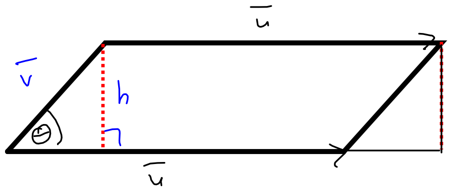
§12.4 exercise #46

Distance from point to plane is

$$\frac{|\bar{a}\cdot(\bar{b}\times\bar{c})|}{\|\bar{b}\times\bar{c}\|} = \frac{|\bar{c}\cdot(\bar{a}\times\bar{b})|}{\|\bar{b}\times\bar{c}\|}$$

Recall $\|\bar{u}\times\bar{v}\|$ = area of parallelogram defined by $\bar{u}$ & $\bar{v}$



Area $= h\,\|\bar{u}\|$

$= (\|\bar{v}\|\sin\theta)\,\|\bar{u}\|$

$= \|\bar{u}\|\,\|\bar{v}\|\sin\theta$

$\|\bar{u}\times\bar{v}\| = \|\bar{u}\|\,\|\bar{v}\|\sin\theta$ if θ is acute $(0 \le \sin\theta \le 1)$

$\|\bar{u}\|\,\|\bar{v}\|\,|\sin\theta|$ $\forall\,\theta$

$\bar{u}\times\bar{v}$ is kind of a sine thing.

§12.3 #45

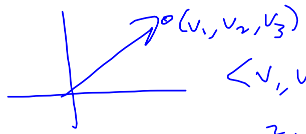
Show that $\mathrm{orth}_{\bar v}\bar u = \bar u - \mathrm{proj}_{\bar v}\bar u$ is $\perp$ to $\bar v$.

$$\left(\bar u - \mathrm{proj}_{\bar v}\bar u\right)\cdot \bar v$$
$$= \left(\bar u - \frac{\bar u\cdot\bar v}{\|\bar v\|^2}\bar v\right)\cdot\bar v$$
$$= \bar u\cdot\bar v - \frac{\bar u\cdot\bar v}{\|\bar v\|^2}\bar v\cdot\bar v$$
$$= \bar u\cdot\bar v - \frac{\bar u\cdot\bar v}{\cancel{\|\bar v\|^2}}\cancel{\|\bar v\|^2} = 0$$

$$= u_1v_1 + u_2v_2 + u_3v_3 \text{, etc.}$$

owie!

$$\|\bar v\| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{\bar v\cdot\bar v}$$



$$\langle v_1, v_2, v_3\rangle \cdot \langle v_1, v_2, v_3\rangle$$
$$= v_1^2 + v_2^2 + v_3^2 = \|\bar v\|^2$$

Low-level (full detail).

Keep high-level & use previous results as much as possible.

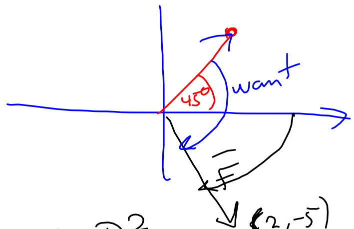
canonical basis $\bar{i}, \bar{j}, \bar{k}$

Other questions 12.1–12.3

12.2 #51

Wrench .5 m long, Force $\bar{F}$ in the direction of $\langle 2,-5 \rangle$

of



want to do cross product (need) for torque, which is 3-D, but this is a 2-D situation. How do we get around this?

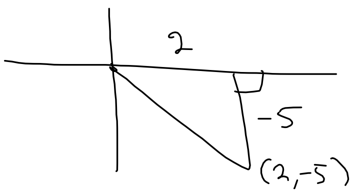
$\langle 2,-5 \rangle$ in $\mathbb{R}^2$ is $\langle 2,-5,0 \rangle$ in $\mathbb{R}^3$

What $\|\bar{F}\|$ is necessary for 100 N-m of torque?

$$|\text{Torque}| = \|\bar{\tau}\|$$

$$= \|\bar{r} \times \bar{F}\| = \|\bar{r}\| \|\bar{F}\| \sin\Theta$$

Direction angle for $\bar{F}$



$-\arctan\left(\frac{-5}{2}\right) \approx 68.19859051^\circ$

So angle between them is $45 + 68....^\circ$

$113....^\circ$

$$\|\bar{r}\| \|\bar{F}\| \sin(113....^\circ)$$

$$= .5 \|\bar{F}\| \sin(113....^\circ) = 100 \text{ N-m}$$

$$\|\bar{F}\| = \frac{100}{.5 \sin(113....^\circ)}$$

$$\approx 217.5935173 \text{ N}$$

$$\boxed{\approx 217.6 \text{ N}}$$

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Ans+45
           113.1985905
100/.5/sin(Ans)
           217.5935173
100/(.5*sin(113.
19859051))
           217.5935173
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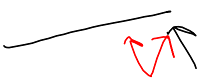

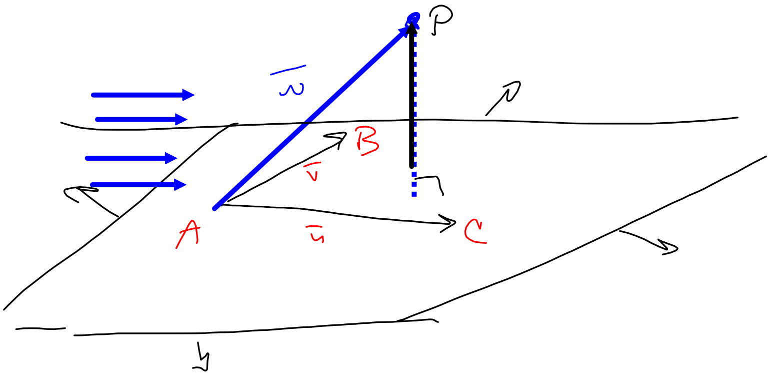

$\bar{n} = \bar{u} \times \bar{v}$ is $\perp$ to the plane in which $\bar{u}$ & $\bar{v}$ reside.

Let $\bar{u} = \overrightarrow{AC}$, $\bar{v} = \overrightarrow{AB}$

$\bar{w} = \overrightarrow{AP}$

$$\text{distance} = |\text{comp}_{\bar{n}}\, \bar{w}| = \frac{|\bar{w} \cdot \bar{n}|}{\|\bar{n}\|} \qquad \frac{|\bar{w} \cdot (\bar{u} \times \bar{v})|}{\|\bar{u} \times \bar{v}\|} \qquad \boxed{\text{T11 (5)}}$$

$$|\text{comp}_{\bar{v}}\, \bar{u}| = \left|\left(\frac{\bar{u} \cdot \bar{v}}{\|\bar{v}\|}\right)\right| = $$

$$= \frac{|\bar{u} \cdot (\bar{w} \times \bar{v})|}{\|\bar{u} \times \bar{v}\|}$$

$$= \frac{|\bar{v} \cdot (\bar{w} \times \bar{v})|}{\|\bar{u} \times \bar{v}\|}$$

§12.5 I #s 2, 6, 7, 10–14, 16, 17, 19, 21, 23, 24, 26, 27, 30, 31, 35, 36

§12.5 II #s 41, 44, 48–51, 53, 58, 59, 61, 63–65, 67–69, 71, 73, 75, 76