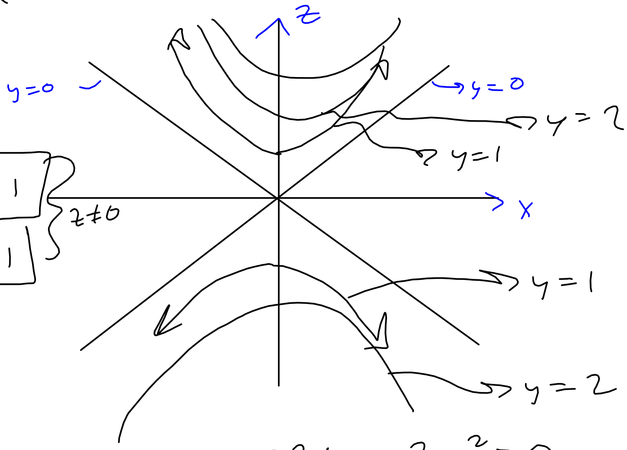
§12.6 # 19 $\quad y = z^2 - x^2$

$y = 0: \quad x^2 = z^2$

$x = \pm z$



$y = 1:$

$z^2 - x^2 = 1$

$\boxed{z^2 = x^2 + 1}$

$\boxed{x^2 = z^2 - 1}$

$z \neq 0$

Hyperbola $\quad \dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

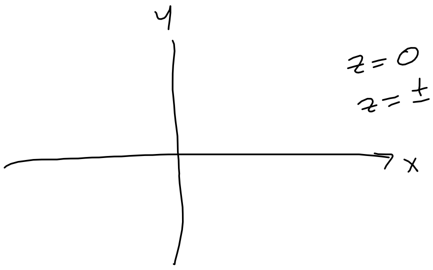
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} = 1 + \frac{y^2}{b^2}$$

$y = 2: \quad z^2 - x^2 = 2$

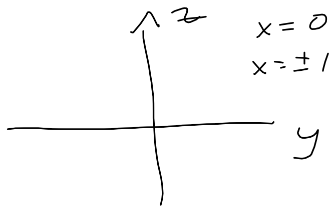
$y = 3 \quad z^2 - x^2 = 3$

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$



$z = 0$

$z = \pm 1$, etc.



$x = 0$

$x = \pm 1$, etc.

Section 12.5 II was brutal. Some selections...

57–58 (a) Find parametric equations for the line of intersection of the planes and (b) find the angle between the planes.

58. $3x - 2y + z = 1, \quad 2x + y - 3z = 3$

Gauss $\left[\begin{array}{ccc|c} a & * & * & a \\ 0 & b & * & * \\ 0 & 0 & c & * \end{array}\right]$ Gauss-Jordan $\left[\begin{array}{ccc|c} 1 & 0 & 0 & f \\ 0 & 1 & 0 & g \\ 0 & 0 & 1 & h \end{array}\right]$

$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 1 \\ 2 & 1 & -3 & 3 \end{array}\right] \begin{array}{c} -2E1 \\ 3E2 \end{array} \left[\begin{array}{ccc|c} -6 & 4 & -2 & -2 \\ 6 & 3 & -9 & 9 \end{array}\right]$

$\begin{array}{c} 0E1 \\ E1+E2 \end{array} \left[\begin{array}{ccc|c} 3 & -2 & 1 & 1 \\ 0 & 7 & -11 & 7 \end{array}\right]$

$7y - 11z = 7$

$7y = 11z + 7$

$y = \frac{11z + 7}{7}$

$3x - 2y + z = 1$

$3x - 2\left(\frac{11z+7}{7}\right) + z = 1$

$21x - 2(11z + 7) + 7z = 7$

$21x - 22z - 14 + 7z = 7$

$21x - 15z = 21$

$x = \frac{15z + 21}{21} = \frac{5}{7}z + 1$

Let $z = t$

$x = \frac{5}{7}t + 1, \; y = \frac{11}{7}t + 1, \; z = t$

59–60 Find symmetric equations for the line of intersection of the planes.

59. $5x - 2y - 2z = 1, \quad 4x + y + z = 6$

$$\left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 4 & 1 & 1 & 6 \end{array}\right] \begin{array}{c} -4R1 \\ 5R2 \end{array} \left[\begin{array}{ccc|c} -20 & 8 & 8 & -4 \\ 20 & 5 & 5 & 30 \end{array}\right]$$

OE1 $\left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 0 & 13 & 13 & 26 \end{array}\right] \sim \left[\begin{array}{ccc|c} 5 & -2 & -2 & 1 \\ 0 & 1 & 1 & 2 \end{array}\right]$

$y + z = 2$, etc.

64. (a) Find the point at which the given lines intersect:

$$\mathbf{r}_1 = \langle 1, 1, 0 \rangle + t\langle 1, -1, 2 \rangle$$

$$\mathbf{r}_2 = \langle 2, 0, 2 \rangle + s\langle -1, 1, 0 \rangle$$

(b) Find an equation of the plane that contains these lines.

(a) $\bar{r}_1 = \langle t+1, -t+1, 2t \rangle$

$\bar{r}_2 = \langle -s+2, s, 2 \rangle$

$t+1 = -s+2$

$-t+1 = s$

$2t = 2 \rightarrow t = 1$ wonderful

$\Rightarrow 1+1 = -s+2$

$2 = -s+2$

$\boxed{s=0}$

$\bar{r}_1(1) = \langle 2, 0, 2 \rangle$ ✓

$\bar{r}_2(0) = \langle 2, 0, 2 \rangle$

Cool.

(b) $\bar{u} = \langle 1, -1, 2 \rangle$, $\bar{v} = \langle -1, 1, 0 \rangle$

$\bar{n} = \bar{u} \times \bar{v}$ & use either one of the 2 pts

$(1,1,0)$ or $(2,0,2)$

65. Find parametric equations for the line through the point (0, 1, 2) that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$.

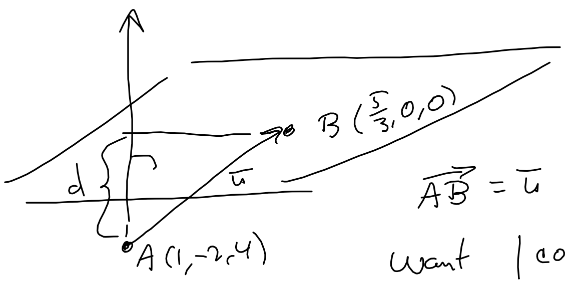
$\bar{u} = \bar{n} \times \bar{v}$

$\bar{n} = \langle 1, 1, 1 \rangle$, $\bar{v} = \langle 1, -1, 2 \rangle$, etc., idiot!

See §12.5 II Sol'ns for 1st version I did in class, before the crash.

71–72 Find the distance from the point to the given plane.

71. $(1, -2, 4)$, $3x + 2y + 6z = 5$



$\vec{AB} = \bar{u} = \langle \frac{2}{3}, 2, -4 \rangle$

want $|\text{comp}_{\bar{n}} \bar{u}| = \dfrac{|\bar{u} \cdot \bar{n}|}{\|\bar{n}\|}$

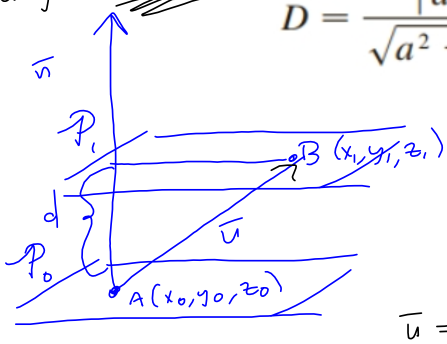
$$\frac{\langle 1, -2, 4\rangle \cdot \langle 3, 2, 6\rangle}{\sqrt{9+4+36}} = \frac{23}{\sqrt{49}} = \frac{23}{7}$$

75. Show that the distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_0 = 0$ is

naught, nought

$$D = \frac{|d_1 - d_0|}{\sqrt{a^2 + b^2 + c^2}}$$

$\bar{n} = \langle a, b, c \rangle$



$$d = \left| \text{comp}_{\bar{n}} \bar{u} \right| = \frac{|\bar{n} \cdot \bar{u}|}{\|\bar{n}\|}$$

$$\bar{u} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

$$\Rightarrow \frac{|\bar{n} \cdot \bar{u}|}{\|\bar{n}\|} = \frac{|a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)|}{\|\bar{n}\|}$$

$$= \frac{|ax_1 - ax_0 + by_1 - by_0 + cz_1 - cz_0|}{\|\bar{n}\|}$$

$$= \frac{|ax_1 + by_1 + cz_1 - (ax_0 + by_0 + cz_0)|}{\|\bar{n}\|}$$

$$= \frac{|-d_1 - (-d_0)|}{\|\bar{n}\|} = \frac{|d_1 - d_0|}{\sqrt{a^2 + b^2 + c^2}}$$

Savor §12.6

Do at least one of the graphs in detail

MAPLE $36.00 4-mo. license

$55.00 12-mo

$75.00 Perpetual.

There's a 15-day free trial on Student Maple.

Play with plots, help, and in the help, there are plotoptions, plot3doptions

Ch 13: Get exposed to it.

$\boxed{13.1 - 13.2}$

§13.3 starts getting hard-core & high detail

TNB-Frames

Arc Length as a parameter!

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$$L = \int_a^b \sqrt{x'^2 + y'^2 + z'^2}\, dt$$

$$= \int_a^b \|\vec{r}\,'(t)\|\, dt$$