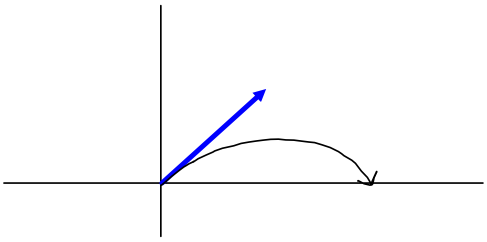
Section 13.4 Assignment

Projectile Motion

$x = (v_0 \cos \alpha)t \qquad y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$



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EDBH 133

Tangential and Normal Components of Acceleration

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|} = \frac{\mathbf{v}}{v}$$

$$\mathbf{v} = v\mathbf{T}$$

6 $$\kappa = \frac{|\mathbf{T}'|}{|\mathbf{r}'|} = \frac{|\mathbf{T}'|}{v} \qquad \text{so} \qquad |\mathbf{T}'| = \kappa v$$

$$\mathbf{T}' = |\mathbf{T}'|\mathbf{N} = \kappa v \mathbf{N}$$

$\frac{\overline{T}'}{\|\overline{T}'\|} = \overline{N}.$

7 $$\boxed{\mathbf{a} = v'\mathbf{T} + \kappa v^2 \mathbf{N}}$$

$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$

8 $$a_T = v' \qquad \text{and} \qquad a_N = \kappa v^2$$

9

$$\frac{dT}{ds} = \frac{\frac{dT}{dt}}{\frac{ds}{dt}} \qquad \frac{T'}{}$$

$$a_T = v' = \frac{\mathbf{v} \cdot \mathbf{a}}{v} = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}$$

10

$$a_N = \kappa v^2 = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} |\mathbf{r}'(t)|^2 = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

Kepler's Laws

1. A planet revolves around the sun in an elliptical orbit with the sun at one focus.
2. The line joining the sun to a planet sweeps out equal areas in equal times.
3. The square of the period of revolution of a planet is proportional to the cube of the length of the major axis of its orbit.

$P^2 = Ka^3$ for some konstant K

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $a > b$.

Second Law of Motion: $\mathbf{F} = m\mathbf{a}$

Law of Gravitation: $\mathbf{F} = -\frac{GMm}{r^3}\mathbf{r} = -\frac{GMm}{r^2}\mathbf{u}$

$F = \frac{Gm_1m_2}{r^2}$

CALC I:

S = position

$\frac{ds}{dt}$ = speed = v

$\frac{d^2s}{dt^2}$ = acceleration = a

Calc III

$\bar r(t)$ = position

$\bar r'(t)$ = velocity

$\bar r''(t)$ = acceleration

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(a) Find the position vector of a particle that has the given acceleration and the specified initial velocity and position.

(b) Use a computer to graph the path of the particle.

17. $\mathbf{a}(t) = 2t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos 2t\,\mathbf{k}, \quad \mathbf{v}(0) = \mathbf{i}, \quad \mathbf{r}(0) = \mathbf{j}$

18. $\mathbf{a}(t) = t\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}, \quad \mathbf{v}(0) = \mathbf{k}, \quad \mathbf{r}(0) = \mathbf{j} + \mathbf{k}$ $= \langle 0, 1, 1\rangle$

18 $\bar a(t) = \langle t, e^t, e^{-t}\rangle$

$\bar v(t) = \int \bar a(t)\,dt + \bar C$

$= \langle \frac{1}{2}t^2, e^t, -e^{-t}\rangle + \bar C$

Given $\bar v(0) = \langle 0, 1, -1\rangle + \bar C = \langle 0, 0, 1\rangle$

$\bar C = \langle 0, -1, 2\rangle$

$\bar v(t) = \langle \frac{1}{2}t^2, e^t - 1, -e^{-t} + 2\rangle$

$\bar r(t) = \int \bar v(t)\,dt + \bar D = \langle \frac{1}{6}t^3, e^t - t, e^{-t} - 2t\rangle + \bar D$

$\bar r(0) = \langle 0, 1, 1\rangle + \bar D = \langle 0, 1, 1\rangle$

$\bar r(t) = \langle \frac{1}{6}t^3, e^t - t, e^{-t} - 2t\rangle$

8. (5 pts) Simplify the derivative: : $\dfrac{d}{dx}\displaystyle\int_0^{6x^2+1} \frac{\sin^2(3\tau)+\tau^5}{\left(\sqrt{6\tau^3}+\cos^2(3\tau)\right)}d\tau$

$$= \frac{\sin^2(3(6x^2+1)) + (6x^2+1)^5}{\sqrt{6(6x^2+1)^3} + \cos^2(3(6x^2+1))} \cdot (12x)$$

FTC I: $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$ $\quad \dfrac{d}{dx}\, G(x)$

FTC I: $\dfrac{d}{dx}\displaystyle\int_a^{u(x)} f(t)\,dt = \dfrac{d}{dx}\left[G(u(x))\right] = \dfrac{dG}{du} \cdot \dfrac{du}{dx}$

$\dfrac{d}{dx}\left[(3x^2+\sin x)^5\right] = 5(3x^2+\sin x)^4(6x+\cos x)$

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$\bar r = \langle \cos t, \sin t, \ln(\cos t) \rangle$

$\bar T, \bar N, \bar B$ @ $\langle 1,0,0 \rangle \leftrightsquigarrow t=0$

$$\bar T = \frac{\bar r'}{\|\bar r'\|} = \frac{\langle -\sin t, \cos t, \frac{-\sin t}{\cos t} \rangle}{\sqrt{\sin^2 t + \cos^2 t + \tan^2 t}} = (\cos t)\langle -\sin t, \cos t, -\tan t \rangle$$

$$\sqrt{1+\tan^2 t} = \sqrt{\sec^2 t} = |\sec t| = \sec t \quad 0 \le t < \frac{\pi}{2}$$

$\frac{\pi}{2} < t < \frac{3\pi}{2}$: $|\sec t| = -\sec t$

$$\bar N = \frac{\bar T'}{\|\bar T'\|}. \quad \bar T' = (-\sin t)\langle -\sin t, \cos t, -\tan t \rangle + (\cos t)\langle -\cos t, -\sin t, -\sec^2 t \rangle$$

$$= \langle \sin^2 t, -\sin t \cos t, \sin t \tan t \rangle + \langle -\cos^2 t, -\sin t \cos t, -\cos t \sec^2 t \rangle$$

$$= \langle \sin^2 t - \cos^2 t, -2\sin t \cos t, \sin t \tan t - \sec t \rangle$$

$$\sin^2 t - (1-\sin^2 t) = 2\sin^2 t - 1 = \cos(2t)$$

$$-2\sin t \cos t = -\sin(2t)$$

$$\frac{\sin^2 t}{\cos t} - \frac{1}{\cos t} = \frac{-\cos^2 t}{\cos t} = -\cos t$$

$$\bar T' = \langle \cos(2t), -\sin 2t, -\cos t \rangle$$

$$\|\bar T'\| = \sqrt{\cos^2(2t) + \sin^2(2t) + \cos^2 t} = \sqrt{1+\cos^2 t}$$

$$\bar N = \frac{1}{\sqrt{1+\cos^2 t}} \langle \cos(2t), -\sin(2t), -\cos(t) \rangle$$

Now, @ $\langle 1,0,0 \rangle$, $t=0$

$(\cos t)\langle -\sin t, \cos t, -\tan t \rangle = \bar T$

$a\bar u \times b\bar v = ab(\bar u \times \bar v)$

$$\boxed{\bar T(0) = 1\langle 0,1,0 \rangle \qquad \bar N(0) = \frac{1}{\sqrt 2}\langle 1,0,-1 \rangle}$$

$\bar B(0) = \bar T \times \bar N$:

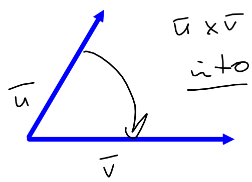
$\langle 0,1,0 \rangle, 0, 1$

$\times \; \langle 1,0,-1 \rangle, 1, 0$

$\langle -1, 0, -1 \rangle$

$$\boxed{\bar B(0) = \frac{1}{\sqrt 2}\langle -1,0,-1 \rangle}$$

"csgn" means what in Maple?



$\bar u \times \bar v$ is into the page!

15.