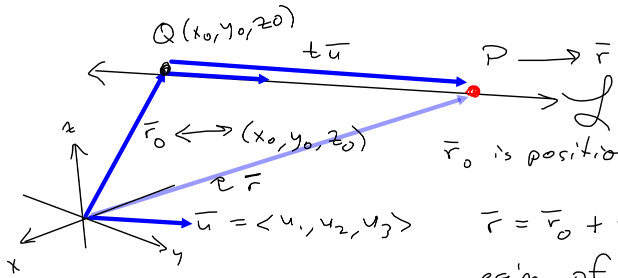
Questions on 12.1 - 12.4?

Today: 12.5 I (and maybe a bit of 12.5 II, depending on questions.)

§12.5 Lines & planes

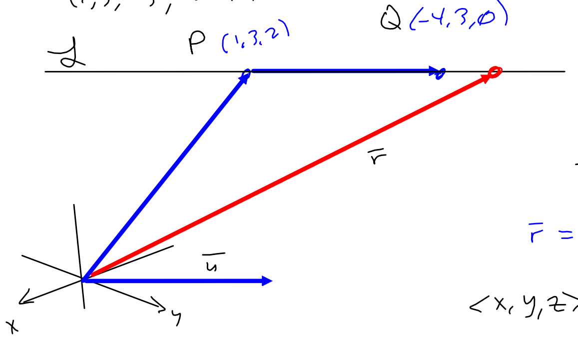
Position vector



$\bar{r}_0$ is position vector for Q

$\bar{r} = \bar{r}_0 + t\bar{u}$ is vector eq'n of line thru $Q(x_0, y_0, z_0)$ and parallel to $\bar{u} = \langle u_1, u_2, u_3 \rangle$

$(1,3,2)$, $(-4,3,0)$



$\bar{u} = \overrightarrow{PQ} = \langle -5, 0, -2 \rangle$ is vector $\parallel$ to $\mathcal{L}$

$\bar{r}_0 = \langle 1, 3, 2 \rangle$

$\bar{r} = \bar{r}_0 + t\bar{u}$ for some t.

$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle u_1, u_2, u_3 \rangle$

$\bar{r} = \langle x, y, z \rangle = \langle 1, 3, 2 \rangle + t \langle -5, 0, -2 \rangle$

Parametric Eq'ns of $\mathcal{L}$. $\Rightarrow$ $\boxed{x = 1 - 5t,\ y = 3,\ z = 2 - 2t}$

Symmetric Equations: Solve each of the above for t: $t = \boxed{\dfrac{x-1}{-5} = \dfrac{z-2}{-2}\ ;\ y = 3}$

https://harryzaims.com/203/203-spring-19/assignments/chapter-12/12-5-part-1-exercises-handout.pdf

10. The line through (2, 1, 0) and perpendicular to both **i** + **j** and **j** + **k**

$\bar{r}_0 = \langle 2, 1, 0 \rangle$ Need $\bar{u}$ in direction of the line.

$\bar{i} + \bar{j} = \langle 1, 1, 0 \rangle, 1, 1$

$\times\ \bar{j} + \bar{k} = \langle 0, 1, 1 \rangle, 0, 1$

$\langle 1, -1, 1 \rangle = \bar{u}$

$\bar{r} = \langle 2, 1, 0 \rangle + t \langle 1, -1, 1 \rangle = \bar{r}_0 + t\bar{u}$

$$\boxed{\begin{array}{l} \text{par}:\ x = 2 + t,\ y = 1 - t,\ z = t \\ \text{sym}:\ t = x - 2 = \dfrac{y-1}{-1} = z \end{array}}$$

12. The line of intersection of the planes $x + y + z = 1$ and $x + z = 0$

Find intersection:

$$\begin{bmatrix} 1 & 2 & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow[-1R1+R2]{R1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

$x + z = 0$

$$\boxed{\begin{aligned} x &= -z \\ y &= 1 \end{aligned}}$$

we got symmetric equations for free!

$(x,y,z) = (-z, 1, z)$

z is the free variable. Replace w/ t.

$(x,y,z) = (-t, 1, t)$

$x = -t,\ y = 1,\ z = t$

$\cancel{t} = \boxed{-x = z,\ y = 1}$

Substitution

$x + y + z = 1$

$x + z = 0$

$x = -z$

$-z + y + z = 1$

$\boxed{y = 1}$

$x + y + z = 1$

$x + 1 + z = 1$

$x + z = 0$

$\boxed{x = -z}$

$(x,y,z) = (-z, 1, z)$

$= (-t, 1, t)$

$\boxed{x = -t,\ y = 1,\ z = t}$

$t = \boxed{-x = z,\ y = 1}$

Is the line through $\overline{u}$ (−4, −6, 1) and (−2, 0, −3) parallel to the line through (10, 18, 4) and (5, 3, 14)? $\overline{v}$

$\cos\theta = -1$

$\cos\theta = +1$

$$\overline{u} = \langle 2, 6, -4\rangle = 2\langle 1, 3, -2\rangle$$
$$\overline{v} = \langle -5, -15, 10\rangle = -5\langle 1, 3, -2\rangle$$

$\parallel$!

$$\frac{\overline{u}\cdot\overline{v}}{\|\overline{u}\|\|\overline{v}\|}$$

$$\cos\theta = \frac{-10 + (-90) - 40}{\sqrt{2^2+6^2+4^2}\sqrt{5^2+15^2+10^2}}$$

$$= \frac{-140}{\sqrt{56}\sqrt{350}}$$

$$= \frac{-140}{\sqrt{56\cdot 350}} = \frac{-140}{\sqrt{2\cdot 2\cdot 2\cdot 7\cdot 2\cdot 5\cdot 5\cdot 7}}$$

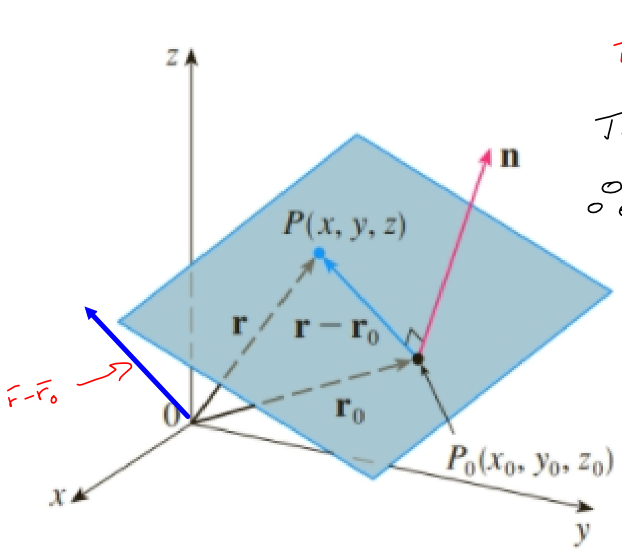
$$= \frac{-140}{2\cdot 2\cdot 7\cdot 5} = \frac{-140}{140} = -1$$

$2\,|\,56$, $2\,|\,28$, $2\,|\,14$, 7

$2\,|\,350$, $5\,|\,175$, $5\,|\,35$, 7

$$\|5\langle 1,3,-2\rangle\| = \sqrt{5^2+15^2+10^2}$$
$$= 5\,\|\langle 1,3,-2\rangle\|$$
$$= 5\sqrt{1^2+3^2+2^2} = 5\sqrt{14}$$
$$\|2\langle 1,3,2\rangle\| = 2\sqrt{14}$$

$$(5\sqrt{14})(2\sqrt{14}) = 10\cdot 14 = 140$$



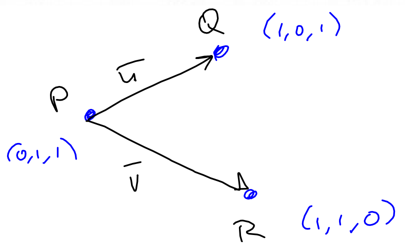
$\bar{n} \perp$ to plane $\mathcal{P}$

Then $\bar{n} \perp (\bar{r} - \bar{r}_0)$

$\therefore \quad \bar{n} \cdot (\bar{r} - \bar{r}_0) = 0$

Basic Vector equation of a plane.

31. The plane through the points (0, 1, 1), (1, 0, 1), and (1, 1, 0)



$$\bar{u} = \langle 1,-1,0\rangle,\ 1,-1$$
$$\times\ \bar{v} = \langle 1,0,-1\rangle,\ 1,0$$
$$\bar{n} = \langle 1,1,1\rangle$$

Let $(x,y,z) \in \mathcal{P}$

$\bar{r} = \langle x,y,z\rangle$ is its position vector.

and $\bar{r}-\bar{r}_0$ is "in the plane (i.e., parallel)

where $\bar{r}_0 = \langle 0,1,1\rangle$ or $\langle 1,0,1\rangle$ or $\langle 1,1,0\rangle$

Then $\bar{n}\cdot(\bar{r}-\bar{r}_0) = 0$

$$\langle 1,1,1\rangle \cdot \langle x-0,\ y-1,\ z-1\rangle$$
$$= x + 1(y-1) + 1(z-1)$$
$$= x+y-1+z-1 = 0$$
$$\Rightarrow \boxed{x+y+z=2}$$

$$ax+by+cz=d$$
$$\bar{n} = \langle a,b,c\rangle$$

TNB frame for space curves!

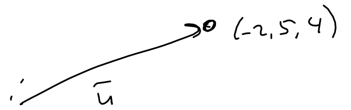
$\bar{r}_0 = \langle 6, 0, -2 \rangle$

35. The plane that passes through the point (6, 0, −2) and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$

Another point in the plane is $t = 0 \Rightarrow (4, 3, 7)$

$\bar{u} = \langle -2, 5, 4 \rangle$ is $\parallel$ to plane.

Want another vector $\parallel$ to the plane



(6,0,-2) & (4,3,7) are points in the plane, so

$\bar{v} = \langle -2, 3, 9 \rangle$ is $\parallel$ to the plane

$\times\ \bar{u} = \langle -2, 5, 4 \rangle$

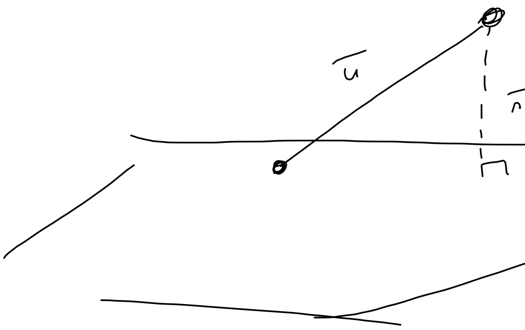
$$\begin{array}{l} \bar{v} = \langle -2, 3, 9 \rangle, -2, 3 \\ \times\ \bar{u} = \langle -2, 5, 4 \rangle, -2, 5 \\ \hline \bar{n} = \langle -33, -10, -4 \rangle \end{array}$$

$\bar{n} \cdot (\bar{r} - \bar{r}_0)$, using $\bar{r}_0 = \langle 6, 0, -2 \rangle$

$$33(x-6) - 10(y-0) - 4(z-2) = 0$$

$$33x - 198 - 10y - 4z + 8 = 0$$

$$\underline{33x - 10y - 4z = 190}$$



$\bar{n}$, $D = |\text{comp}_{\bar{n}}\,\bar{u}| =$

$$\frac{|\bar{u}\cdot\bar{n}|}{\|\bar{n}\|} = D$$

Think with pics more than formulas.