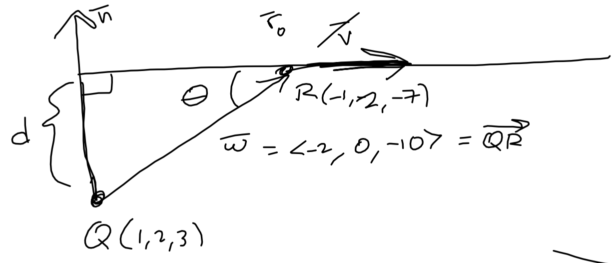


L:

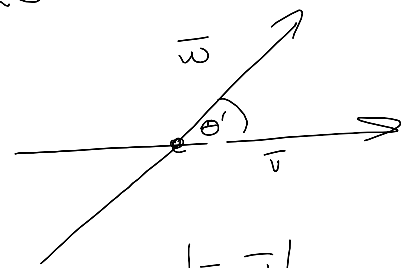
$\bar{r} = \langle -1, 2, -7\rangle + \langle 3, 2, 1\rangle t$

$x = 3t - 1,\ y = 2t + 2,\ z = t - 7$

$\theta = \theta'$

$\cancel{\theta = \pi - \theta'}$

$\frac{d}{\|\bar{w}\|} = \sin\theta$



$$\cos\theta' = \frac{\bar{w}\cdot\bar{v}}{\|\bar{w}\|\|\bar{v}\|} = \frac{\langle -2, 0, -10\rangle \cdot \langle 3, 2, 1\rangle}{\sqrt{104}\sqrt{14}}$$

$$\cos^{-1}\left(\frac{|-6-10|}{\sqrt{104}\sqrt{14}}\right) = \cos^{-1}\frac{16}{\sqrt{104\cdot 14}}$$

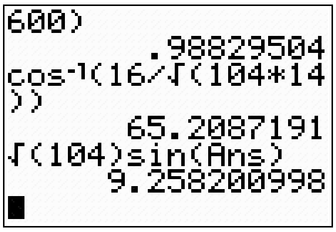
$\approx$

$$d = |\mathrm{comp}_{\bar{n}}\,\bar{w}| = \frac{|\bar{n}\cdot\bar{w}|}{\|\bar{n}\|}$$

$$\frac{d}{\|\bar{w}\|} = \sin\theta$$

$$d = \|\bar{w}\|\sin\theta = \sqrt{104}\,\sin(65.2087\ldots^\circ)$$

≈ 9.258200998



Point to a line formula.

That's slicker.

Eq'n of line segment from $P(1,2,1)$ to $Q(-3,5,7)$

$P(1,2,1)$ $\overrightarrow{PQ} = \langle -4, 3, 6\rangle$ is vector between them

$(1-t)\langle 1,2,1\rangle + t\langle -3,5,7\rangle$

Needn't simplify on test.

$$m \frac{d\bar{v}}{dt} = \frac{dm}{dt} \bar{v}_e$$

$$\frac{d\bar{v}}{dt} = \frac{\frac{dm}{dt}}{m} \bar{v}_e$$

$$\int \frac{d\bar{v}}{dt} dt = \int \frac{\frac{dm}{dt}}{m} \bar{v}_e dt$$

$$\int_0^t d\bar{v} = \bar{v}_e \int_0^t \frac{dm}{m}$$

m_0

$\bar{v}_0$

$$\Rightarrow \bar{v}(t) - \bar{v}(0) = \bar{v}_e \left(\ln(m(t)) - \ln(m(0)) \right)$$

$$\bar{v}(t) = \bar{v}(0) + \ln\left(\frac{m(t)}{m_0}\right) \bar{v}_e$$

$$= \bar{v}(0) - \ln\left(\frac{m_0}{m(t)}\right) \bar{v}_e$$

$$\ln\left(\frac{A}{B}\right) = \ln\left(\left(\frac{B}{A}\right)^{-1}\right) = -\ln\left(\frac{B}{A}\right)$$

$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$$

$$S = \int_a^b \sqrt{\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dx}\right)^2}\ dx$$

$$= \int_a^b \sqrt{f'(x)^2 + 1}\ dx$$

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arc length

Parametric version

$$\int_c^d \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\ dt$$

Yes, can (may) treat $\frac{dy}{dx}$ as a fraction for practical purposes.

$$\frac{dy}{dx} = x^2+2$$

$$dy = (x^2+2)dx$$

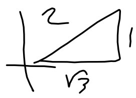
$$\int dy = \int (x^2+2)dx$$

$$y = \frac{x^3}{3} + 2x + C$$

$\left(\frac{5}{2}, \frac{3\sqrt{3}}{2}, \frac{4\sqrt{3}}{2}\right)$

$5\sin t = \frac{5}{2}$

$\sin t = \frac{1}{2}$, $t = \frac{\pi}{6}$



$\bar{r}(t) = \langle 5\sin(t), 3\cos(t), 4\cos(t)\rangle$

$\bar{r}' = \langle 5\cos(t), -3\sin(t), -4\sin(t)\rangle \Rightarrow \bar{r}'(\frac{\pi}{6}) = \langle \frac{5\sqrt{3}}{2}, -\frac{3}{2}, -2\rangle$

$\bar{r}'' = \langle -5\sin(t), -3\cos(t), -4\cos(t)\rangle \Rightarrow \bar{r}''(\frac{\pi}{6}) = \langle -\frac{5}{2}, -\frac{3\sqrt{3}}{2}, -\frac{4\sqrt{3}}{2}\rangle$

$\kappa = \frac{\|\bar{r}' \times \bar{r}''\|}{\|\bar{r}'\|^3}$

$\frac{1}{2}\langle 5\sqrt{3}, -3, -4\rangle$, $5\sqrt{3}, -3$

$\times \quad \frac{1}{2}\langle -5, -3\sqrt{3}, -4\sqrt{3}\rangle$, $-5, -3\sqrt{3}$

$\frac{1}{4}\langle 12\sqrt{3} - 12\sqrt{3}, 20 + 20\sqrt{3}, 45 - 15\rangle$

$= \frac{1}{4}\langle 0, 20\sqrt{3} + 20, 30\rangle$

$= \bar{r}' \times \bar{r}''$

$\|\bar{r}'\| = \frac{1}{2}\sqrt{75 + 9 + 16} = (\sqrt{100})\frac{1}{2} = \frac{10}{2} = 5 = \|\bar{r}'\|$

5 Norm in Maple

Norm(vbar, Euclidean)

with(Student[VectorCalculus]):

with(LinearAlgebra):

$(a\bar{v} \times b\bar{w})$

$= ab(\bar{v} \times \bar{w})$

So $\kappa_{t=\frac{\pi}{6}} = \frac{\|\frac{1}{4}\langle 0, 20\sqrt{3}, 30\rangle\|}{5^3}$

$= \frac{\frac{1}{4}(\sqrt{1200 + 900})}{125} = \frac{\frac{1}{4}(2100)^{\frac{1}{2}}}{(125)} = \frac{\frac{1}{4}\cdot 10\sqrt{21}}{125}$

$= \frac{\frac{5}{2}\sqrt{21}}{125} = \frac{\frac{1}{2}\sqrt{21}}{25}$

$= \frac{\sqrt{21}}{50} = \kappa$

$r = \frac{50}{\sqrt{21}} = \frac{50\sqrt{21}}{21}$

$$\bar{u} \times \bar{v} = \left\langle \tfrac{2}{3}, \tfrac{8}{9}, -\tfrac{1}{3} \right\rangle \times \left\langle \tfrac{1}{2}, 5, \tfrac{3}{4} \right\rangle$$

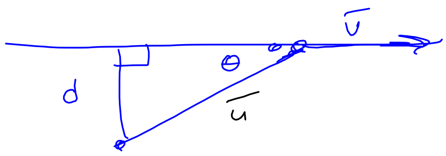
$$= \left(\tfrac{1}{9} \langle 6, 8, -3 \rangle\right) \times \left(\tfrac{1}{4} \langle 2, 20, 3 \rangle\right)$$

$$= \tfrac{1}{36} \left(\langle 6,8,-3 \rangle \times \langle 2,20,3 \rangle \right)$$

$$\begin{array}{r} \langle 6, 8, -3 \rangle, 6, 8 \\ \times \ \langle 2, 20, 3 \rangle, 2, 20 \\ \hline \langle 84, -24, 104 \rangle \end{array} \Rightarrow \bar{u} \times \bar{v} = \tfrac{1}{36} \langle 84, -24, 104 \rangle$$

1-page cheat sheet.

Might want to see the picture for one of those distance problems



$$\|\bar{u} \times \bar{v}\| = \|\bar{u}\| \|\bar{v}\| \sin\theta$$

$$\sin\theta = \frac{\|\bar{u} \times \bar{v}\|}{\|\bar{u}\|\|\bar{v}\|}$$

$$\frac{d}{\|\bar{u}\|} = \sin\theta \Rightarrow d = \|\bar{u}\| \sin\theta$$

$$d = \cancel{\|\bar{u}\|} \frac{\|\bar{u} \times \bar{v}\|}{\cancel{\|\bar{u}\|}\|\bar{v}\|} = \frac{\|\bar{u} \times \bar{v}\|}{\|\bar{v}\|}$$

Distance between pt & line

$\bar{u}$ = vector from point to any point on the line.

$\bar{v}$ = direction vector for the line

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