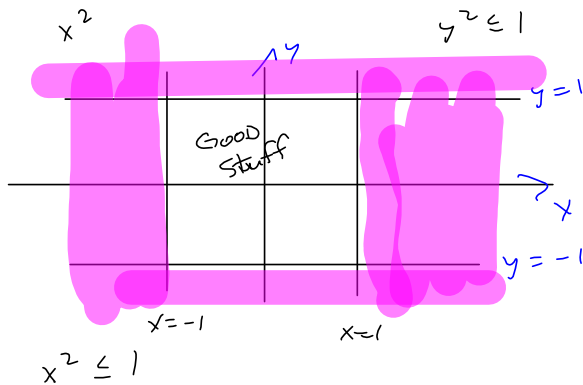
$\frac{x^2}{9} + y^2 < 1$

Stay inside the ellipse.

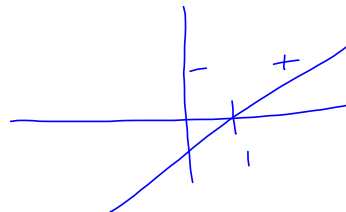
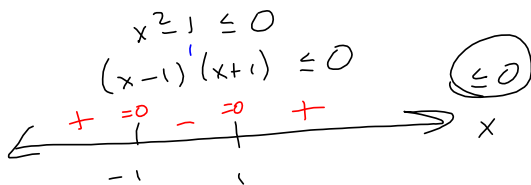


15. $f(x, y) = \sqrt{1 - x^2} - \sqrt{1 - y^2}$

Need $1 - x^2 \geq 0$ AND $1 - y^2 \geq 0$



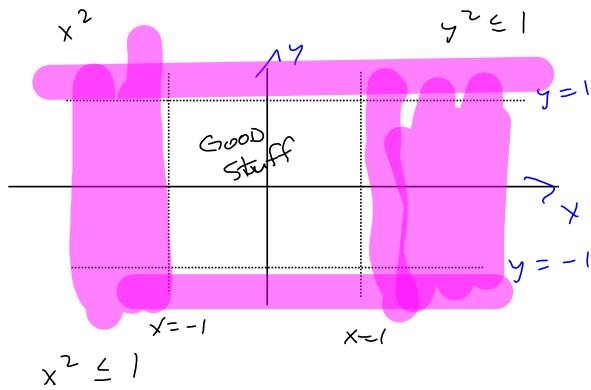
Scratch out the bad stuff!
The square with boundary.



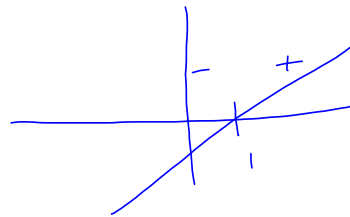
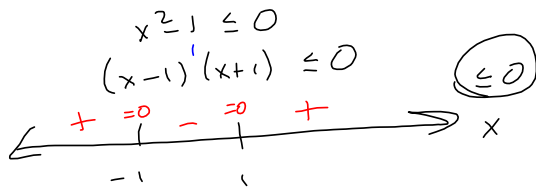
15. ~~$f(x,y) = \sqrt{1-x^2} - \sqrt{1-y^2}$~~

$f(x,y) = \log(1-x^2) + \log(1-y^2)$

Need $1-x^2 \geq 0$ AND $1-y^2 \geq 0$



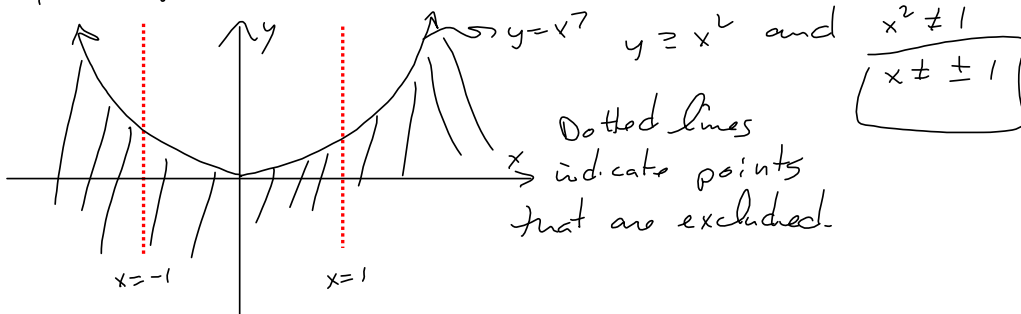
Scratch out
the bad stuff!
The square with **OUT**
boundary.



17. $f(x, y) = \frac{\sqrt{y-x^2}}{1-x^2}$

Scratch out the bad stuff. Keep the good stuff.

Need $y-x^2 \geq 0$ AND $1-x^2 \neq 0$



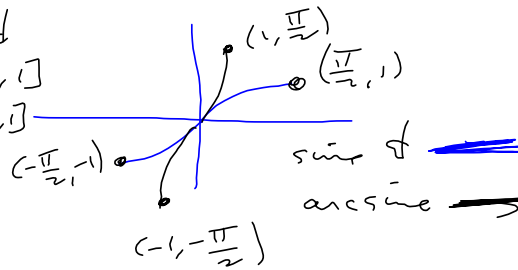
18. $f(x, y) = \arcsin(x^2 + y^2 - 2)$

Need $x \in [-1, 1]$

$x^2 + y^2 - 2 \in [-1, 1]$

$-1 \leq x^2 + y^2 - 2 \leq 1$

$1 \leq x^2 + y^2 \leq 3$
Annulus

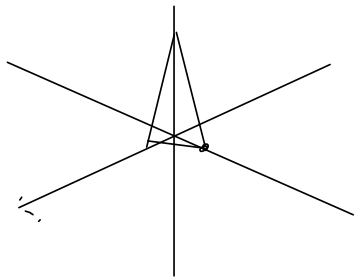


19. $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$

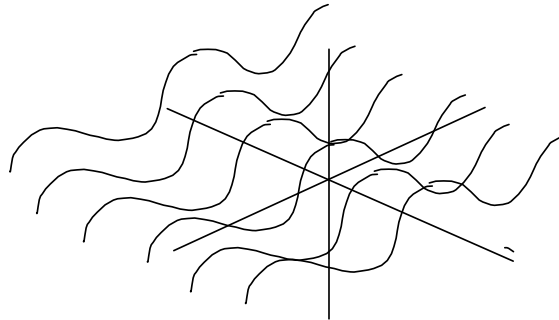
21-29 Sketch the graph of the function.

23. $f(x, y) = 10 - 4x - 5y$

$z = 10 - 4x - 5y$

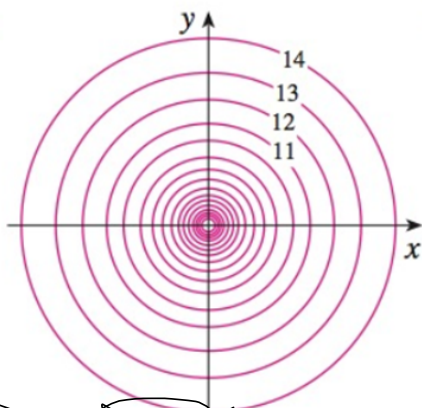


24. $f(x, y) = \cos x$ $z = \cos x$

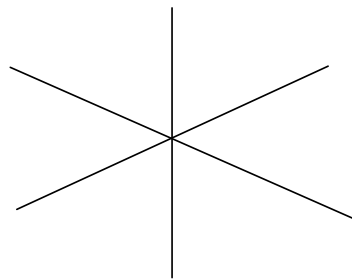
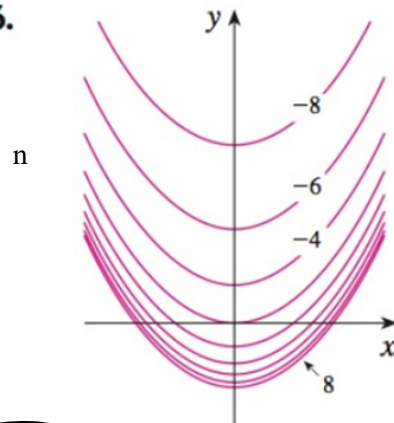


35-38 A contour map of a function is shown. Use it to make a rough sketch of the graph of f . #35 NA

35.



36.

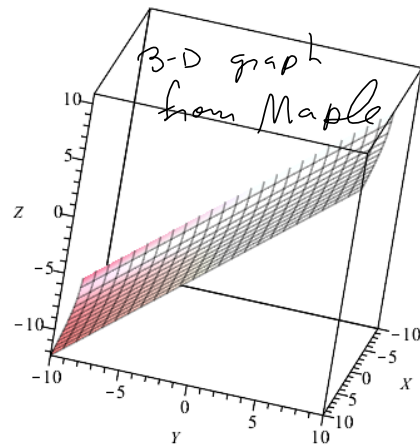
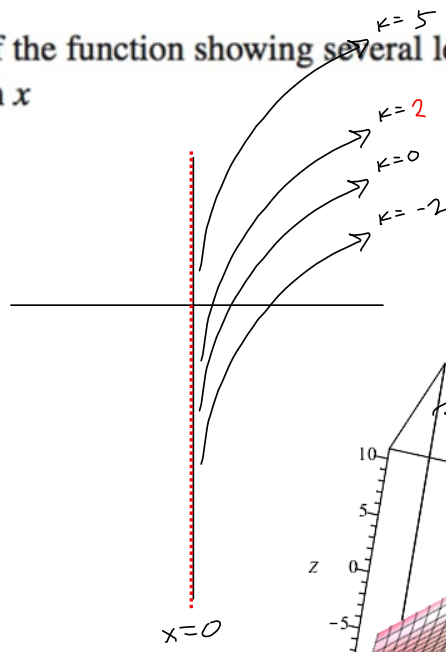


39-46 Draw a contour map of the function showing several level curves. **41.** $f(x, y) = y - \ln x$

$$y - \ln(x) = k$$

$$y = \ln(x) + k$$

$$y = \ln(x) + 2$$



43. $f(x, y) = ye^x$

51–54 Use a computer to graph the function using various domains and viewpoints. Get a printout of one that, in your opinion, gives a good view. If your software also produces level curves, then plot some contour lines of the same function and compare with the graph. #s 51-2 NA

51. $f(x, y) = e^{-x^2} + e^{-2y^2}$ **52.** $f(x, y) = (1 - 3x^2 + y^2)e^{1-x^2-y^2}$

