$z = x^2 - y^2$

$x^2 - y^2 = 1$ $y^2 = x^2 - 1$

$x^2 = y^2 + 1$ $y = \pm\sqrt{x^2 - 1}$

Maple Coming, Today!

§12.6 example $x^2 - y^2 = z$

Looking at level curves on, e.g. $z = 1$

$z = 1$: $x^2 - y^2 = 1$

Plot trick: solve for y:

$y^2 = x^2 - 1$

$y = \pm\sqrt{x^2 - 1}$

Plotted:

$\bar{r}_1(t) = \langle t, \sqrt{t^2 - 1}, 1 \rangle$

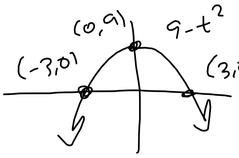
$\bar{r}_2(t) = \langle t, -\sqrt{t^2 - 1}, 1 \rangle$

Send matching questions for 13.1 #s 21-27

S 13.1 #1 $\langle \ln(t+1), \frac{1}{\sqrt{9-t^2}}, 2^t \rangle = \vec{r}(t)$

$\mathcal{D}$: Need $9-t^2 \geq 0$ & $9-t^2 \neq 0$ $\Big\}$ $9-t^2>0$ & $t+1>0$

$9-t^2>0$



(0,9) $9-t^2$

(-3,0) (3,0)

>0

$=0$ $=0$

$-$ $+$ $-$

-3 3

N N Y N N

$(-3,3)$ &

$t+1>0$

$t>-1$

$-$ $=0$ $+$

-1

N N Y

$(-1,\infty)$

-3 3

-1

AND

-3 -1 3

N N N N ~~N~~ N N

$\mathcal{D} = (-1,3)$

$(fg) = f'g + fg'$

Book does $gf' + fg'$ OR

$fg' + gf'$.

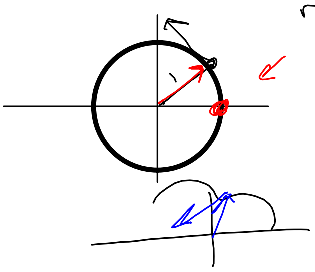
Example 4, §13.2 is big for §13.3

If $\|\bar r(t)\| = c =$ constant, then

$\bar r \perp \bar r'$

$\bar r(t) = \langle \cos(t), \sin(t) \rangle$ $\quad \|\bar r\| = 1 \ \forall t.$

$\bar r'(t) = \langle -\sin(t), \cos(t) \rangle$ $\quad \|\bar r'\| = 1 \ \forall t.$



Crucial for §13.3, b/c $\bar T$ = unit ~~tan~~ tangent. So $\|\bar T\| = 1 \ \forall t$

$\|\bar T(t)\| = 1 \ \forall t$

So, $\bar T \perp \bar T' \equiv \bar N$ → NOT Quite!

$\bar N = \frac{\bar T'}{\|\bar T'\|}$

= unit Normal to curve

$\bar B = \bar T \times \bar N$ = unit Binormal

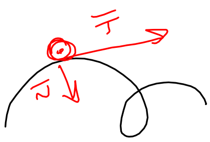
$\bar T$ points in direction of increasing t.

$\bar N$ " to inside of curve.

$\bar B$ is $\perp$ to both.

Unicycle in zero g on a rigid curved wire.

There's a weight LIGHTER than the cycle. It points in direction of $\bar N$.



Then $\bar B$ is always on your left

You're on top. $\bar N$ BOTTOM

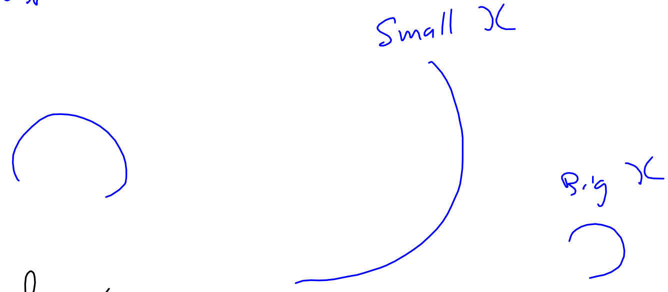
$\bar B$ to left.

$$\bar{T} = \frac{\bar{r}'}{\|\bar{r}'\|} = \frac{1}{\|\bar{r}'\|}\bar{r}' = fg$$

$$\bar{T}' = (fg)' = f'g + fg' = \left(\frac{1}{\|\bar{r}'\|}\right)'\bar{r}' + \frac{1}{\|\bar{r}'\|}\bar{r}''$$

$$\bar{N} = \frac{\bar{T}'}{\|\bar{T}'\|} = \text{Nasty!}$$

Curvature $\kappa = \left\|\frac{d\bar{T}}{ds}\right\|$ = rate of change of tangent with respect to arc length



Chain Rule helps us big-time. Assume $s = s(t)$ is a function of t. To the extent this is an invertible function, we can say $t = t(s)$ is a function of s

$$\frac{d\bar{T}}{dt} = \frac{d\bar{T}}{ds}\cdot\frac{ds}{dt} \Longrightarrow \frac{d\bar{T}}{ds} = \frac{\frac{d\bar{T}}{dt}}{\frac{ds}{dt}}$$

are from original problem

$$f = f(u(x))$$

$$\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$$

$$s(t) = \text{arc length} = \int_a^t \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,dt$$

$$= \int_a^t \|\bar{r}'(t)\|\,dt$$

$$\Longrightarrow \frac{ds}{dt} = \|\bar{r}'(t)\|$$ Cool!

$\langle e^t \sin t, e^t \cos t, \sqrt{2} e^t \rangle = \bar{r}(t)$

$P(0, 1, \sqrt{2}) \Rightarrow$

$t = 0$ is start point.

$s = S(t) = \int_0^t 2e^t dt = 2e^t - 2e^0$

$= 2e^t - 2 = s$

$\Rightarrow 2e^t = s+2 \Rightarrow e^t = \frac{s+2}{2} \Rightarrow t = \ln\left(\frac{s+2}{2}\right)$

$\bar{r}' = \langle e^t \sin t + e^t \cos t, e^t \cos t - e^t \sin t, \sqrt{2} e^t \rangle$

$\Rightarrow \|\bar{r}'\| = \sqrt{(e^t \sin t + e^t \cos t)^2 + (e^t \cos t - e^t \sin t)^2 + 2e^{2t}}$

$= (e^{2t} \sin^2 t + 2e^{2t} \sin t \cos t + e^{2t} \cos^2 t$

$+ e^{2t} \cos^2 t - 2e^{2t} \sin t \cos t + e^{2t} \sin^2 t$

$+ 2e^t)^{1/2}$

$= \sqrt{e^{2t} + e^{2t} + 2e^t} = \sqrt{4e^{2t}} = 2e^t = \|\bar{r}'(t)\|$

$e^{2t} \sin^2 t + e^{2t} \cos^2 t = e^{2t} (\sin^2 t + \cos^2 t)$

$\vec{r} = \langle \arctan(t), 2e^{2t}, 8te^{t} \rangle$ @ $t=0$ find $\vec{T}$.

$\frac{d}{dt}\left[\arctan(t)\right] = \frac{1}{t^2+1}$ $\qquad \vec{T} = \frac{\vec{r}\,'}{\|\vec{r}\,'\|}$, where

$\vec{r}\,' = \langle \frac{1}{t^2+1}, 4e^{2t}, \underbrace{8e^{t} + 8te^{t}} \rangle$

$\Rightarrow \|\vec{r}\,'\| = \left(\left(\frac{1}{t^2+1}\right)^2 + 16e^{4t} + 64e^{2t} + 2(8e^{t})(8te^{t}) + 64t^2e^{2t} \right)^{\frac{1}{2}}$

is a mess!

$\boxed{\vec{T} = \frac{\vec{r}\,'}{\|\vec{r}\,'\|}}$

$\vec{T}(0) = \frac{\langle 1, 4, 8 \rangle}{\sqrt{1+16+64}} = \frac{\langle 1,4,8 \rangle}{\sqrt{81}} \boxed{= \frac{1}{9}\langle 1,4,8 \rangle = \vec{T}(0)}$

Arc length increment can get very messy.

§13.3
#s 1, 4, 7, 8, 10, 13, 21, 26, 47, 48

https://harryzaims.com/203/203-fall-20/solutions/12-5-I-solns.pdf

My style is a fairly good indication of what I want from you, except it's a little bit cramped, due to being distributed to multiple persons (to save paper). Don't save paper on YOUR homework.